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CONTENTS

NUMBER I

NUMBER II

NUMBER III

NUMBER IV

NUMBER V

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ASTROPHYSICAL JOURNAL
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VOLUME XXXI JANUARY 1910 NUMBER 1

EUGENE v. GOTHARD

By BARON BÉLA HARKÁNYI

Eugene v. Gothard was born in Herény (Hungary) as the eldest son of the landed proprietor Stephen v. Gothard on May 31, 1857. He inherited his love for natural sciences, and especially for physics, from his grandfather, Francis v. Gothard, who occupied himself a great deal from 1780 to 1830 with botany and physical experiments. The nice old electric machine still to be found in the Herény observatory was often used by him in those days. The father of Eugene spoke very often to his eldest son about these old experiments and encouraged the boy in his predilection for physics, when he studied at the gymnasium at Szombathely, and gave him opportunity to make experiments with steam engines and electric motors in a small laboratory for his own use. After leaving the gymnasium, in 1875, Eugene went to Vienna and for four years pursued his studies at the Polytechnic in the department of mechanical engineering. He attended the lectures of Professor Herr on geodesy and of Professor Tinter on astronomy with great assiduity, and worked a great deal in the course in practical mechanics. After completing his course in Vienna he traveled in foreign countries. Upon returning home, he intended to build a physical laboratory on his own estate in Herény, but later on—partly under the influence of his friend Nicolaus v. Konkoly—he changed his plan and founded the Astrophysical Observatory in Herény.

The buildings of the observatory were finished in 1881, the equip-

ment completed in 1882; observations had already commenced in the autumn of 1881, in the dome then ready for use. This contained the principal instrument of the observatory, a $10\frac{1}{4}$-inch (260 mm) Newtonian reflector of 77 inches (1967 mm) focal length, with a silver-on-glass mirror. This instrument was originally constructed by Browning for N. v. Konkoly of Ó-Gyalla, and had proved excellent during seven years of spectroscopic and micrometric work there. Von Gothard made several important changes in it, and attached to it a new $4\frac{1}{2}$-inch guiding telescope, thus transforming it into an instrument well adapted for astrophotographic use. The smaller instruments were a portable transit instrument, two clocks, and several spectroscopes, with all the auxiliary apparatus needed, and a set of meteorological instruments. Besides these, we must mention a rich collection of more than 200 physical instruments for different uses, built mostly in the workshops of the observatory in the early stage of its development; most of them served only for learning the methods of making and using them and for training in physical laboratory work. A few of them were often used in later researches. The whole outfit was completed by an electric-light plant, a gas apparatus for laboratory use, and a complete workshop. Besides E. v. Gothard, the staff of the observatory consisted until 1883 of his brother Alexander, who was also owner of the establishment, and of their youngest brother Stephen who, while a student at the gymnasium, assisted occasionally in the observations, and of J. Molnár as mechanician.

The first larger publication of the institution, entitled *Publikationen des astrophysikalischen Observatorium zu Herény in Ungarn*, I. Heft, appeared in 1884 and contains the detailed description of the observatory, observations of spectra of the brighter stars made in 1881–1882 with a spectroscope attached to the eyepiece of the instrument, without slit and cylindric lens, and observations of the surface of *Mars* and *Jupiter* (1881–1882) made by Alexander v. Gothard, and a few other miscellaneous observations of the same epoch.

Later on, von Gothard devoted himself mainly to the study of the spectra of heavenly bodies, and published his first paper on the spectrum of Comet 1882 II in the *Astronomische Nachrichten* (**103**, 377, 1882); then followed the researches on the spectra of Comet

Brooks-Swift (1883), of Comet 1884 I, of γ *Cassiopeiae* and β *Lyrae*. In the paper on the new star in the *Andromeda* nebula (*Astronomische Nachrichten*, **112**, 390, 1885) he mentions his first experiments on the use of photography for observations of nebulae, a method he had previously employed for the solar eclipse in 1882. The interesting photograph of a meteor accidentally crossing the field of his reflector during an exposure on some other object, and forming a trail like a comet, was made in his earliest photographic researches. His first success in comet photography came in the autumn of 1886, when he obtained a photograph of Comet 1886 IX. After the observations on the spectrum of *U Orionis*, he published his first account of his most interesting photographs of nebulae, to which were devoted many years of his assiduous work, and increased a great deal our knowledge of the constitution of the nebulae. The discovery of the small star in the *Lyra* nebula was considered somewhat doubtful at first by several observers, but was universally recognized as certain after having been thoroughly verified by others. His first photographs showed the great advantage of the new method over visual observations, and on his photographs of comparatively small size von Gothard obtained delicate details which were lacking on the best drawings made with large instruments, or appeared there only as slight traces. Professor Vogel in Potsdam examined very closely some of von Gothard's photographs, and requested Mr. S. Widt to make elaborate drawings of five nebulae, believing that only well-made drawings are able to show all the fine details of the originals (*Astronomische Nachrichten*, **119**, 337, 1888). He emphasizes the fact that small condensations of the nebulous matter can always be easily distinguished from star-images by examining carefully the silver deposit on the plate. He could make very exact measures of well-marked details on the photographs. The excellence of von Gothard's photographs can also be seen on his photographs of the small nebula near *B.D.*$+34^\circ 980$ and of the small nebula, *M 57*, discovered by Barnard; in this case he succeeded in verifying the existence of these very faint objects with his comparatively small instrument, though only the Lick telescopes seemed to be large enough to show them visually (*Astronomische Nachrichten*, **135**, 11, 1894; *Astronomy and Astrophysics*, **13**, 190, 1894).

The photographs of the spectrum of Comet 1892 I show great progress in this line of research (*Astronomische Nachrichten*, **129**, 405, 1892). After some unsuccessful attempts, the plate taken on April 9, 1892, with four hours' exposure, showed a fine, sharp spectrum of the comet from λ 3873 to λ 5673, with much detail, and permitted the identification of the bands in the comet's spectrum with the hydrocarbon bands of the Swan spectrum, as carefully investigated by Professor Eder (1890).

Then came von Gothard's most important investigations of the spectra of gaseous nebulae and the new star in *Auriga*, which led to the interesting discovery of the identity of the nebular spectrum and the spectrum of the *Nova*. The first spectra of the *Nova* were taken with his quartz spectrograph (1892); afterward, on September 12, he tried a 10-inch objective-prism with great success (*Astronomische Nachrichten*, **129**, 93, 1892). He found a spectrum with bright lines showing a great resemblance to the spectrum of the Wolf-Rayet stars and to that of the gaseous nebulae. From the photographs of the nebulae *G. C.* 4964, 4447, 4373, 4514, 4628, and *N. G. C.* 7027, 6891, and 6884, taken with the objective-prism; the plates of the great *Orion* nebula, *G. C.* 1179, and the nebulae *G. C.* 4964 and 4628, taken with the quartz spectrograph, he could determine the wave-lengths of many nebular lines very exactly, and prove in a satisfactory way that seven *Nova* lines were absolutely identical with the corresponding nebular lines, and even the intensities of the lines showed a similar behavior in both cases. Only two lines, near λ 3720 and λ 4642, were not found in some of the nebular spectra. The detailed account of these investigations appeared in Hungarian in the *Memoirs* of the Hungarian Academy of Sciences, III Section, October 17, 1892, and was translated for other scientific journals (*Astronomy and Astrophysics*, **12**, 51, 1893; *Monthly Notices*, **53**, 55, 1893; *Memorie della Società degli Spettroscopisti Italiani*, **21**, 169, 1892).

Von Gothard also carried on similar researches on the spectrum of *Nova Persei* of 1901 with the objective-prism. Many photographs in the spring of 1901 show periodic changes: on certain days the spectrum seemed to be continuous; on other days bright lines appeared, which are characteristic of gaseous nebulae. The length

of the period was about nine days, and the star remained longest in the stage showing the nebular spectrum (*Astronomische Nachrichten*, **155**, 269, 1901, and **157**, 141, 1901). The observations in August 1901 show no further changes in the spectrum, which was then very similar to the gaseous spectrum observed in April, and remained unchanged during all of the following observations. The difference between the gaseous spectrum in April and this spectrum of the later stage was the enhancement of the nebular lines of the first spectrum, the hydrogen lines remaining unchanged. Von Gothard's last paper on astronomical subjects appeared September 6, 1901, in the *Astronomische Nachrichten* (**156**, 283, 1901), concerning the photographic aureola observed around *Nova Persei*. He attributes this phenomenon to the great intensity of the nebular lines λ 3867 and λ 3970, thinking that the photographic objectives used to photograph the *Nova* were not sufficiently corrected for these rays and therefore gave chromatic dispersion-circles around the image of the star.

Besides these astrophysical researches, von Gothard did much spectroscopic work in the laboratory. He used, at first, a spectrograph with a direct-vision Wernicke prism; later on, a fine Rowland concave grating, mounted in Rowland's manner. He published very little about these researches; the most important paper on them is his "Spectrographic Studies," which appeared in Hungarian in 1891 in the *Memoirs* of the Hungarian Academy of Sciences (III Section). It contains a detailed description of the instruments, and extensive measures made on the nitrogen spectrum from λ 3650 to λ 4060. He used the lines of the iron arc as standards, and deduced the wave-lengths of the nitrogen spectrum by a graphical method from the spectrograms taken with the Wernicke prism spectrograph. The final results agree very well with Hasselberg's measures. Other investigations carried on in his laboratory were his experiments on the photography of the electric spark. He began these researches in 1887, and obtained beautiful figures by conducting the discharge of a Wimshurst machine, in a dark room, directly upon the sensitive layer of the plate. The figures were the finest when Leyden jars were used and the back of the plate covered with tinfoil. The figures around the positive pole resembled the roots of a tree; those around

the negative pole were like feathers with very fine detail (Eder's *Jahrbuch für Photographie*, 1889).

Von Gothard occupied himself very much with scientific photography and was thoroughly acquainted with all its methods. He was a real master of this art. He used to try every new material and process in his laboratory, to determine whether they might not prove useful in their application to astronomy or spectroscopy. A large number of papers dealing with these questions appeared from 1888 onward in Eder's *Jahrbuch für Photographie und Reproduktions-Technik*, in the *Photographische Correspondenz*, and *Photographische Rundschau*. The usefulness of photography for astronomy had not been universally acknowledged up to the time of von Gothard's early researches and therefore he was very anxious to prove to the skeptics the great importance of this new method.

Besides his observational work, he found great pleasure in making new instruments and spent much of his time in the workshop, constructing many fine instruments by the most exact and modern methods. He not only made all the apparatus for his own use, but furnished, also, many instruments—transit instruments, spectrographs, and photographic cameras—to outside scientific establishments. Among his new constructions we mention only his wedge photometer with type-printing apparatus, which has served as a model for the photometer of Toepfer used in Potsdam (cf. Müller, *Photometrie der Gestirne*, p. 185), and the fine spectrographs continually used since by Professors Eder and Valenta in Vienna. Some of his new models von Gothard described in the *Zeitschrift für Instrumentenkunde* from 1883 onward; the first of his constructions are treated also in N. von Konkoly's *Praktische Anleitung zur Himmelsphotographie* (1887).

In 1895 von Gothard was made director of the new electric works founded in Szombathely. For many years he remained in this post, devoting his whole energy to the development of the new enterprise, and succeeded in bringing it technically and financially to great perfection. Under these circumstances he could devote himself to scientific work only in leisure hours, and found no more time for researches of any considerable length.

The first signs of a serious heart disease appeared about 1899;

failing health forced him to give up all laborious occupations, and from this time he worked only on rare occasions in his observatory and workshops. He traveled in autumn and winter in the South, mostly in Italy, studying there with great enthusiasm the treasures of ancient art; he visited also Algeria and Egypt. The greater part of the year he spent quietly in retirement in Herény, collecting books and doing some scientific work from time to time. He died May 29, 1909, quite unexpectedly, and was mourned by his relatives and many friends and colleagues. E. v. Gothard was a very kind, reserved man, with great energy, and helpful to everybody who asked aid or advice. He was the recipient of many honors: in 1886 the Voigtländer silver medal from the Vienna photographic society; in 1887 the gold medal of the Vienna photographic exhibition; the highest distinctions of the photographic exhibition of 1889 in Berlin and of 1889 in Moscow. In 1890, he was elected corresponding member of the Hungarian Academy of Sciences in Budapest. He was also a member of the Royal Astronomical Society, of the Astronomische Gesellschaft, and of several other learned societies.

BUDAPEST
October 1909

ON A GREAT NEBULOUS REGION AND ON THE QUESTION OF ABSORBING MATTER IN SPACE AND THE TRANSPARENCY OF THE NEBULAE

By E. E. BARNARD

While photographing the region of the great nebula of ρ *Ophiuchi* (which I had found with the Willard lens) at the Lick Observatory in 1893, the plates with the small lantern lens ($1\frac{1}{2}$ inches diameter, also attached to the Willard mounting) showed a remarkable nebula involving the 4.5 magnitude star ν *Scorpii* (Plate I). It had not been noticed on the Willard lens photograph, where it was very faint and near the edge of the plate. The discovery of this object therefore is due to the small lantern lens.

Roughly this nebula is bounded by the figure formed by the following places (for 1855.0):

α	δ
$15^h\ 59^m$	$-18^\circ\ 20'$
16 4	−18 0
and	
16 10	−21 00
16 16	−18 50

In its fainter portions it involves to the southeast the stars *B. D.* $-19^\circ.4357$, $-19^\circ.4359$, and $-19^\circ.4361$. The last two are in a dense nebulous mass in which on the north following side close to the stars are a thin dark lane and a narrow strip of brighter nebulosity. These two stars are joined to $-19^\circ.4357$, which is itself nebulous, by a thin thread of nebulosity which is well shown in Plate II A. North and following these objects are dark regions where there are apparently very few stars. The extensions of this great nebula reach to, and in a feeble manner connect with, the great nebula of ρ *Ophiuchi*.

The greatest interest in this nebula, however, lies in the fact that it seems to show a veiling of the stars in certain of its portions. Especially is this noticeable at its northern and western end, near the stars $-17^\circ.4511$, $-17^\circ.4502$, and $-18^\circ.4240$. It is quite evident that the thinning out or dimming of the stars in this region, that are apparently in the nebula, is not due to a chance vacancy. The line of demarka-

PLATE I

N

E

W

GREAT NEBULA OF NU SCORPII

[illegible] Inch Lens. [illegible], April 4, [illegible]^h 25^m to 24^h 2[illegible] G. M. T. Scale, [illegible] 57 mm

The plate covers [illegible]

tion between the rich and poor portions of the sky here is too definitely and suddenly drawn by the edges of the nebula to assume the appearance due to an actual thinning out of the stars. It looks, where this part of the nebula spreads out, as if the fainter stars were lost, and the brightness of the others reduced at least a magnitude or more. This remarkable feature of the nebula is very important to a proper understanding of the region of the great nebula of ρ *Ophiuchi*, which is five degrees south of ν *Scorpii*. In the region of ρ *Ophiuchi* there is every appearance of a blotting-out of the stars by the fainter portions of the nebula, but from its complicated and irregular form the hiding of the stars is not so clearly evident as in the case of the ν *Scorpii* nebula. At present we have no means of determining whether a nebula is transparent or not. The assumption has always been that they are transparent like the comets. The proof of the transparency of comets is easy, but for obvious reasons there can be no similar proof with respect to the nebulae. I think in the present case, however, that the nebula of ν *Scorpii* is shown to be at least partially transparent, but the absorption of the light of the stars behind it must be considerable. The picture is quite conclusive evidence that the nebula is nearer to us than the general background of stars at this point. This fact, unfortunately, is not so evident in the reproduction as it is in the original, an inspection of which would at once lead to the above conclusion.

In connection with the present subject I would call attention to a paper of mine in *Astrophysical Journal*, **23**, 144, March 1906, which describes a very intricate and straggling nebula in this region, connecting the stars π and δ *Scorpii*. I believe this object will ultimately be found, with more sensitive plates and longer exposures, to be connected with the ν *Scorpii* and ρ *Ophiuchi* nebulosities. The accompanying chart, which covers parts of the constellations *Ophiuchus*, *Scorpio*, *Libra*, and *Lupus*, is intended to show the relation of these various nebulosities to each other. There is strong evidence that they are but the brighter parts of one enormous nebula that covers all this region. I have indicated only the brighter portions of these nebulosities, especially in the case of ρ *Ophiuchi*, for that nebula extends in a strongly marked manner for some distance to the east and can be traced for at least 5° in α and $6\frac{1}{2}$° in δ. Indeed I am

convinced that all this region as far east as θ *Ophiuchi* and beyond is affected with this diffused nebulosity.

The ρ *Ophiuchi* nebula is far more remarkable than that of ν *Scorpii*. Indeed I do not think there is a finer nebula in the entire sky. Even in comparison with the great nebula of *Orion* in some

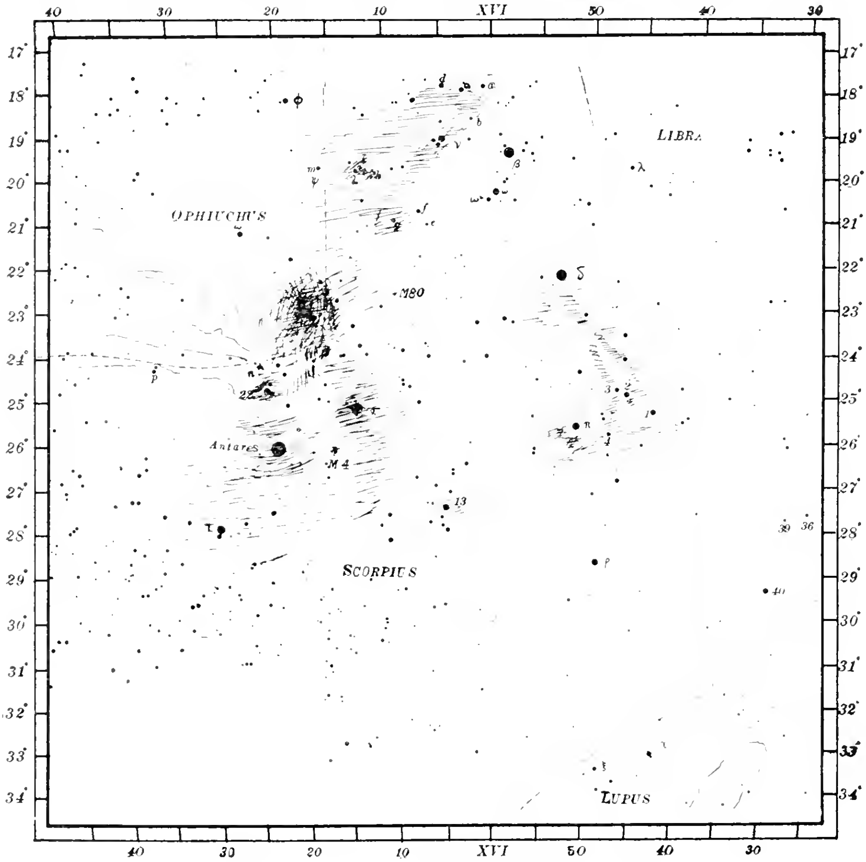
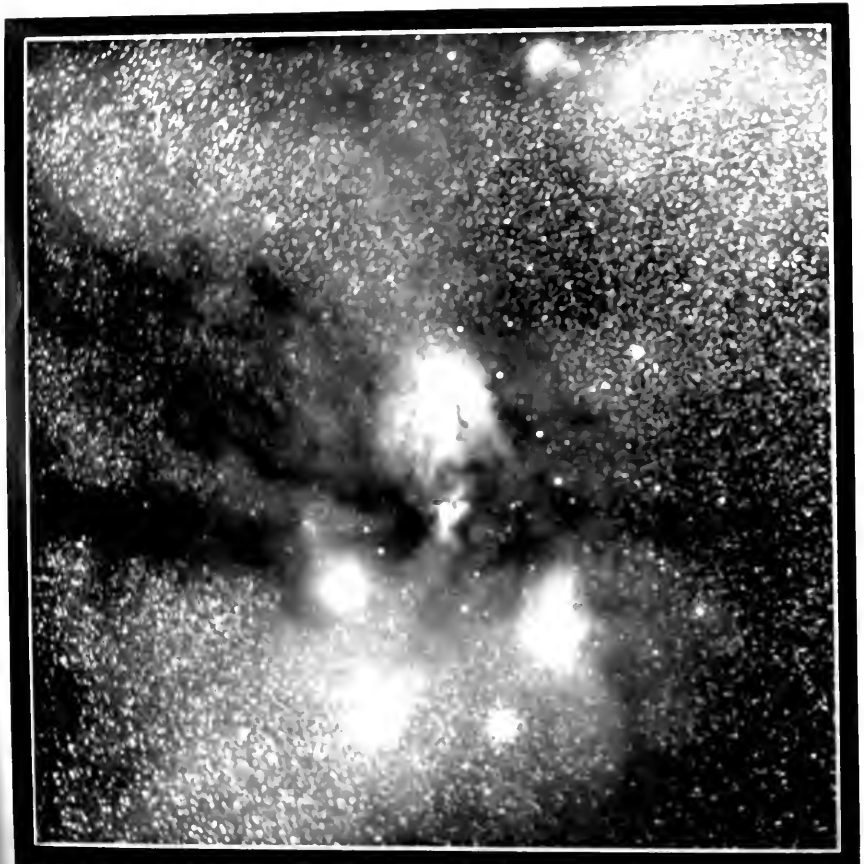
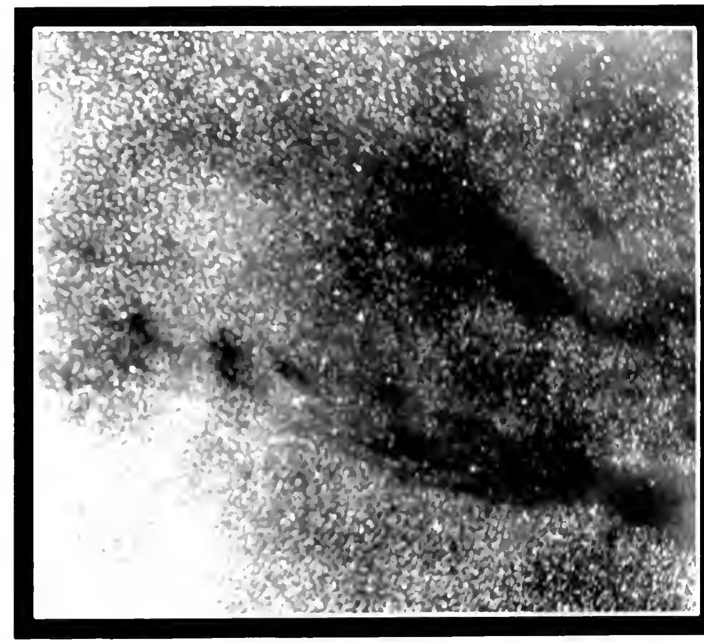


CHART OF THE GREAT NEBULOUS REGION

respects it has a deeper interest because of the aspect of the sky near it. It is impossible adequately to describe in detail its extraordinary nature and that of the surrounding region. The reproduction falls far short of doing justice to this subject, though it shows the brighter parts of the nebula fairly well. The dark lanes which run eastward





from it contain very striking black markings, especially the northern one of the two. To me these singular dark features are of as much importance as the bright portions of the nebula, but it has been impossible to bring them out, in the half-tone. The picture shows, however, the striking absence of stars in the space occupied by the main portions of the nebula. To all appearance, the great nebula is located in a hole in a very dense part of the Milky Way, from which vacant lanes extend far to the east. Besides the two main condensations, σ *Scorpii* is involved in a very strong and irregular condensation which is marked by a considerable amount of detail and which spreads in a faint diffusion for several degrees to the south. In addition to the main great condensations there is another one, equally remarkable, 1° due south of ρ at the star *C. D.* $-24^\circ.12684$. This presents a very singular and striking appearance. From the star as a center issue four bright whorls of nebulosity, which are each about 20′ or 30′ long, the two running north and south being the longest. About 14′ north and slightly east is a singular U-shaped dark marking that is so distinct as to appear almost like a defect. Immediately following this condensation is a dark whirlpool appearance which is formed by the beginning of the vacant lanes running to the east.

The two main and largest condensations lie, one, about the triple star ρ *Ophiuchi*, and the other, equally important, precedes it to the west about 30′. This last does not seem to center at any particular star. These condensations are separated by an irregular dark rift 20′ to 30′ long, which runs north and south. North of ρ the nebula assumes a beautiful ribbed appearance which is but feebly represented in the half-tone. The star *22 Ophiuchi* lies between two diverging strips of nebulosity, the northern and upper strip curving around the star. This star and the nebulous strips singularly resemble a human eye, from which fact I have called it "the eye." Waves of nebulosity extend to and beyond *Antares*, diffusing as far south as τ *Scorpii*. At its upper edge this plate (Plate II A) shows the three nebulous stars which are near the lower left-hand corner of Plate I. It also shows a portion of the nebula of ν *Scorpii* about ¾ inch or 19 mm (1° 8′) to the west of these stars. Part of the illumination in the extreme upper right-hand corner of Plate II A is due to the

reproduction. The portion of the great *ν Scorpii* nebula which is shown at this point is readily made out (because of its great intensity) 0.9 inch from the right-hand outer edge of the block and 1.1 inches from the upper edge. The rest in this corner is unreal. The diffused nebulosity south of *Antares* is relatively too bright in the half-tone, though it is real.

The star *n* (which is *C. D.*$-24^{\circ}.12698$) is 0.41 inch north and 0.25 inch east of *22 Scorpii.* It has a narrow strip of nebulosity extending west and south from it for about 6′. This is noticeable on Plate II A.

Among the most remarkable features of this marvelous region are the vacant lanes or streams, previously referred to, extending to the east. The lower or southern of these, which is ½° broad, is the strongest marked. Its full extent is beautifully shown in Plate II B, which overlaps Plate II A. Its edges are very clearly defined for about 7°, after which it becomes broken and shattered and ends 10° to the east in an irregular group of small holes. The northern and shorter of the two most conspicuous lanes is marked for about 2° with very black, irregular, and sharply defined rifts and perforations which unfortunately are lost in the reproduction. For a history of the discovery of this great nebulous region see *Popular Astronomy*, **5**, 227, September 1897.

I have at other times called attention to the fact that the real connection of this great nebula with such bright stars as *σ Scorpii*, *ρ Ophiuchi*, and others, and its connection with the substratum of small stars of the Milky Way, in which the lanes occur, was a proof of the actual smallness of the stars forming the groundwork of the Milky Way at this point and elsewhere. This must necessarily be true, for the connection with the bright and small stars would imply that the small stars are roughly as near to us as the large ones in this part of the sky, and hence relatively small bodies. If, however, the connection with the small stars is only apparent and the lanes and holes are due to absorbing media between us and the Milky Way, the supposition of smallness would not hold true.

While speaking of these strange dark forms, such as are connected with the *ρ Ophiuchi* nebula, and which are so wonderfully shown on the photographs of the region of *θ Ophiuchi* (*Astrophysical Journal*,

9, 157, 1899, and *Popular Astronomy*, **14**, 579, Dec. 1906), I would call special attention to an object of this class which has been shown on a number of my photographs for the past fifteen years or more. It is a small black hole in the sky, very much like a black planetary nebula. It is round and sharply defined. Its measured diameter on the negative is 2!6. The position is closely:

$$1875.0 \ \alpha = 18^h 25^m 31^s, \ \delta = -26^\circ 9'.$$

On account of its sharpness and smallness and its isolation, this is perhaps the most remarkable of all the black holes with which I am acquainted. It lies in an ordinary part of the Milky Way and is not due to the presence or absence of stars, but seems really to be a marking on the sky itself.

If these dark spaces of the sky are due to absorbing matter between us and the stars—and I must confess that their looks tempt one to this belief—such matter must, in many cases, be perfectly opaque, for in certain parts of the sky the stars are apparently entirely blotted out. It is hard to believe in the existence of such matter on such a tremendous scale as is implied by the photographs. As to its nature if it does exist, it must in some way be related to the nebulae, for we find them in most cases to be intimately connected. Is it an ultimate condition of nebulous matter or is it something wholly different from the ordinary nebulosity of the sky?

To those who may be interested in the subject of possible masses of dark absorbing matter in space in connection with visible nebulosities I would refer to a paper of mine, "On a Nebulous Groundwork in the Constellation *Taurus*," *Astrophysical Journal*, **25**, 218, April 1907, where a system of dark lanes and holes in *Taurus* is shown to exist in the sky independently of the stars.

The accompanying photographs were made by me with the 10-inch Brashear lens of the Bruce photographic doublet which, through the courtesy of Professor Hale, was temporarily stationed at the Solar Observatory of the Carnegie Institution on Mount Wilson, California, in 1905.

As will be noticed in the photographs the great lane extending to the east from "the eye" on Plate II A and continued in Plate II B runs almost due east and west. While at the Lick Observatory,

I once showed a plate of this region to Professor Tucker, who had such a large part in the making of the *Cordoba Durchmusterung*. He said that this picture made clear an experience in his observing work at Cordoba that had always been a puzzle to him. One night he had set his telescope in the region a little north of *Antares* and prepared to record the transits of stars as they passed through the field. Presently no stars came into the field of his telescope. After watching for some time he finally concluded the sky had clouded over, but on looking out he found it perfectly clear. He returned and watched a long time before any stars appeared. His telescope had been pointed to this lane and nothing but blank sky had passed.

LIST OF STARS REFERRED TO ON THE CHART

Bonner Durchmusterung, EPOCH 1855.0

	No.		Mag.	α			δ		
b.	−18°	4240	7.5	16h	0m	39.s1	−18°	36.′4	
c.	−17	4502	6.5	16	1	34.4	−17	57.1	
ν Scorpii	−19	4333	4.5	16	3	33.9	−19	4.4	
d	−17	4511	7.3	16	3	40.6	−17	51.0	
e	−21	4305	7.0	16	5	9.4	−21	1.5	
f	−20	4444	6.8	16	5	56.9	−20	43.8	
g	−20	4454	6.8	16	8	26.8	−20	56.2	
h	−19	4357	6.0	16	10	38.3	−19	51.5	
k	−19	4359	7.7	16	11	36.3	−19	41.9	
l	−19	4361	7.3	16	12	0.8	−19	45.7	
ψ Ophiuchi = m	−19	4365	5.0	16	15	37.3	−19	4.6	

Cordoba Durchmusterung, EPOCH 1875.0

	No.		Mag.	α			δ		
χ Lupi	−33°	10754	4.2	15h	43m	1.s8	−33°	14.′7	
π Scorpii	−25	11228	3.4	15	51	18.0	−25	45.3	
δ Scorpii	−22	11292	2.7	15	52	57.1	−22	16.2	
13 Scorpii	−27	10841	5.3	16	4	36.6	−27	35.7	
σ Scorpii	−25	11484	3.4	16	13	37.2	−25	17.1	
	−24	12684	8.0	16	17	52.1	−24	10.3	The star with the 4 whorls
ρ Ophiuchi	−23	12861	4.8	16	18	6.1	−23	9.2	
Antares	−25	11359	1.4	16	21	46.0	−26	8.9	
22 Scorpii	−24	12695	5.5	16	22	37.6	−24	50.5	
n	−24	12698	9.3	16	24	7.4	−24	8.8	
τ Scorpii	−27	11015	3.2	16	28	6.8	−27	57.2	
p	−24	12765	6.3	16	34	2.4	−24	13.5	

YERKES OBSERVATORY
November 30, 1909

PRECAUTIONS NECESSARY IN PHOTOGRAPHIC PHOTOMETRY

By J. A. PARKHURST

In trying to get precise photometric results from stellar photographs, many sources of error have been encountered, some of which have not been thoroughly investigated. In the photometric work done at this observatory during the past five years, it has been found necessary not only to be on guard against unknown sources of error but also to ascertain the possible amounts of the errors arising from the better-known causes for the particular instruments, plates, and developers used. As the work progressed it became more and more evident that the plates contained precise data, which could be deduced by proper handling; a rough estimate of the accuracy possible was two to four times that found in the better class of visual work with photometric instruments. The present paper summarizes the investigations, much of the work being done in collaboration with Frank C. Jordan, while he was Fellow at this observatory.

COMPARISON OF DEVELOPERS

Pairs of plates were exposed in a Scheiner sector-machine and developed with hydroquinone, pyro, and rodinal. Fig. 1 shows specimens of the resulting development-curves obtained from measures of opacities in a Brace spectrophotometer as modified by Wallace for such work. In the figure the abscissas represent the logarithm of the light, calculated from the sector-angles. The ordinates are Hurter and Driffeld "density-units." It has been found (by means to be explained later) that on the straighter parts of these curves, one density-unit corresponds to about 1.7 stellar magnitudes. It follows from the figure that at log light=0.6, for example, the pyro curve lies above the rodinal curve by more than 0.3 magnitude, while the hydro curve is nearly 0.9 above the rodinal. Errors of such amounts might therefore arise from comparison of plates developed with different agents.

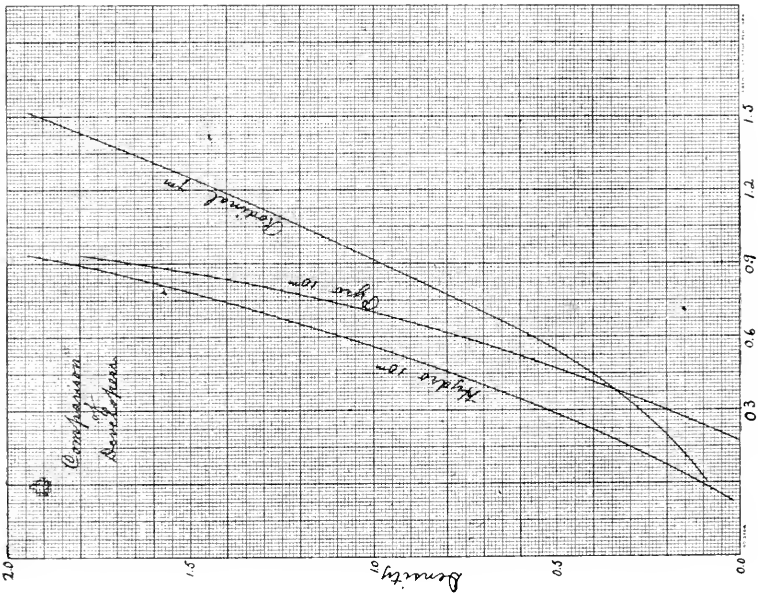


FIG. 1.—Comparison of Developers

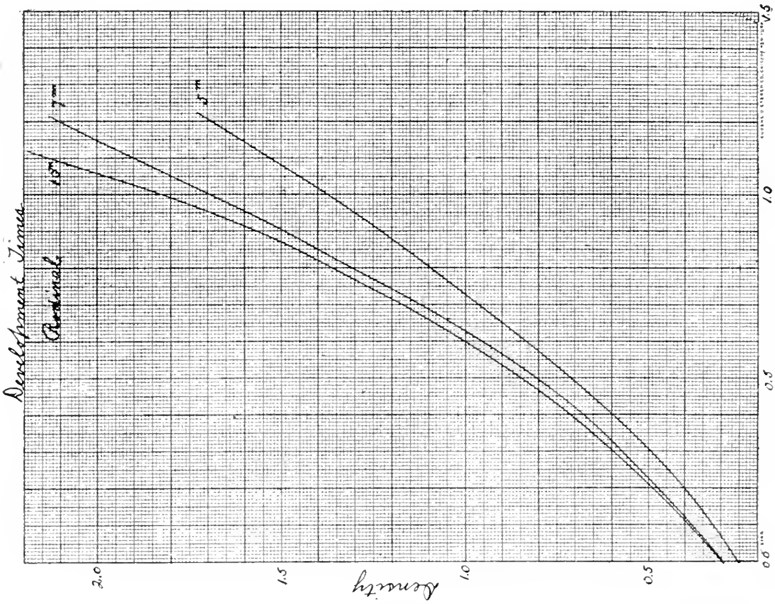


FIG. 2.—Time of Development

TIME OF DEVELOPMENT

The effects of different development-times for Seed plates were tested for the three agents mentioned above. Fig. 2 shows sample curves for rodinal with 5, 7, and 10 minutes' development. Using the above-mentioned conversion factor, it will be seen from the curves that errors of 0.35 and 0.52 magnitude might arise from comparison of the 7 and 10 minutes' development, respectively, with the 5 minute; while on the denser images the difference might easily be double these amounts. In actual practice the development of our stellar plates, which is carried on in total darkness, is timed by an interval-timer which rings a bell after the interval for which it is set.

TEMPERATURE OF DEVELOPMENT

The effect of temperature was quite thoroughly investigated for the different developers and development-times. Fig. 3 shows the differences in light-effect on the Seed plates developed for 7 minutes with rodinal at 55° and 65° F. In this figure the ordinates are expressed in stellar magnitudes, which renders it easy to see the amount of the error possible from a range of 10° F. in the development-temperature—in this case 0.75 magnitude at log light = 1.5.

The literature of stellar photometry indicates that this point has frequently been neglected, or at least not sufficiently controlled. For example, King at Harvard,[1] in his standard tests of photographic plates, developed at "from 70° to 75° Fahrenheit," though his notes show the temperature of the developing room to have been as high as 87° F.[2] It is evident that such ranges are sure to introduce considerable errors, and it seems conservative to put the maximum allowable range as less than one degree F. in order to keep the outstanding errors within 0.05 magnitude. At this observatory the photometric plates are developed with hydroquinone in a tank at 20° C. for ten minutes, thus keeping the above-mentioned factors under close control.

EFFECT OF SKY-FOG, OR SUPPLEMENTARY EXPOSURE

With the extra-focal plates used for determining absolute photographic magnitudes,[3] the effect of sky-fog on the fainter images was

[1] *Harvard Annals*, **59**, 3. [2] *Ibid.*, **59**, 14.

[3] *Astrophysical Journal*, **26**, 244, 1907.

very marked. Fig. 4 shows in the full line the reduction-curve found for the plates used, where the film was clear (scale-reading not above 7), while the dotted line shows the curve needed where the sky-fog was enough to give a scale-reading of 10. The actual difference between these film readings is only 0.03 of a density-unit, but the

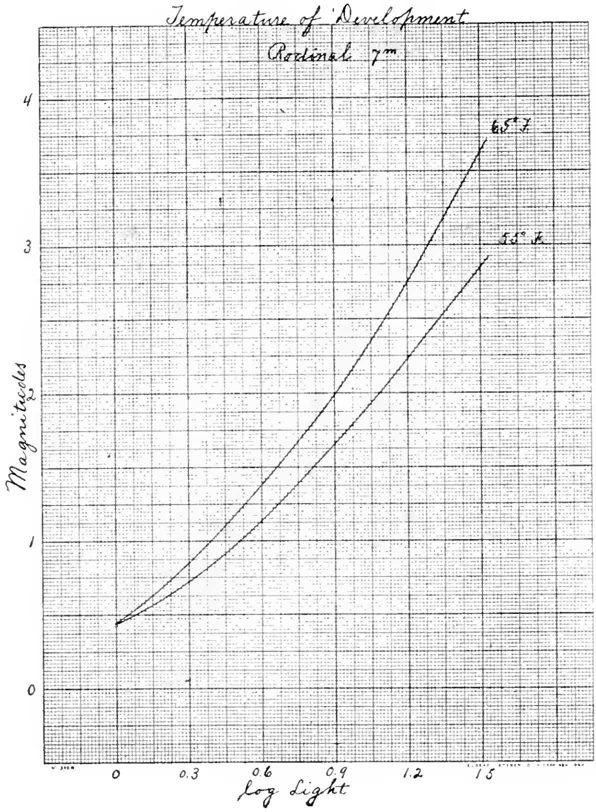


FIG. 3.—Temperature of Development

error caused by neglecting it would amount to 0.6 magnitude at scale-reading 12. Errors of similar amounts will be found in the measurement of the diameters of focal images, since the sky-fog will strengthen the penumbra around the fainter stars to an appreciable extent. This will be especially marked on refractor plates, but the effect cannot be neglected even on plates taken with a reflector, where the penumbra is weaker.

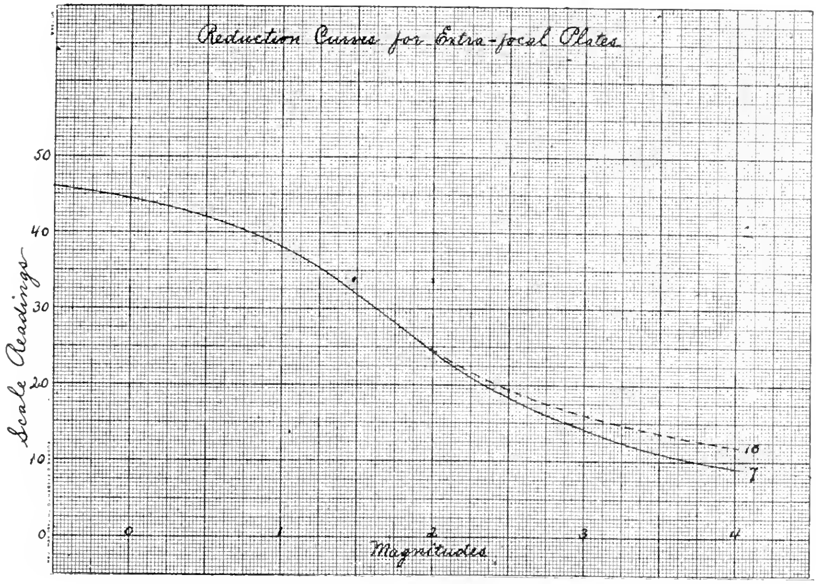


FIG. 4.—Effect of Sky-Fog on Reduction-Curves

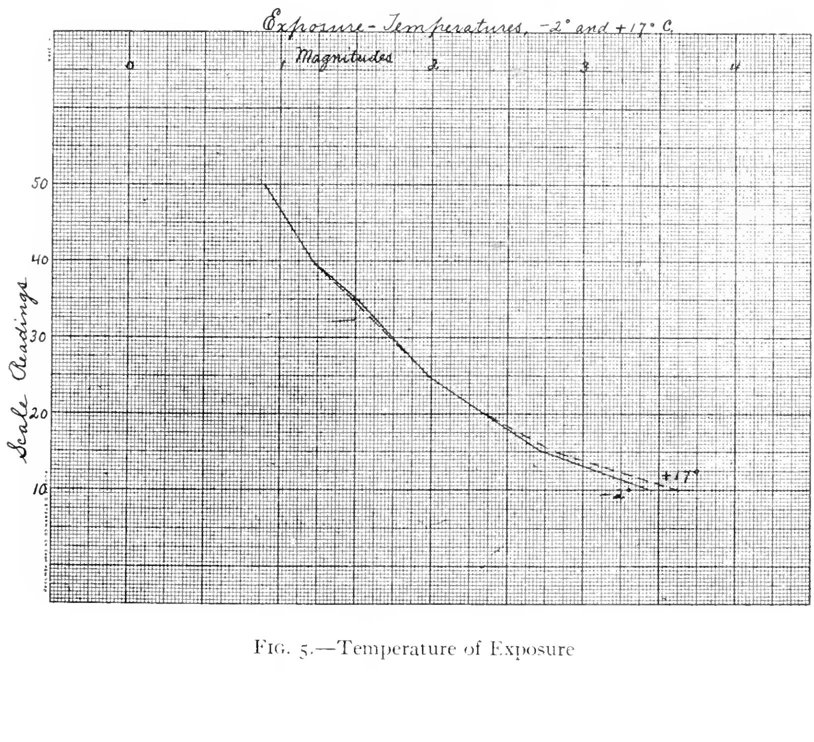


FIG. 5.—Temperature of Exposure

EXPOSURE-TEMPERATURE

Tests were made of the effect of exposure-temperatures ranging from $-2°$ to $+17°$ C. Fig. 5 shows the curves platted from two plates at the extreme temperatures. These plates were exposed in a sensitometer box and measured with the Hartmann "Mikrophotometer" which is described in the *Astrophysical Journal* article last quoted. In the figure measured points are connected by straight lines (as the scarcity of data does not warrant the drawing of a curve) and show that the effect of temperature is not evident within the measured portion of the plates, which did not include the dense and thin extremes.

LACK OF UNIFORMITY IN THE PHOTOGRAPHIC FILM

This is quite certain to be present if the film is irregular in thickness, and may also arise from differences in sensitiveness. The Seed and Cramer plates used, $3\frac{1}{4}\times 4\frac{1}{4}$ and 4×5 inches in size (8×11 and 10×12.5 cm), were cut at the factory after coating, from larger plates. The film within half an inch of the edge of the original large plate is usually thinner than the rest, enough so to introduce errors as large as half a magnitude; agreeing with Hartmann's[1] investigations. No such errors were found near the cut edges of the small plates, but as it is usually impossible to tell which are the cut edges, all the edges were discarded. The small "local errors" remaining are checked by measures of the unexposed film near each star-image. A consideration of several hundred such measures showed that the variations seldom exceeded 0.2 mm in the scale-reading of the Hartmann photometer used with the extra-focal plates. The maximum effect on the star-magnitudes is 0.1 in the most unfavorable case, and would seldom exceed 0.04 for the stars measured. If the variation in the film was greater than 0.3 mm the star was rejected. These errors are much smaller than might be expected on commercial plates, and seem to justify their use, rather than those coated on plate-glass, which are not on the market in this country.

[1] "Ueber die Konstanz der Empfindlichkeit innerhalb einer photographischen Platte," Eder's *Jahrbuch für Photographie*, 1906.

CURVATURE OF THE SURFACE

The commercial plates of the above-mentioned makers are usually coated on the concave side. Measures of a large number of plates made with the feet of the spherometer 48 mm from the screw, showed a concavity ranging from 0.05 to 0.14 mm, the uncertainty being usually less than 0.04. This is about the uncertainty in focusing. It would have no appreciable effect on the magnitudes from the extra-focal plates, and would seldom introduce errors greater than 0.01 or 0.02 magnitude on the focal plates.

REDUCTION FORMULAE FOR FOCAL PLATES

Two reduction formulae have been in common use, the logarithmic and the square root. The former,

$$\text{Mag.} = a - b \log D,$$

where D is the disk-diameter, and a and b are constants determined for the plate, is given by Scheiner in his *Photographie der Gestirne*. The other,

$$\text{Mag.} = a - b\sqrt{D},$$

is used at Greenwich and at other English observatories. Fig. 6 shows how well the two formulae fit the reflector and doublet plates taken here. The platted points are from measures of *Pleiades* plates, using magnitudes determined by the extra-focal method, and agreeing with Schwarzschild's photographic magnitudes. In the upper part of the figure are platted in round dots the square-roots from the co-ordinates given at the top and right. In the lower part, the crosses represent the logs of the same diameters, platted on a slightly smaller scale of abscissas. It is evident that the magnitudes are very well represented by the square-root formula. On the contrary, the use of the log formula would cause errors varying from half a magnitude to two or more magnitudes, according as the standards fell near the middle or the end of the curve. The conclusion is that the log formula cannot be used with the Seed or Cramer plates taken with the reflector or the Zeiss doublet (both kinds of plates gave the above result, also Cramer plates taken with the 40-inch refractor).

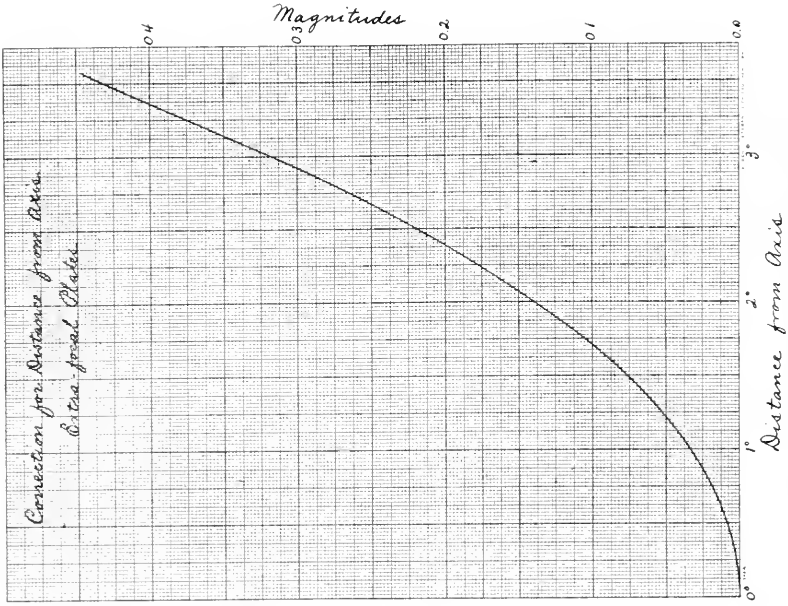


Fig. 8. Correction for Distance from Axis on Extra-focal Plates

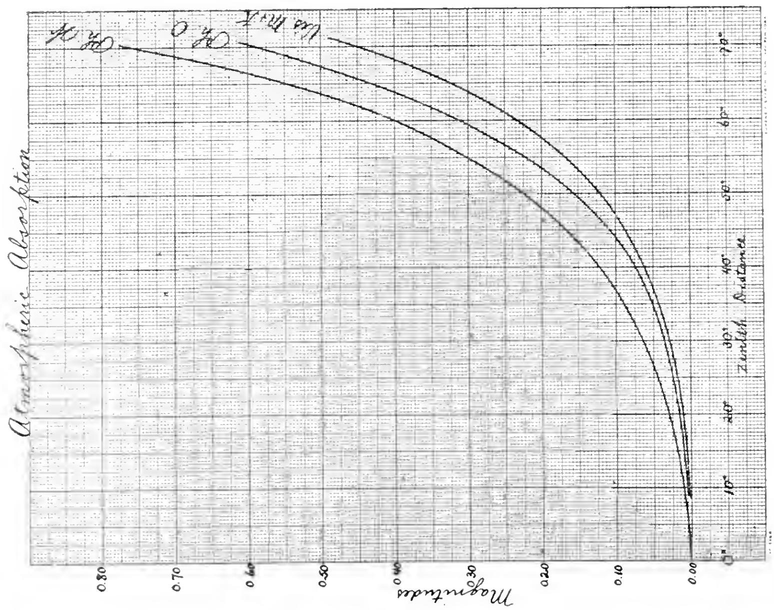


Fig. 7.—Atmospheric Absorption

CORRECTION FOR ATMOSPHERIC ABSORPTION

The three curves drawn in Fig. 7 show the atmospheric absorption as far as 70° zenith distance. The lower curve, marked "Vis. M. & K.," gives the absorption of the visual rays from the Potsdam tables. These values were used with the plates taken with the "visual luminosity" filter, giving visual magnitudes. The second curve, marked "Ph. O.," gives the absorption of the photographic rays deduced by Oppolzer[1] from Schaeberle's measures of star images taken on Seed plates with a 6-inch Dallmeyer doublet. Oppolzer used the log formula in the reductions.

The third curve, marked "Ph. W.," gives the absorption determined by Wirtz[2] from extra-focal plates reduced with the "absolute" scale. This curve gives values of the absorption considerably greater than those found by Oppolzer, the probable reason being the use of the log formula by the latter, as our experience with the same brand of plates and similar doublets make the applicability of that formula very doubtful. The values given by Wirtz were therefore used here.

CORRECTION FOR DISTANCE FROM THE AXIS

The method used for finding this correction was by making uniform exposures on *Polaris*, moving the telescope by means of the declination screw between exposures. This gave a row of about 20 images across the axis of the lens. Measurements of these images with the corresponding distance from the center of the plate will give the correction-curve directly, *provided that the sky was uniform during the exposures.* The only possible control over this condition lies in a comparison of the curves from many different plates, as accidental errors, or temporary changes in the transparency of the air, will cause local deviations from the curve, while a progressive change in transparency will show by the inclination of the curve. The curve for extra-focal images, shown in Fig. 8, was drawn from the mean of 14 rows, rejecting those which were shown to be bad by the above criterion. This is the only part of the work where the time element enters. The plates were taken 6 mm inside the focus,

[1] "Photographic Extinction," *Sitzungsberichte der K. Akademie der Wiss. in Wien*, CVII, Abth. II, December 1898; also *Astrophysical Journal*, **9**, 317, 1899.

[2] *Astronomische Nachrichten*, **154**, 349, 1900.

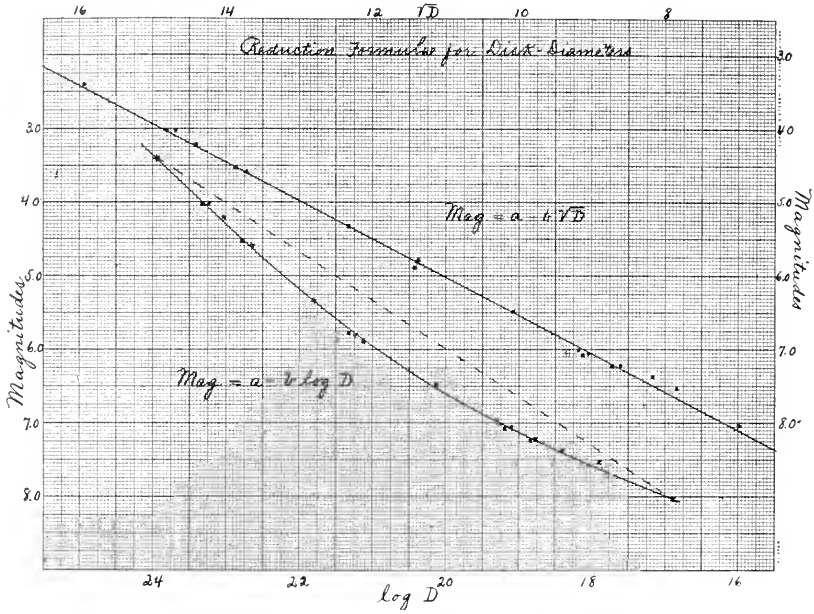


FIG. 6.—Reduction Formulae for Focal Plates

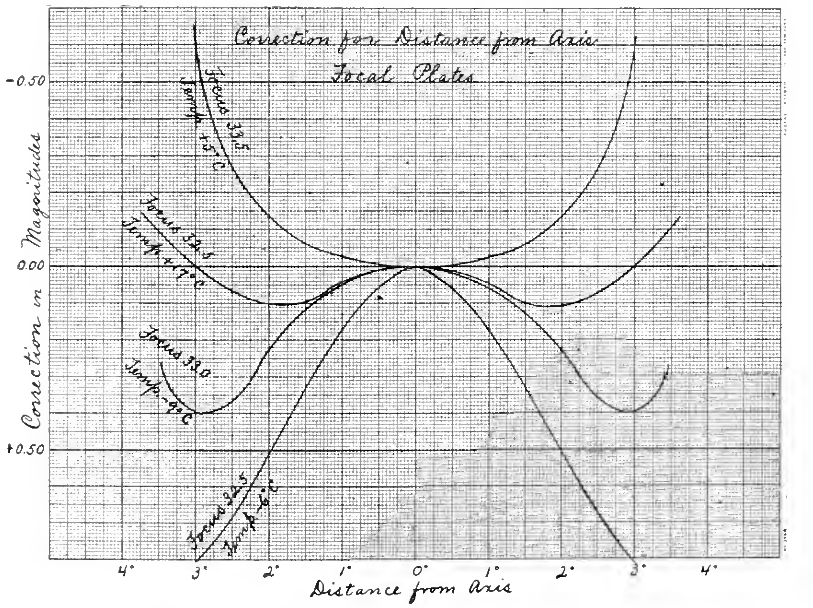


FIG. 9.—Correction for Distance from Axis on Focal Plates

giving images 1.2 mm in diameter at the center of the plate. The correction at 3° from the axis amounts to 0.32 magnitude. Beyond this distance the measures are not very reliable.

Fig. 9 shows some of the correction-curves for focal plates taken with the Zeiss doublet at different focal settings and temperatures. The focal length of the doublet is 814 mm. The camera tube is brass, and has an inconveniently large expansion coefficient, giving a change of 1 mm for 34° C. The difference in the correction-curves for a change of 1 mm in the focus is shown by the upper and lower curves, made at settings 32.5 and 33.5. At three degrees from the axis the difference in the correction amounts to 1.4 magnitudes. This value applies only to these particular plates, which were taken through the "visual luminosity" filter and have very sharp images, the penumbra being almost absent. On plates taken without a filter, having more penumbra around the images, the actual amount of the correction would be different, perhaps more, perhaps less. The setting chosen for the focal plates was nearly that shown in the curve "Focus 32.5, Temp. +17° C." At this setting the maximum correction is at 2° from the axis, and an uncertainty of 0.05 mm in the setting would cause an error not exceeding 0.05 magnitude at 3° from the axis.

The ideal material for the camera tube would be "Invar," but whatever the material, the focal setting should correspond with the temperature within 0.05 mm for precise work.

YERKES OBSERVATORY
December 1909

THE SUN-SPOTS OF SEPTEMBER 25, 1909

By FREDERICK SLOCUM

As there is evidently some connection between terrestrial magnetism and solar activity, a statement in regard to observations of the sun about September 25, 1909, the date of the recent magnetic storm and auroral display, may be of interest.

The greatest disturbance of the magnetic storm, as reported by Mr. J. E. Burbank, of the magnetic observatory at Cheltenham, Md.,[1] lasted from $11^h 39^m$ A. M. to about 9^h P. M., G. M. T., September 25. The sun was observed here from 3^h to $8^h 30^m$, G. M. T., on that date. The most conspicuous feature on the disk was a large spot in latitude $-5°$, longitude 307°. At 4^h, this spot was 25° west of the central meridian of the sun. Visual observations showed the spot to be of a normal type, the umbra pear-shaped, about 29,000 km long, crossed by several bridges; the penumbra approximately circular, 38,000 km in diameter. Farther to the west, in latitude $-18°$ and $-16°$, longitude 320° and 329°, was a pair of small spots, connected by a stream of faculae, and to the north, in latitude $+18°$, longitude 333°, was an area of faculae without visible spot.

Calcium plates taken with the Rumford spectroheliograph, both before and after local noon, show a moderate amount of activity in the area around the large spot and in the region between the pair. Several eruptions are shown on the bridges and around the edge of the penumbra of the main spot, and four eruptions of considerable energy appear between the two spots of the pair. A series of five exposures, made in rapid succession just before noon, show no apparent difference, but a plate taken two hours after noon shows a new eruption on the northwest edge of the penumbra. Fig. 11 (Plate IV) is a portion of the morning plate, showing three of the five exposures, all of which were made on the same plate.

A prominence plate taken with the H line of calcium at $3^h 55^m$, G. M. T., shows a moderately large prominence in position-angle

[1] *Science*, **30**, 598, October 29, 1909.

PLATE III

N

1
Sept. 17
$9^h\ 21^m.6$
G. M. T.
H_1

4
Sept. 20
$9^h\ 38^m.4$
G. M. T.
H_1

2
Sept. 18
$9^h\ 27^m.4$
H_2

E

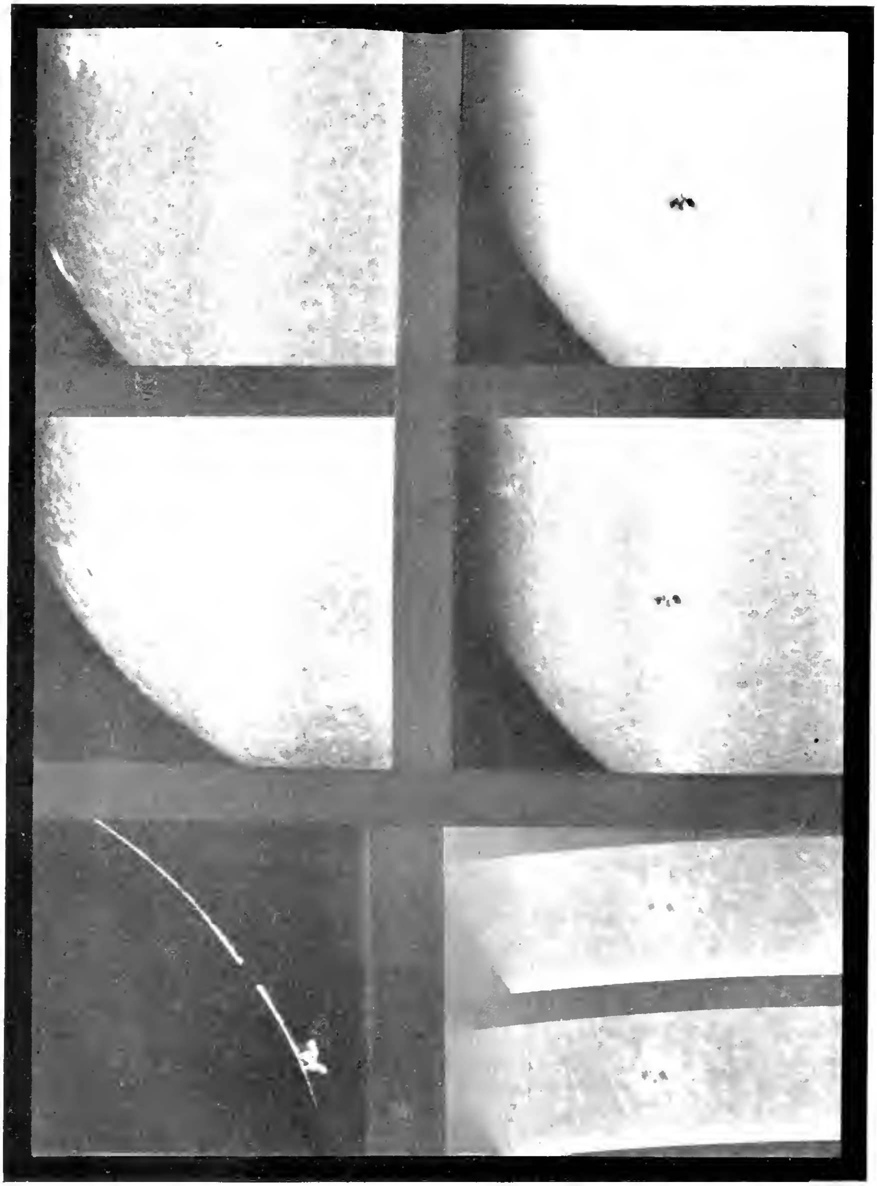
W

5
Sept. 20
$9^h\ 28^m.8$
H_3

3
Sept. 20
$10^h\ 57^m.3$
H_2

6
Sept. 20
B $11^h\ 20^m.5$
H_2

A $11^h\ 19^m.5$
H_2



SPECTROHELIOGRAMS OF SUN-SPOTS
1909, September 17–20
Scale: Sun's Diameter = 140 mm

140°, which extends 8° along the limb and rises to an approximate height of 58,000 km. Eight other small prominences appear scattered around the edge of the sun.

These facts are mentioned simply to give an idea of the activity of the sun as a whole. If the sun is responsible for the great magnetic storm of September 25, it would seem as if the source of that disturbance must lie in the region of the great spot, on this date or earlier. A more detailed history of the spot will, therefore, be given.

The first trace of the spot was noted on September 1, when a small area of flocculi appeared near the western limb of the sun. This passed out of sight in a day or two, but reappeared on the eastern limb on September 17, Fig. 1 (Plate III). A prominence plate taken at 3^h 59^m, G. M. T., shows a small eruptive prominence in position-angle 118°, just over the northern edge of the spot. On September 18 the rotation of the sun had brought this region well into view, and it was at once noted that a good-sized spot had developed, Fig. 2. There are two umbrae, inclosed within a single penumbra, or perhaps it might be considered one umbra crossed by a broad heavy bridge. A chain of five or six eruptions extends along the northern edge of the preceding part of the umbra and down along the whole length of the bridge.

When the next observations were made, on September 20, a marked transformation had taken place. Figs. 3, 4, and 5 (Plate III) are taken from low, intermediate, and high-level calcium plates. The most interesting features of these plates are the broad bridges, one running from N. E. to S. W., the other from S. E. to N. W. On all three plates the latter seems to overlap and rest directly upon the former. Slight changes in the structure of these bridges are noted on plates taken only a few minutes apart. A few small eruptions are observed, particularly on the upper bridge and near its terminals, but these are neither as numerous nor as vigorous as on the 18th. On the H_2 plate, Fig. 6, the whole umbra appears veiled.

The spot passed the central meridian of the sun about 6^h, G. M. T., on September 23. Unfortunately no observations could be made on that date, but on the 24th, the plates show that the spot was still undergoing interesting changes. The high-level calcium flocculi over the spot present a marked spiral form, with several brilliant

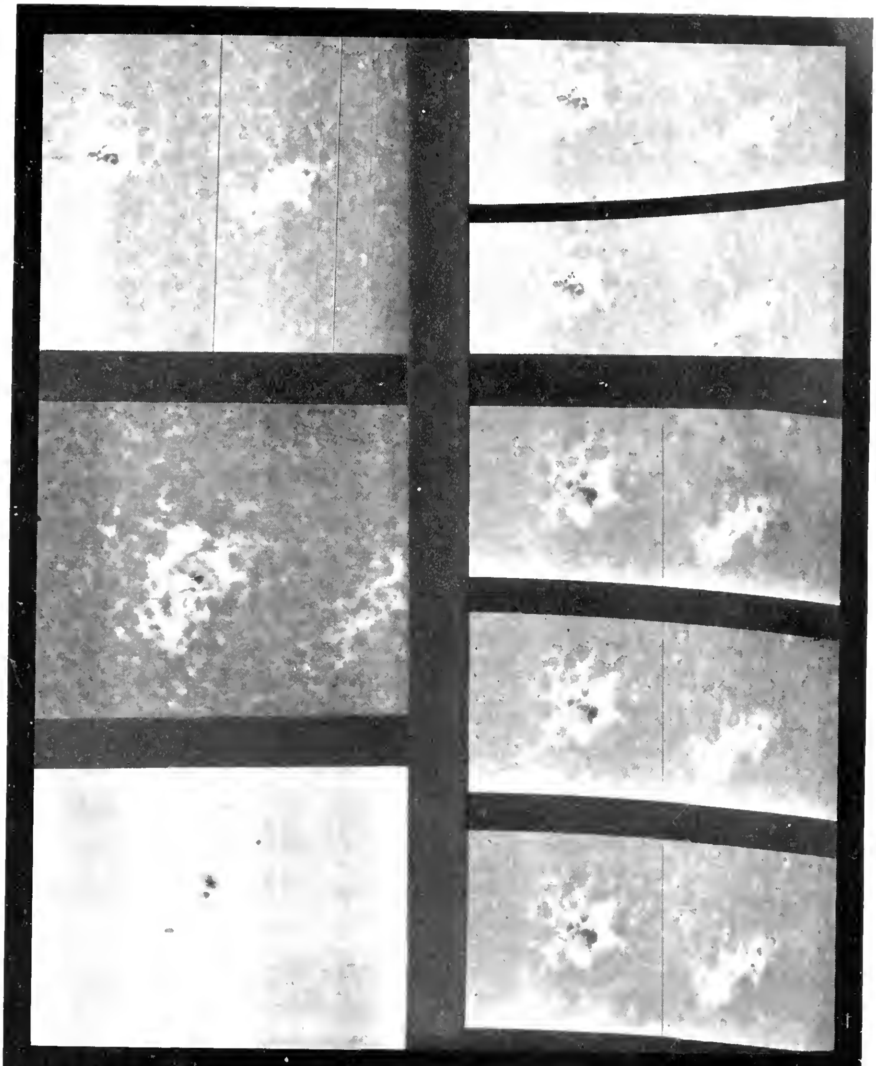
eruptions on the branches and outside the spiral, Figs. 7 and 10. Subsequent observations showed that the spot maintained its activity. On the 25th it had lost its spiral structure but was crossed by several bridges, as noted above, Fig. 11. Plates of the 27th show a complete change in the arrangement of the bridges, with again a slight trace of spiral structure. They also indicate eruptions of considerable energy in various parts of the disturbed region.

No observations were obtained on the 28th or 29th. During the night of September 29–30, the spot passed around the western limb, and no trace of it could be seen visually on the morning of September 30, but its history was not yet completed. A prominence plate, taken at $3^h\ 48^m$, G. M. T., shows well several prominences of moderate size, and one small and inconspicuous jet at the point where the great spot disappeared. A second prominence plate, taken at $4^h\ 57^m$, G. M. T., shows that the spot was still active. Some time within the hour between the exposures of the two plates, a violent eruption had occurred. Where the one lone jet appears on the first plate is a brilliant prominence on the second plate, Fig. 3. In intensity, it is more brilliant than the chromosphere, a fine line of which appears on the prominence plate. It rises in several arches to a height of about 32,000 km, and extends some 5° or 6° along the limb. The observations of the next day, October 1, showed that this last violent eruption was still in progress. The spot had been carried some distance around the limb, but the tops of the arches of the prominence were still visible over the edge.

Two weeks later the spot appeared once more on the eastern limb. A long spell of cloudy weather seriously interfered with observations, but it was photographed on October 16, 18, 19, and 26. A companion spot had developed 2° south, and 7° east of the original spot, and both were inclosed by an extensive area of flocculi. Fig. 9 is taken from a low-level plate of October 19 and shows the spot nearly as it would appear in a direct view. Fig. 8 is from a high-level plate taken eight minutes earlier. This shows an extensive area of calcium flocculi with six eruptions following the leading spot and two just preceding the following spot. At the time of the brilliant auroral display of October 18, the spot was about 20° east of the central meridian.

PLATE IV

N



7
Sept. 24
10h 20m.7
G. M. T.
H_2

8
Oct. 10
9h 22m.5
H_2

E

9
Oct. 10
9h 30m.3
H_1

I
Sept. 24
B 2h 30m.6
G. M. T.
H_2

A 2h 38m.7
H_2

II
C Sept. 25
11h 2m.0
W H_2

B 11h 1m.3
H_2

A 11h 0m.5
H_2

SPECTROHELIOGRAMS OF SUN-SPOTS
1909, September 24 and 25 and October 10
Scale: Sun's Diameter = 140 mm

On October 26, the spot could be seen very near the western limb of the sun and on the next day it had disappeared around the edge. On November 9 it appeared for the third time on the eastern limb, greatly diminished in area and activity. It was further observed on November 15 and 19. On the latter date it was apparently fast waning. There remained but a single small umbra and some scattered patches of calcium flocculi. This was the last seen of this interesting spot. It passed around the western limb a few days later, but failed to reappear on the eastern limb.

Yerkes Observatory
December 9, 1909

AN INVESTIGATION OF THE DISPLACEMENTS OF THE SPECTRUM LINES AT THE SUN'S LIMB[1]

By WALTER S. ADAMS

In an important communication published in 1907, Halm announced the discovery of small displacements of the lines of the solar spectrum at the sun's limb which are independent of the motion of rotation.[2] The results of an investigation of two iron lines in the less refrangible part of the spectrum indicated a displacement toward the red of +0.012 Ångström as compared with their position at the sun's center. On the other hand, an enhanced line of iron at λ 6516 showed no displacement. The cause of these shifts Halm concluded to be the somewhat greater effective pressure in the sun's reversing layer at the limb than at the center, the lowest strata, which are subject to the greatest pressure, contributing a relatively greater length of path to the light at the limb than they do at the center. Halm accordingly explained the absence of displacement for the enhanced iron line by assigning it to a higher level in the sun's atmosphere. In such a case it is evident that the effect of the relative increase in the length of path of the light in the lower strata will affect the wave-length of the line but slightly.

A detailed study of these displacements was taken up by Mr. Hale and myself soon after the appearance of Halm's announcement, and one of our first results was the discovery that the whole solar spectrum is greatly changed in appearance at the sun's limb.[3] Three features were commented upon especially in connection with these results. First, the great weakening, and in some cases the almost complete disappearance of the wings shown by many of the strongest lines at the center of the sun. Second, a slight widening, which is characteristic of practically all of the lines in the spectrum, accompanied in most cases by a decided modification of their intensity-curves. Third, a strengthening and weakening of the lines which in

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 43.

[2] *Astronomische Nachrichten*, **173**, 273, 1907.

[3] *Contributions from the Mount Wilson Solar Observatory*, No. 17; *Astrophysical Journal*, **25**, 300, 1907.

general closely corresponds to that found in sun-spots. These main features of the spectrum of the sun's limb, together with some others, were summarized briefly in a later communication.[1] Some provisional results of the displacements measured were also given at the same time.

Recently MM. Buisson and Fabry undertook a comparison of the spectrum of the center and the limb of the sun with the aid of interference apparatus.[2] Their results relate to a region at about λ 4400, and give both the displacements and the widths of the lines observed. Perhaps their most important conclusion is that the displacements are due to the widening of the lines upon their red edges, the violet edges retaining their normal positions. This widening is ascribed by them to the relatively greater effect of pressure in the lower strata of the reversing layer, their explanation being identical with that of Halm.

The first plates obtained at Mount Wilson, and those upon which the results published by Mr. Hale and myself were based, were taken with the 18-foot spectrograph and the Snow telescope. These plates, though excellently suited for qualitative investigation of the character of the spectra and for relative displacements of the lines, were not well adapted for measurements of absolute displacements. This is evident from the fact that in order to pass from the center of the sun to the limb it was necessary to shift the image upon the slit, thus introducing a change in illumination of the grating, which could easily lead to systematic errors in the amount of the displacements upon the photographs. With the completion of the tower telescope the investigation was continued by myself with the 30-foot (9.1 m) spectrograph used in conjunction with it, and in order to avoid shifting the sun's image upon the slit a small diagonal prism attachment was designed to allow of photographing center and limb simultaneously. This is very similar to that used in the study of the chromosphere and described in a recent paper by Mr. Hale and myself,[3] except that opposite the central prism there are two small

[1] *Publications of the Astronomical Society of the Pacific*, **20**, 27, 1908.

[2] *Comptes Rendus*, **148**, 1741, 1909.

[3] *Contributions from the Mount Wilson Solar Observatory*, No. 41; *Astrophysical Journal*, **30**, 222, 1909.

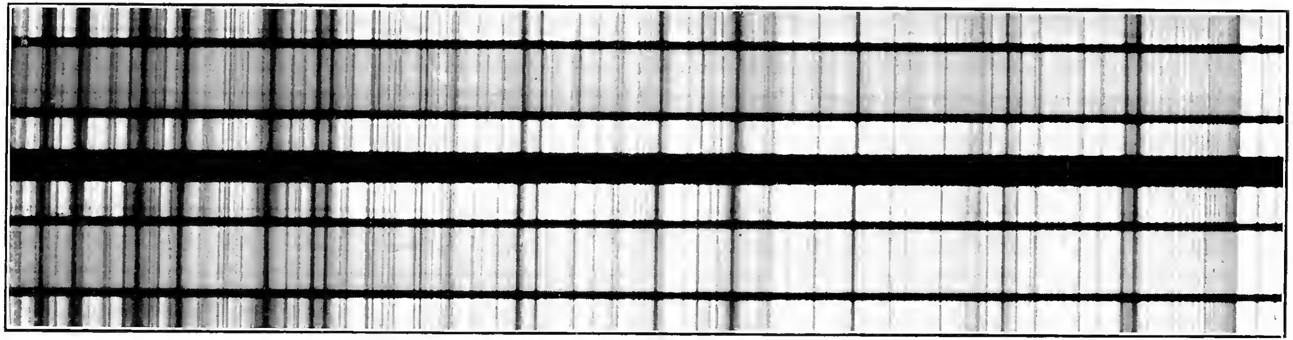
openings through which the light from two regions of the sun symmetrical about the center falls directly upon the slit. The exposure upon the sun's limb is continuous, while that upon the center consists of a series of short exposures distributed throughout the entire exposure on the limb. This has, of course, the advantage of tending to eliminate possible effects of temperature variation during the exposure. The ratio of exposure time at the center to that at the limb varies greatly with the distance of the slit from the limb and with the region of the spectrum observed. On the average, a ratio of about 1 to 5 in the red to 1 to 15 in the ultra-violet is necessary to give equal intensities to the two spectra.

The plan followed in making the exposures has been to compare the spectrum of the center with that of the east limb and then with that of the west limb, the points selected lying at the extremities of the solar equator or at 0° of heliographic latitude. This, of course, introduces into the results the maximum displacements due to the rotation of the sun. These are readily eliminated, however, and the selection of points at the solar equator has two marked advantages. The first is that the change of the rotational velocity is smallest near the equator, so that slight errors in the setting of the instrument in position-angle have their least effect at this point. The second is that the measured displacements are large, and all positive at the west limb, and negative at the east limb. Judging from my personal experience in measuring displacements of this kind, they are much less liable to subjective error than small displacements of varying sign. It is clear that if δ is the displacement between center and west limb, and δ' the displacement between center and east limb, both taken without regard to sign,[1] $(\delta+\delta')/2$ and $(\delta-\delta')/2$ will be the displacements due to the rotation of the sun, and to pressure, respectively. The value of $(\delta+\delta')/2$ when converted into radial velocity should, of course, be constant for the different plates, and equal to the sun's equatorial linear velocity. This is about 1.86 km a second, uncorrected for reduction to the sun's edge or for the inclination of the sun's axis,[2] and corresponds to a linear displacement of 0.044 mm

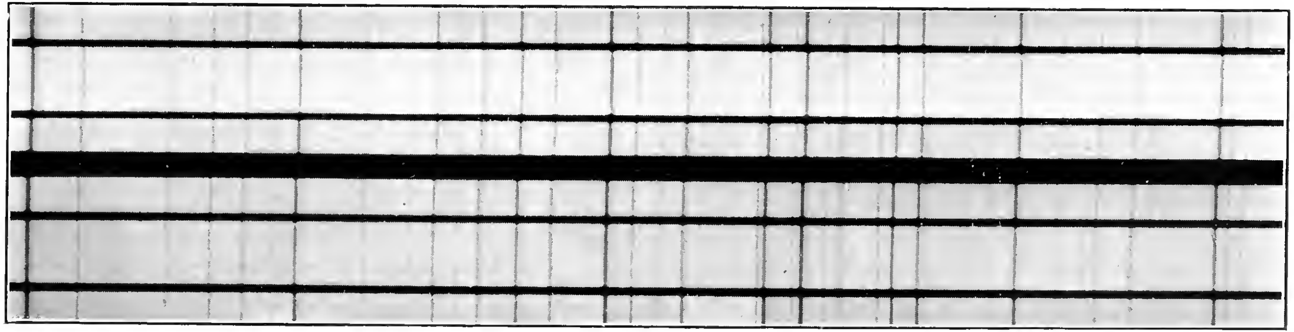
[1] *Contributions from the Mount Wilson Solar Observatory*, No. 24; *Astrophysical Journal*, **27**, 213, 1908.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 33; *Astrophysical Journal*, **29**, 110, 1909.

PLATE V

1
2


Region λ 3830–λ 3885

1
2


Region λ 5370–λ 5425

SPECTRA OF CENTER AND LIMB OF SUN

Scale: 1 Ångström = 3 mm, approximately

1. Center and West Limbs

2. Center and East Limbs

(at λ 4250) on the photographs in the third order of the 30-foot spectrograph. In the great range of wave-length covered by the measures given in this communication the linear value of $(\delta+\delta')/2$

Plate	Center of Region λ	Order	Observed $\delta+\delta'$	Computed $\delta+\delta'$	Rotational Velocity km
P 112........	3818	3	0.086	0.080	1.04
P 155........	3910	3	0.081	0.082	1.86
P 104........	3940	3	0.084	0.083	1.91
P 133........	3995	3	0.078	0.084	1.75
P 134........	3995	3	0.084	0.084	1.89
P 114........	4070	3	0.086	0.085	1.90
P 104........	4110	3	0.088	0.086	1.92
P 135........	4260	3	0.087	0.089	1.83
P 114........	4300	3	0.094	0.090	1.95
P 103........	4320	3	0.088	0.090	1.83
P 75........	4320	3	0.094	0.090	1.95
P 95........	4430	3	0.095	0.093	1.92
P 122........	4430	3	0.092	0.093	1.86
P 147........	4500	3	0.085	0.094	1.70
P 103........	4520	3	0.089	0.095	1.77
P 89........	4780	2	0.066	0.066	1.87
P 78........	4780	3	0.100	0.100	1.88
P 105........	4890	3	0.094	0.102	1.73
P 73........	4980	3	0.103	0.104	1.86
P 73........	4980	3	0.098	0.104	1.77
P 143........	4980	2	0.069	0.069	1.88
P 106........	5050	2	0.064	0.071	1.71
P 91........	5250	2	0.075	0.073	1.92
P 106........	5400	2	0.078	0.075	1.93
P 101........	5400	2	0.079	0.075	1.96
P 84........	5450	3	0.114	0.114	1.88
P 140........	5480	3	0.113	0.115	1.84
P 91........	5600	2	0.076	0.078	1.83
P 141........	5670	3	0.109	0.118	1.73
P 101........	5700	2	0.084	0.079	1.98
P 105........	5770	2	0.084	0.081	1.96
P 115........	5950	3	0.126	0.125	1.89
P 92........	6050	2	0.086	0.085	1.93
P 100........	6100	2	0.088	0.085	1.94
P 116........	6100	3	0.133	0.128	1.96
P 105........	6140	2	0.089	0.086	1.95
P 116........	6240	3	0.132	0.131	1.91
P 92........	6400	2	0.094	0.089	1.98
P 100........	6420	2	0.086	0.089	1.80
P 124........	6420	2	0.083	0.089	1.74
P 139........	6420	2	0.085	0.089	1.79
P 100........	6420	2	0.091	0.089	1.91

corresponding to a velocity of 1.86 km varies from 0.040 mm to 0.068 mm. The constancy of the value of $\delta+\delta'$ when reduced to velocity has furnished an extremely valuable check upon the observations and the adjustment of the instrument. In the present discus-

sion no plates have been included for which this value has differed from the theoretical value to an extent beyond the reasonable limits of accidental error, but among all the plates taken since the attachment for obtaining the spectra of center and limb simultaneously has been employed only three have been discarded for this cause.

The following table contains a list of the plates employed in this discussion. The second column gives the mean wave-length of the lines measured, the fourth column the corresponding values of $\delta+\delta'$, the fifth column the theoretical values based on a value of 0.088 mm for λ 4250, and the last column the observed values of $(\delta+\delta')/2$ reduced to radial velocity. The mean of all the plates gives a determination of the sun's equatorial rate of rotation which is of considerable weight, although inferior to one obtained from the same number of plates in which the spectra of the two limbs are compared directly. The value obtained from the mean of all the plates is 1.87 km as compared with about 1.86 km found from the investigation of the rotation of the sun during 1908.

The selection of the lines to be measured upon these plates was a subject which gave rise to a considerable amount of difficulty on account of the large number of questions involved. The great variety in the behavior of the lines of different elements at the sun's limb, the wide range of level attained by various gases in the solar atmosphere, the abnormal behavior of the enhanced lines, as well as the necessity of including a sufficient number of lines in different parts of the spectrum to furnish material for a discussion of the change of displacement with wave-length, all made it difficult to keep the list within reasonable limits. Finally a total of 470 lines was selected, ranging in wave-length from λ 3741 to λ 6573. This list includes:

1. All prominent enhanced lines.
2. All lines of calcium, magnesium, and sodium.
3. The measurable lines in the cyanogen flutings.
4. The α and γ lines of hydrogen.
5. The measurable lines of elements of very high atomic weight, such as lanthanum and cerium.
6. A large number of lines of iron, titanium, vanadium, chromium, and other elements, distributed as uniformly as possible throughout the spectrum.

7. Lines especially strengthened or weakened in the spectrum of the sun's limb.

In the list of these lines which follows, the mean values of the displacements are given, it being impossible within the limits of this discussion to give the individual values for the different plates. The first three columns of the table give the wave-lengths, intensities, and identifications of the lines as taken from Rowland's table. The fourth column gives the intensity and behavior of the lines at the limb. The widening, indicated by "w," is on a scale of 1 to 3, "w_1" representing a comparatively small amount. As is well known, in the case of winged lines, the wings are greatly reduced at the limb. This behavior is indicated by the abbreviation "sh" for "sharpened," the degree to which this takes place being indicated by the subscripts 1, 2, and 3. The fifth column contains the number of measures, and the sixth column, Δ, the mean displacement in Ångström units. It is, of course, equivalent to the quantity $(\delta-\delta')/2$, referred to earlier in this discussion, converted into Ångström units. The positive sign, as usual, denotes displacement toward the red.

The seventh column, Δ', requires discussion. It was found early in the series of measures that an occasional plate which gave a correct value for the rotational velocity would give considerably larger values for the displacements at the limb than did the majority of the plates. This applies as well to the lines in the cyanogen flutings at λ 3883 and λ 4216, which were shifted by small amounts toward the red. If, however, a correction was applied to these lines to reduce their shifts to zero it was found that the values for the other lines agreed satisfactorily with those obtained from the remainder of the plates. As is well known, the cyanogen flutings are not shifted by pressure, at least to any appreciable extent. In fact, the only cause with which we are familiar that is adequate to produce displacements of fluting spectra is motion in the line of sight. Accordingly, since instrumental sources of error for these plates are made extremely improbable by the fact that correct values for the sun's rotational velocity are given by them, the conclusion seems practically inevitable that they are affected by currents in the sun's reversing layer, giving rise to small motions in the line of sight. In the case of cyanogen, the average displacement toward the red of 14 lines in the ultra-violet

fluting for the plate showing the largest discrepancies amounts to 0.003 Ångström. A current in the reversing layer at the center of the sun which is ascending at a rate of 0.12 km a second, or sufficient to produce a displacement of 0.0015 Ångström, would be adequate to explain this result. Since the convection currents in the solar atmosphere are almost certainly directed radially, small upward motions at the sun's center are rendered very probable by considerations quite apart from those discussed here. The largest difference of this kind encountered upon any of the plates is in the case of P106 for which the center of the spectrum falls at about λ 5400. The difference from the mean in this case amounts to +0.006 Ångström. This would indicate a motion of ascent at the sun's center of 0.23 km a second.

It has seemed desirable to show the results of a correction for this effect upon the observed displacements. This has been done by obtaining from the mean of the cyanogen lines in the ultra-violet fluting, measured upon all the plates and given under Δ in the table, a value of the correction necessary to reduce their displacements to zero, and applying it throughout the spectrum. The value obtained was 0.0020 Ångström, and this quantity, increased in proportion to wave-length, has been subtracted from all of the measured displacements. The results are given in the column Δ'. It is, of course, evident that this procedure is not strictly correct, since the plates in the different parts of the spectrum were not all taken at the same time as the plates containing the cyanogen flutings, and hence are not subject necessarily to the same correction. Since, apart from the cyanogen lines, however, we have no standards for reference, it has seemed best to apply this value as giving at least a reasonable approximation to a mean condition. In the discussion of the results, both the directly observed values, Δ, and the corrected values, Δ', will be considered.

Before taking up the more extended discussion of these results and the question of pressure as the effective agent in producing them, I wish to call attention to some of the prominent characteristics of the list which are evident from a simple inspection.

1. The lines of hydrogen, the H and K lines of calcium, λ 4227 of calcium, the D lines of sodium, and the *b* lines of magnesium

λ	Substance	Intensity	Intensity at Limb	No. of Meas.	Δ	Δ′	Remarks
3741.791	*Ti*	4	3	2	+0.006	+0.004	Enhanced line of *Ti*
3748.144	*Ti*	1	0	2	+0.001	−0.001	Enhanced line of *Ti*
3748.408	*Fe*	10	sh_2	2	+0.002	0.000	
3753.003	*Ti*	4	w_1	2	+0.002	0.000	
3757.824	*Cr–Ti*	4	3	2	+0.003	+0.001	Enhanced line of *Ti*
3759.447	*Ti*	12d?	$8sh_1$	2	+0.003	+0.001	Enhanced line of *Ti*
3760.679	*Fe*	4	w_1	2	+0.005	+0.003	
3762.012	*Ti*	3	$2w_1$	2	+0.004	+0.002	Enhanced line of *Ti*
3774.473	*Y*	3	$2w_1$	2	+0.001	−0.001	
3774.971	*Fe*	4	w_1	2	+0.004	+0.002	
3775.717	*Ni*	7	w_1	2	+0.005	+0.003	
3778.203	*Ni*	2	$3w_1$	2	+0.004	+0.002	
3778.939	*CN*	1	0–1	2	+0.002	+0.001	
3783.674	*Ni*	6	6	2	+0.006	+0.004	
3791.246	*CN*	0	0	2	+0.002	0.000	
3792.824	—	2	1	2	+0.009	+0.007	
3801.5	*CN*	2	$0–1w_1$	2	0.000	−0.002	
3806.357	*Fe–CN*	2	$1w_1$	3	+0.007	+0.005	
3815.038	*CN*	0	000	3	0.000	−0.002	
3817.059	*Co*	1	0	3	+0.005	+0.003	Enhanced line of *Co*
3818.759	*CN*	1	0–1	3	+0.004	+0.002	
3829.501	*Mg*	10	sh_3 w_2	3	−0.002	−0.004	
3832.450	*Mg*	15	sh_3 w_2	3	0.000	−0.002	
3834.364	*Fe*	10	$8sh_2$	3	+0.004	+0.002	
3836.229	*Ti*	2	1–2	4	+0.004	+0.002	Enhanced line of *Ti*
3836.66	*CN*	2d	1	4	+0.004	+0.002	
3842.191	*Co*	3	$2w_1$	4	+0.010	+0.008	
3843.854	*Co*,—	2d	1	4	+0.006	+0.004	Enhanced line of *Co*
3844.378	*CN*	4d?	$3w_1$	4	+0.002	0.000	
3846.131	*CN*	2	1	4	+0.005	+0.003	
3846.554	*Fe*	2	1–2	4	+0.005	+0.003	Enhanced line of *Fe*
3855.989	*V*	4	2	4	+0.006	+0.004	
3863.533	*CN*	3N	$2w_1$	4	0.000	−0.002	
3864.438	*CN*	3	$4w_2$	5	+0.004	+0.002	
3865.005	*V*	3Nd?	$2w_1$	5	−0.001	−0.003	
3866.960	*CN–V*	2	$1w_2$	5	+0.003	+0.001	Enhanced line of *V*
3868.539	*CN*	1	$0w_1$	6	0.000	−0.002	
3871.963	*Fe*	2	1–2	5	+0.008	+0.006	Enhanced line of *Fe*
3880.87	*CN*	2	$1–2w_2$	6	+0.004	+0.002	
3881.78	*CN*	2	$1w_2$	5	+0.003	+0.001	
3882.650	*CN*	1	0	5	−0.001	−0.003	
3895.119	*Co*	3	$4w_1$	6	+0.002	0.000	
3900.681	*Ti–Fe*	5	4–5	6	+0.008	+0.006	Enhanced line of *Ti*
3902.002	*Nd*–	3	1	5	+0.006	+0.004	
3904.926	*Ti*	3	2	6	+0.002	0.000	
3905.660	*Si*	12	$8sh_3$	6	+0.004	+0.002	Possibly enhanced line of *Cr* falls here
3906.044	*Nd*–	3N	1	6	+0.010	+0.008	
3913.609	*Ti*–	5d?	w_1	6	+0.006	+0.004	Enhanced line of *Ti*
3916.545	*V*	3	2–3	6	+0.008	+0.006	Enhanced line of *V*
3920.410	*Fe*	10	$9sh_2$	6	+0.006	+0.004	
3921.563	*Ti*	1	2	5	+0.002	0.000	
3921.695	*La*–	4	1	6	+0.008	+0.006	

λ	Substance	Intensity	Intensity at Limb	No. of Meas.	Δ	Δ′	Remarks
3924.673	*Ti*	4	4	6	+0.006	+0.004	
3926.597	*Mn*	2n	1	6	+0.009	+0.007	
3933.825	*Ca*	1000	w_3 sh_3	5	+0.001	−0.001	Central absorption line measured
3941.878	*Co*	3	w_1	6	+0.005	+0.003	
3944.160	*Al*	15	w_1 sh_3	4	+0.001	−0.002	
3948.818	*Ti, V*	4	4	5	−0.001	−0.003	
3950.102	*Fe*	5	$6w_1$	5	+0.006	+0.004	
3950.497	*Y*	2	2	5	+0.004	+0.002	
3954.002	*Fe*–	3	4	5	+0.005	+0.003	
3956.819	*Fe*	6	5	5	+0.007	+0.005	
3958.073	*Co*	2	$3w_1$	5	+0.002	0.000	
3961.674	*Al*	20	w_1 sh_3	5	+0.003	+0.001	
3962.995	*Ti*	3	3	5	+0.003	+0.001	
3968.625	*Ca*	700	w_3 sh_3	3	0.000	−0.002	Central absorption line measured
3977.891	*Fe*	6	5	5	+0.006	+0.004	
3984.294	*Mn*	2	$1–2w_1$	5	+0.005	+0.003	
3987.755	*Ti?*	2	$2–3w_1$	5	+0.006	+0.004	Enhanced line of *Ti*
3988.6	*La*	1	0	3	+0.002	0.000	
3994.828	*Nd?*	2	$1w_1$	6	+0.006	+0.004	
3995.463	*Co*	5	$6w_1$	6	+0.006	+0.003	
3995.899	*La*	1nd?	00	5	+0.002	0.000	
3996.140	*Fe*	3	3	6	+0.005	+0.003	
3997.258	*Cr? V*	1	1–2	6	+0.007	+0.005	
3998.790	*Ti*	4	4	6	+0.006	+0.004	
4003.012	*Ce–Fe–Ti*	3	3	6	+0.001	−0.001	
4005.408	*Fe*	7	$6sh_2$	6	+0.008	+0.006	
4005.856	*V*	3	$2–3w_1$	6	+0.005	+0.003	Enhanced line of *V*
4007.429	*Fe*	3	$4w_1$	6	+0.006	+0.004	
4012.541	*Ti, Ce*	4	$3w_1$	5	+0.004	+0.002	Enhanced line of *Ti*
4014.420	*Fe*	1	2	5	+0.005	+0.003	
4015.760	*Ni*	3	2–3	5	+0.006	+0.004	Enhanced line of *Ni*
4018.25	*Mn*	7	$6w_1$	5	+0.005	+0.003	
4022.018	*Ti–Fe–V*	5d?	4	5	+0.009	+0.007	
4023.533	*V, Co*	3	w_2	5	+0.005	+0.003	Enhanced line of *Co*
4024.726	*Ti*	3	4	5	+0.005	+0.002	
4025.286	*Ti–Ce*	3	3	5	+0.007	+0.005	Enhanced line of *Ti*
4028.497	*Ti–Ce*	4	3	5	+0.004	+0.002	Enhanced line of *Ti*
4034.644	*Mn*–	6d	$5sh_1$	5	+0.006	+0.004	
4035.883	*Mn*	4d?	3	5	+0.007	+0.005	
4041.803	*Zr*–	1	1	5	+0.005	+0.003	
4044.294	*K*	0	1	5	+0.005	+0.003	
4047.461	*Ce–Fe*	2	w_1	4	+0.007	+0.005	
4049.482	*Fe*	2	$2–3w_1$	4	+0.006	+0.004	
4053.981	*Cr–Fe–Ti*	3	w_1	4	+0.006	+0.004	Enhanced line of *Ti*
4055.701	*Mn–Fe*	6	w_1	4	+0.006	+0.004	Enhanced line of *Fe*
4058.748	*Ce*	0	0	4	+0.003	+0.001	
4059.081	*Mn*	3	w_2	4	+0.007	+0.005	
4060.415	*Ti*	1	1–2	3	+0.003	+0.001	
4061.244	*Nd*–	3	2	4	+0.007	+0.005	
4063.759	*Fe*	20	$18sh_3$	4	+0.008	+0.006	
4070.431	*Mn*	3	w_1	4	+0.006	+0.004	

λ	Substance	Intensity	Intensity at Limb	No. of Meas.	Δ	Δ′	Remarks
4083.095	*V–Mn*	4	w_1	4	+0.008	+0.005	
4086.861	*La*	1	0	4	+0.001	−0.001	
4095.094	*Ca ?*	4	$3–4w_1$	4	+0.004	+0.002	
4103.097	*Si, Mn*	5	4	4	+0.009	+0.007	
4105.318	*V*	2	1	4	+0.004	+0.002	
4111.509	*Ce ?*	1	0–1	4	+0.002	0.000	
4111.940	*V*	4	$3w_1$	4	+0.006	+0.004	
4112.869	*Ti*	1	2	4	+0.004	+0.002	
4115.330	*V*	3	w_1	4	+0.002	0.000	
4123.384	*La*	1	1	3	+0.001	−0.001	
4128.251	*Ce–V*, –	6d	$5w_1$	2	+0.006	+0.004	
4129.337	*Ce–*	3	2	2	+0.004	+0.002	
4130.804	*Ba*	2	0	2	+0.001	−0.001	
4137.809	*Ce*	1	0	4	+0.007	+0.005	
4144.038	*Fe*	15	$12sh_3$	4	+0.008	+0.006	
4165.759	–, *Ce*	1	1–2	4	+0.007	+0.005	
4167.884	*CN*	1N	0–1	4	+0.004	+0.002	
4171.213	*Ti*, –	4	3	5	+0.011	+0.009	
4172.066	*Ti, Fe*	2	$1–2w_1$	5	+0.007	+0.005	Enhanced line of *Ti*
4179.025	*Fe*	3	2	6	+0.009	+0.007	Enhanced line of *Fe*
4185.058	*Fe, Cr*	4	w_1	6	+0.006	+0.004	
4189.723	*CN*, –	2	w_1	6	+0.005	+0.003	
4196.699	*La*	2	1	6	0.000	−0.002	
4197.257	*CN*	2	2–3	6	+0.004	+0.002	
4202.198	*Fe*	8	$7sh_2$	6	+0.005	+0.003	
4203.730	*Cr*	2	$3w_2$	5	+0.005	+0.003	
4207.566	*CN*	1N	$0–1w_1$	6	+0.005	+0.003	
4212.801	*Cr ?*	3N	$2w_2$	6	+0.009	+0.007	
4216.136	*CN*	1	$1–2w_1$	6	+0.003	0.000	
4220.509	*Fe*	3	$4w_1$	6	+0.008	+0.006	
4226.904	*Ca*	20d ?	sh_1	6	+0.001	−0.001	
4232.887	*Fe*	2	3–4	6	+0.006	+0.004	
4233.328	*Mn–Fe*	4	3	6	+0.009	+0.007	Enhanced line of *Fe*
4233.772	*Fe*	6	$5sh_2$	6	+0.008	+0.006	
4238.070	*Fe*	5	4	6	+0.009	+0.007	
4240.872	*Cr*	1	1–2	6	+0.004	+0.002	
4254.505	*Cr*	8	sh_3	5	+0.005	+0.002	
4258.477	*Fe*	2	$4w_1$	5	+0.006	+0.004	
4260.640	*Fe*	10	sh_3	5	+0.004	+0.002	
4266.081	*Mn*	2	1–2	5	+0.006	+0.004	
4271.934	*Fe*	15	sh_3	5	+0.006	+0.004	
4274.958	*Cr*	7d ?	$6sh_2$	5	+0.005	+0.003	
4281.530	*Ti*	0	1	5	+0.004	+0.002	
4282.565	*Fe*	5	$4–5sh_1$	5	+0.007	+0.005	
4283.169	*Ca*	4	$5sh_1$	5	+0.009	+0.007	
4284.382	*Cr*	2Nd ?	0–1	5	+0.011	+0.009	Enhanced line of *Cr*
4287.566	*Ti*	1	$1–2w_1$	5	+0.006	+0.004	
4288.310	*Ti, Fe*	1	w_1	5	+0.008	+0.006	
4289.525	*Ca*	4	$4–5w_1$	5	+0.005	+0.003	
4289.885	*Cr*	5	sh_2	5	+0.007	+0.004	
4290.377	*Ti*	2	1–2	5	+0.007	+0.005	Enhanced line of *Ti*
4291.630	*Fe*	2	$3w_2$	5	+0.007	+0.005	
4294.936	*Zr*	2	2	5	+0.006	+0.004	

λ	Substance	Intensity	Intensity of Limb	No. of Meas.	Δ	Δ′	Remarks
4299.149	*Ca*	3	3	5	+0.006	+0.004	
4300.211	*Ti*	3	2	5	+0.008	+0.005	Enhanced line of *Ti*
4302.085	*Ti*	2	1	5	+0.012	+0.010	Enhanced line of *Ti*
4302.692	*Ca*	4	4–5sh$_1$	5	+0.005	+0.002	
4313.034	*Ti*	3	2	5	+0.004	+0.002	Enhanced line of *Ti*
4314.248	*Sc*	3	w$_1$	5	+0.004	+0.002	
4315.262	*Fe*	4	4	5	+0.008	+0.006	
4316.962	*Ti?*	1	0	5	+0.008	+0.006	Enhanced line of *Ti*
4318.817	*Ca, Mn?*	4	4–5sh$_1$	6	+0.005	+0.003	
4320.907	*Sc*	3	w$_2$	6	+0.003	+0.001	
4325.152	*Sc*	4	3	6	+0.004	+0.002	
4325.939	*Fe*	8	9sh$_3$	6	+0.003	+0.001	
4328.080	*Fe*	2	1–2	6	+0.003	+0.001	
4330.866	*Ti, Ni*	2	1	6	+0.008	+0.005	Enhanced line of *Ti*
4337.216	*Fe*	5	sh$_1$	6	+0.008	+0.006	
4338.084	*Ti*	4	3w$_1$	6	+0.004	+0.002	Enhanced line of *Ti*
4340.634	*H*	20N	15sh$_2$	6	+0.002	0.000	*Hγ* narrower at limb
4341.530	*Ti?*	2	1–2	6	+0.005	+0.003	Enhanced line of *Ti*
4344.451	*Ti–*	2	1–2	6	+0.005	+0.002	Enhanced line of *Ti*
4344.670	*Cr*	4	4–5	6	+0.005	+0.003	
4352.083	*Mg*	5Nd?	4sh$_2$	6	+0.004	+0.002	
4352.908	*Fe*	4	4	6	+0.006	+0.004	
4355.257	*Ca?*	2	1	6	+0.003	+0.001	
4376.107	*Fe*	6	6	10	+0.005	+0.003	
4378.419	—	2Nd?	w$_1$	10	+0.004	+0.002	
4379.396	*V*	4	3w$_2$	10	+0.005	+0.002	
4380.325	*Co*	2Nd?	1–2w$_2$	10	+0.004	+0.002	
4385.548	*Fe*	2	0	10	+0.013	+0.010	Enhanced line of *Fe*
4387.007	*Ti?*	1	0–1	8	+0.009	+0.007	Enhanced line of *Ti*
4387.220	—	1N	0–1	8	+0.006	+0.003	Possibly enhanced line of *Pb*
4395.201	*Ti*	3	2–3	8	+0.007	+0.005	Enhanced line of *Ti*
4399.935	*Ti, Cr*	3	2	8	+0.006	+0.004	Enhanced line of *Ti*
4400.555	*Sc*	3	w$_2$	8	+0.004	+0.002	
4415.722	—	3	2–3w$_2$	10	+0.006	+0.004	
4417.884	*Ti–*	3	w$_2$	10	+0.008	+0.005	Enhanced line of *Ti*
4425.608	*Ca*	4	sh$_1$	10	+0.004	+0.002	
4427.266	*Ti*	2	1–2	8	+0.004	+0.001	
4430.785	*Fe*	3	3	8	+0.008	+0.006	
4435.129	*Ca*	5	4sh$_1$	8	+0.006	+0.004	
4435.851	*Ca*	4	sh$_1$	8	+0.006	+0.004	
4441.881	*V–*	3Nd?	2–3w$_1$	8	+0.007	+0.005	
4442.510	*Fe*	6	5sh$_2$	8	+0.006	+0.004	
4443.365	*Fe*	3	w$_2$	8	+0.007	+0.005	
4443.976	*Ti*	5	4	8	+0.008	+0.005	Enhanced line of *Ti*
4447.892	*Fe*	6	5sh$_1$	8	+0.005	+0.003	
4449.313	*Ti*	2	1–2w$_1$	8	+0.005	+0.003	
4450.654	*Ti?*	2	2	8	+0.008	+0.006	Enhanced line of *Ti*
4453.486	*Ti*	2	w$_1$	8	+0.004	+0.002	
4454.552	*Fe*	3	3	8	+0.007	+0.004	
4456.794	*Ca*	2	2–3	8	+0.005	+0.003	
4461.818	*Fe*	4	5w$_1$	7	+0.007	+0.005	
4464.617	*Ti?*	2	w$_2$	7	+0.007	+0.005	Enhanced line of *Ti*

λ	Substance	Intensity	Intensity at Limb	No. of Meas.	Δ	Δ′	Remarks
4468.663	*Ti*–	5	4	7	+0.007	+0.005	Enhanced line of *Ti*
4469.316	*Ti*	1	1–2w$_1$	7	+0.004	+0.001	
4470.648	*Ni–Zr*	2	w$_2$	7	+0.007	+0.004	
4482.3	*Fe*, –	8	10w$_2$	6	+0.004	+0.002	
4489.911	*Fe*	4	6w$_2$	6	+0.007	+0.005	
4491.570	*Fe*	2	1w$_1$	6	+0.010	+0.008	Enhanced line of *Fe*
4494.738	*Fe*	6	5sh$_1$	6	+0.010	+0.007	
4497.023	*Cr*	3	w$_1$	6	+0.005	+0.003	
4501.445	*Ti*,–	5	4	6	+0.007	+0.005	Enhanced line of *Ti*
4508.455	*Fe ?*, –	4	3	6	+0.011	+0.008	Enhanced line of *Fe*
4512.906	*Ti*	3	4	6	+0.004	+0.002	
4515.508	*Fe*	3	2	6	+0.011	+0.008	Enhanced line of *Fe*
4518.198	*Ti*	3	4	6	+0.004	+0.001	
4520.397	*Fe ?*, –	3	2	6	+0.010	+0.008	Enhanced line of *Fe*
4522.802	*Fe*	3	1	6	+0.010	+0.008	Enhanced line of *Fe*
4522.974	*Ti*	2	2–3	6	+0.005	+0.003	
4527.101	*Ca ?*	3	w$_2$	6	+0.007	+0.004	
4527.490	*Ti*	3	2–3	6	–0.004	–0.006	
4528.798	*Fe*	8	7sh$_2$	6	+0.005	+0.003	
4531.327	*Fe*	5	6	6	+0.007	+0.005	
4533.419	*Ti*	4	5	6	+0.005	+0.003	
4534.139	*Ti–Co*	6	5	6	+0.007	+0.004	Enhanced line of *Ti*
4534.953	*Ti*	4	w$_1$	6	+0.004	+0.001	
4536.094	*Ti*	2	1–2	5	+0.004	+0.001	
4546.129	*Cr*	3	4w$_1$	5	+0.007	+0.005	
4548.024	*Fe*	3	w$_1$	5	+0.008	+0.006	
4548.938	*Ti*	2	2–3w$_1$	5	+0.005	+0.003	
4549.642	*Fe*	2	0	5	+0.008	+0.006	Enhanced line of *Fe*
4549.808	*Ti–Co*	6d ?	5	5	+0.005	+0.003	Enhanced line of *Ti*
4554.211	*Ba*	8	w$_1$	5	+0.008	+0.006	
4555.162	*Cr*	2	1	5	+0.009	+0.007	Enhanced line of *Cr*
4555.662	*Ti*	3	4w$_1$	5	+0.005	+0.003	
4556.063	*Fe*	3	1	5	+0.011	+0.008	Enhanced line of *Fe*
4558.827	*Cr ?*	3	2	5	+0.009	+0.007	Enhanced line of *Cr*
4563.939	*Ti*	4	3–4	5	+0.008	+0.006	Enhanced line of *Ti*
4571.275	*Mg*	5	6w$_1$	5	+0.007	+0.005	
4572.156	*Ti*–	6	w$_1$	5	+0.007	+0.005	Enhanced line of *Ti*
4576.512	*Fe*	2	1	5	+0.009	+0.007	Enhanced line of *Fe*
4584.018	*Fe*–	4	2	5	+0.012	+0.009	Enhanced line of *Fe*
4586.047	*Ca*	4	3	5	+0.006	+0.003	
4586.552	*V*	1	0–1w$_1$	5	+0.007	+0.004	
4588.381	*Cr*	3	2–3	5	+0.010	+0.008	Enhanced line of *Cr*
4590.126	*Ti*	3	2–3	5	+0.008	+0.006	Enhanced line of *Ti*
4592.707	*Ni*	2	1–2	5	+0.005	+0.003	
4600.541	*Ni*	2	1–2	5	+0.006	+0.004	
4605.171	*Ni*	3	2	5	+0.009	+0.006	
4607.510	*Sr*	1	0–1w$_1$	5	+0.001	–0.001	
4679.409	*Ni*	2N	1–2w$_1$	2	+0.008	+0.006	Enhanced line of *Ni*
4680.317	*Zn*	1	0	2	+0.005	+0.003	
4682.088	*Ti*	3	4	2	+0.006	+0.004	
4686.395	*Ni*	3	2	2	+0.011	+0.008	
4703.177	*Mg*	10	8sh$_2$	2	+0.008	+0.006	
4703.994	*Ni*	3	2	2	+0.010	+0.007	

λ	Substance	Intensity	Intensity at Limb	No. of Meas.	Δ	Δ′	Remarks
4708.846	*Ti*	2	2	2	+0.008	+0.005	Enhanced line of *Ti*
4714.5	—*Ni*	7	5sh$_1$	2	+0.012	+0.010	
4715.946	*Ni*	4	4	2	+0.008	+0.005	
4722.342	*Zn*	3	2–3	2	+0.009	+0.007	
4729.864	*Fe? Cr*	1	1	2	+0.006	+0.004	
4733.779	*Fe*	4	w$_1$	2	+0.010	+0.007	
4736.963	*Fe*	6	5–6sh$_1$	2	+0.008	+0.006	
4737.817	*Fe?*	1	1–2	2	+0.004	+0.002	
4741.718	*Fe*	3	w$_1$	2	+0.008	+0.006	
4745.092	*Fe*	4	3–4	2	+0.009	+0.007	
4752.613	*Ni*	3	w$_1$	2	+0.006	+0.004	
4754.225	*Mn*	7	w$_1$	2	+0.006	+0.004	
4756.300	*Cr*	2	1	2	+0.008	+0.006	
4756.705	*Ni*	3	2–3	2	+0.008	+0.005	
4762.567	*Mn*	5	4–5sh$_1$	2	+0.007	+0.005	
4766.621	*Mn*	4	w$_1$	4	+0.007	+0.004	
4780.169	*Ti, Co*	2	1–2	4	+0.007	+0.004	Enhanced line of *Ti*
4783.613	*Mn*	6	w$_1$	4	+0.007	+0.005	
4787.003	*Fe*	2	2	4	+0.008	+0.005	
4789.849	*Fe*	3	w$_1$	4	+0.007	+0.005	
4810.724	*Zn*	3	2	4	+0.006	+0.004	
4823.697	*Mn*	5	w$_1$	4	+0.008	+0.005	
4841.074	*Ti*	3	3–4w$_1$	4	+0.004	+0.002	
4859.928	*Fe*	4	3–4	8	+0.009	+0.006	
4871.512	*Fe*	5	4	8	+0.009	+0.007	
4876.060	*Fe*	2	2	8	+0.007	+0.004	
4883.867	*Yt*	2	2	8	+0.004	+0.002	
4885.620	*Fe*	3	w$_1$	8	+0.009	+0.007	
4886.522	*Fe*	3	3	8	+0.008	+0.005	
4913.803	*Ti*	2	w$_1$	6	+0.005	+0.003	
4919.174	*Fe*	6	5	6	+0.009	+0.006	
4924.107	*Fe*	5	3–4	6	+0.009	+0.006	Enhanced line of *Fe*
4924.956	*Fe*	3	w$_1$	6	+0.008	+0.006	
4934.2	*Ba–Fe*	6	w$_1$	6	+0.014	+0.011	
4946.568	*Fe*	3	2–3	6	+0.009	+0.007	
4981.912	*Ti*	4	4–5	6	+0.006	+0.003	
4991.247	*Ti*	3	2–3	6	+0.004	+0.001	
4994.316	*Fe*	3	w$_1$	6	+0.008	+0.006	
4999.689	*Ti, La*	3	w$_1$	6	+0.006	+0.004	
5018.629	*Fe*	4	3	6	+0.008	+0.006	Enhanced line of *Fe*
5023.052	*Ti*	2	w$_1$	6	+0.006	+0.004	
5041.069	*Fe*	3d?	2	6	+0.006	+0.004	
5041.795	*Ca*	2	1	6	+0.003	+0.001	
5060.258	*Fe*	3	3	6	+0.010	+0.008	
5064.836	*Ti*	3	w$_1$	6	+0.008	+0.006	
5083.518	*Fe*	4	5w$_1$	9	+0.009	+0.006	
5084.279	*Ni*	3	2–3	9	+0.008	+0.005	
5087.601	*Y?*	1	0–1	5	+0.006	+0.003	
5097.175	*Fe, Cr*	3	2	5	+0.010	+0.008	Enhanced line of *Fe*
5103.142	*Ni*	1	2w$_1$	5	+0.005	+0.003	
5110.574	*Fe*	5d	5w$_1$	5	+0.012	+0.009	
5129.336	*Ti?*	3	2–3	5	+0.008	+0.006	Enhanced line of *Ti*
5131.942	*Ni*	1	w$_1$	5	+0.007	+0.004	

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5137.250	*Ni, Cr*	3	w_2	5	+0.007	+0.004	
5139.644	*Fe*	4	$3sh_1$	5	+0.007	+0.004	
5152.087	*Fe*	3	$3–4w_1$	5	+0.007	+0.005	
5154.244	*Ti–Co*	2	2	5	+0.007	+0.005	Enhanced line of *Ti*
5155.035	*Ni*	2	$1–2w_1$	5	+0.008	+0.005	
5159.231	*Fe*	2	1–2	3	+0.006	+0.004	
5166.454	*Cr–Fe*	3	$4w_1$	3	+0.012	+0.010	
5167.497	*Mg*	15	$20sh_3$	3	+0.002	−0.001	
5172.856	*Mg*	20	$25sh_3$	3	+0.001	−0.002	
5173.917	*Ti*	2	2–3	3	+0.006	+0.003	
5183.791	*Mg*	30	$35sh_3$	3	+0.001	−0.001	
5188.863	*Ti*	2	2	3	+0.004	+0.001	Enhanced line of *Ti*
5189.018	*Ca*	3	2	3	+0.008	+0.006	
5193.139	*Ti*	2	2–3	3	+0.005	+0.003	
5195.113	*Fe*	4	$4–5w_1$	3	+0.008	+0.005	
5197.743	—	2	1	3	+0.010	+0.007	Strong chromo-spheric line
5198.888	*Fe*	3	w_1	4	+0.007	+0.005	
5208.596	*Cr*	5	sh_1	4	+0.006	+0.004	
5210.555	*Ti*	3	3	5	+0.007	+0.004	
5225.695	*Fe*	2	$3w_1$	6	+0.008	+0.005	
5226.707	*Ti–*	2	1–2	6	+0.009	+0.007	Enhanced line of *Ti*
5234.791	—	2	1	6	+0.012	+0.010	Strong chromo-spheric line
5237.493	*Cr?*	1	0–1	6	+0.008	+0.005	Enhanced line of *Cr*
5247.737	*Cr*	2	$3w_1$	6	+0.007	+0.004	
5250.385	*Fe*	2	3	6	+0.008	+0.005	
5260.561	*Ca*	0	$0–1w_1$	4	+0.004	+0.001	
5265.720	*Ca*	3	3	7	+0.005	+0.003	
5284.281	*Ti*	1	0	9	+0.012	+0.009	Probably not *Ti*
5288.705	*Fe*	2	w_2	9	+0.009	+0.006	
5298.455	*Cr*	4	$5w_1$	9	+0.008	+0.005	
5316.790	*Fe*	4	2	9	+0.012	+0.009	Enhanced line of *Fe*
5333.089	*Fe*	4	$4–5w_1$	9	+0.010	+0.008	
5336.974	*Ti*, —	4	3–4	9	+0.011	+0.008	Enhanced line of *Ti*
5342.890	*Co*	1	0–1	9	+0.007	+0.004	
5348.511	*Cr*	4	w_1	9	+0.008	+0.005	
5349.653	*Ca*	4	4	9	+0.007	+0.004	
5367.669	*Fe*	6	$5sh_1$	11	+0.008	+0.006	
5377.800	*Mn*	2N	$1–2w_2$	11	+0.007	+0.004	
5381.221	*Ti*	2	2	11	+0.009	+0.006	Enhanced line of *Ti*
5394.87	*Mn*	2	$3w_2$	11	+0.008	+0.006	
5400.711	*Fe*	3	2	11	+0.006	+0.004	
5405.089	*Fe*	6	sh_1	11	+0.009	+0.007	
5410.000	*Cr*	4	4–5	11	+0.007	+0.004	
5420.56	*Mn*	1	$2w_2$	11	+0.005	+0.002	
5424.290	*Fe*	6	sh_1	11	+0.007	+0.004	
5429.911	*Fe*	6d?	$6–7sh_1$	11	+0.009	+0.006	
5432.753	*Mn*	1Nd?	$2w_2$	11	+0.007	+0.005	
5433.160	*Fe*	2	1–2	11	+0.008	+0.006	
5434.740	*Fe*	5	5–6	11	+0.010	+0.007	
5446.797	*Ti*	2	$1–2w_1$	11	+0.006	+0.004	
5447.130	*Fe*	6d?	sh_1	11	+0.010	+0.007	

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5455.671	*Fe?*	2	$1w_2$	10	+0.006	+0.004	
5455.834	*Fe*	4	$4-5w_1$	10	+0.008	+0.005	
5470.84	*Mn*	1	$2w_2$	10	+0.007	+0.004	
5477.123	*Ni*	5	4–5	9	+0.010	+0.007	
5483.566	*Co*	1d?	$1-2w_2$	10	+0.010	+0.007	
5497.735	*Fe*	5	5–6	9	+0.012	+0.009	
5501.683	*Fe*	5	$5-6w_1$	9	+0.010	+0.007	
5507.000	*Fe*	5	$5-6w_1$	9	+0.011	+0.008	
5512.741	*Ti*	2	w_1	7	+0.009	+0.006	
5514.753	*Ti*	2	2	7	+0.008	+0.005	
5528.641	*Mg*	8	sh_2	8	+0.008	+0.005	
5569.848	*Fe*	6	$5sh_1$	10	+0.011	+0.008	
5576.320	*Fe*	4	3	10	+0.011	+0.008	
5582.198	*Ca*	4	5	8	+0.007	+0.004	
5586.991	*Fe*	7	$6sh_1$	8	+0.010	+0.007	
5588.985	*Ca*	6	sh_1	8	+0.007	+0.005	
5590.343	*Ca*	3	3	8	+0.007	+0.004	
5594.691	*Ca*	4	3–4	7	+0.009	+0.006	
5594.884	*Fe*	1	0	7	+0.015	+0.012	
5598.711	*Ca*	4	4	5	+0.006	+0.003	
5601.505	*Ca*	3	3	5	+0.008	+0.005	
5615.877	*Fe*	6	$5sh_1$	5	+0.009	+0.006	
5682.869	*Na*	5	$7w_1$	6	+0.007	+0.004	
5684.710	*Si*	3	$2w_1$	6	+0.010	+0.008	
5688.436	*Na*	6	$7w_1$	6	+0.006	+0.003	
5690.646	*Si*	3	$2w_1$	6	+0.008	+0.005	
5701.323	*Si*	1N	w_1	6	+0.007	+0.004	
5708.622	*Si*	3N	2	6	+0.009	+0.006	
5709.775	*Ni*	5	4–5	6	+0.012	+0.009	
5727.271	*Ti–V*	2N	$2-3w_1$	4	+0.004	+0.001	
5772.364	*Si*	3	$2-3w_1$	6	+0.008	+0.005	
5798.077	—	3	2–3	5	+0.009	+0.006	
5853.902	*Ba?*	5	$6w_1$	7	+0.006	+0.003	
5857.976	*Ni*	3	$2w_1$	6	+0.011	+0.008	
5862.582	*Fe*	6	6	6	+0.009	+0.006	
5866.675	*Ti*	3	$3-4w_2$	6	+0.007	+0.004	
5890.186	*Na*	30	$50sh_3$	7	+0.001	−0.002	
5893.097	*Ni*	4d?	3–4	8	+0.012	+0.009	
5896.155	*Na*	20	$40sh_3$	8	+0.001	−0.002	
5934.881	*Fe*	5	5	6	+0.011	+0.008	
5948.765	*Si*	6	5–6	6	+0.010	+0.007	
5953.386	*Ti*	1	w_1	6	+0.006	+0.003	
5956.923	*Fe*	4	4–5	6	+0.010	+0.007	
5977.007	*Fe*	4	4	6	+0.010	+0.007	
6008.785	*Fe*	6	w_1	7	+0.011	+0.008	
6013.715	*Mn*	6	$6-7w_2$	7	+0.007	+0.004	
6016.861	*Mn*	6	w_2	7	+0.007	+0.004	
6020.401	*Fe*	4	3	7	+0.013	+0.010	
6022.016	*Mn*	6	w_2	7	+0.007	+0.004	
6024.281	*Fe*	7	7	7	+0.011	+0.008	
6042.315	*Fe*	3	w_1	7	+0.010	+0.007	Enhanced line of *Fe*
6065.709	*Fe*	7	7	5	+0.012	+0.009	
6079.227	*Fe*	2	1–2	5	+0.012	+0.009	Enhanced line of *Fe*

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6082.930	*Fe*	1	1	5	+0.010	+0.007	
6102.392	*Fe*	6	$5w_2$	4	+0.013	+0.010	
6102.937	*Ca*	9	10	5	+0.008	+0.005	
6122.434	*Ca*	10	12	5	+0.007	+0.004	
6136.829	*Fe*	8	7	5	+0.013	+0.010	
6141.938	*Fe, Ba*	7	9	5	+0.004	+0.001	
6149.458	*Fe*	2	1	5	+0.013	+0.010	Enhanced line of *Fe*
6151.834	*Fe*	4	4	5	+0.012	+0.009	
6154.438	*Na*	2	2–3	5	+0.011	+0.008	
6160.956	*Na*	3	4	7	+0.012	+0.008	
6162.390	*Ca*	15	sh_1	7	+0.008	+0.005	
6166.651	*Ca*	5	5	7	+0.009	+0.006	
6169.249	*Ca*	6	6	7	+0.010	+0.006	
6169.778	*Ca*	7	7	6	+0.009	+0.006	
6173.553	*Fe*	5	w_1	7	+0.014	+0.011	
6175.584	*Ni*	3	2–3	7	+0.011	+0.008	
6177.027	*Ni*–	5	4	7	+0.011	+0.007	
6191.393	*Ni*	6	6	7	+0.011	+0.008	
6191.779	*Fe*	9	$8w_2$	7	+0.014	+0.010	
6213.644	*Fe*	6	w_2	7	+0.014	+0.011	
6230.943	*V–Fe*	8	8	9	+0.011	+0.008	
6238.598	*Fe*	2	1	8	+0.012	+0.009	Enhanced line of *Fe*
6246.535	*Fe*	8	7–8	12	+0.014	+0.011	
6247.774	*Fe*	2	1	12	+0.014	+0.011	Enhanced line of *Fe*
6252.773	*Fe*	7	w_1	12	+0.013	+0.010	
6258.322	*Ti*	2	$2–3w_1$	12	+0.007	+0.004	
6258.927	*Ti*	3	$3–4w_1$	12	+0.008	+0.005	
6265.348	*Fe*	5	w_2	12	+0.013	+0.010	
6301.718	*Fe*	7	6	11	+0.012	+0.009	
6302.709	*Fe*	5	5	11	+0.012	+0.009	
6318.239	*Fe*	6	5–6	11	+0.014	+0.010	
6337.048	*Fe*	7	7	11	+0.012	+0.009	
6380.958	*Fe*	4	4	6	+0.008	+0.005	
6408.233	*Fe*	5	4	9	+0.012	+0.008	
6417.133	*Fe?*	1	0	9	+0.012	+0.009	Enhanced line of *Fe*
6420.169	*Fe*	4	3	9	+0.010	+0.007	
6431.066	*Fe*	5	$5w_1$	9	+0.012	+0.008	
6439.293	*Ca*	8	8	9	+0.006	+0.003	
6450.033	*Ca*	6	6	9	+0.006	+0.003	
6455.820	*Ca*	2	2	9	+0.008	+0.005	
6456.603	*Fe*	3	1	9	+0.013	+0.010	Enhanced line of *Fe*
6494.004	*Ca*	6	$5–6w_1$	9	+0.007	+0.003	
6495.213	*Fe*	8	6	9	+0.013	+0.009	
6497.128	*Fe*	4	5	9	+0.005	+0.002	
6499.168	*Fe*	1	1–2	9	+0.008	+0.005	
6499.880	*Ca*	4	$4w_1$	9	+0.008	+0.005	
6516.311	*Fe*	2	0	9	+0.013	+0.010	Enhanced line of *Fe*
6546.479	*Ti–Fe*	6	5	9	+0.011	+0.007	
6563.045	*H*	40	$60w_3sh_2$	8	+0.002	−0.002	Width of $H\alpha$ at limb = 1.15 Ångströms
6569.460	*Fe*	5	4	9	+0.010	+0.007	
6573.030	*Ca?*	1	1–2	9	+0.008	+0.005	

show no appreciable displacement. The other lines of calcium, sodium, and magnesium show displacements, which, though varying considerably among themselves, are, as a rule, smaller than those for most of the other elements.

2. The displacements for titanium, vanadium, and scandium are considerably smaller than those for iron or nickel.

3. The lines of the elements of very high atomic weight, such as lanthanum and cerium, show very small displacements.

4. The lines of the cyanogen flutings are shifted by small but appreciable amounts.

5. One or two lines in the list are shifted unmistakably toward the violet. The most pronounced case of this sort is λ 4527.490 of titanium.

6. The lines which are considerably strengthened at the limb usually show small displacements.

7. The enhanced lines as a class show decidedly larger shifts than do the arc lines, especially in the more refrangible part of the spectrum. The enhanced lines of titanium are shifted less than those of iron, the latter giving the largest average displacements of any lines in the list.

It is hardly necessary to comment extensively on (1) and (2) in this list, except so far as to call attention to the fact that they point to a low-level cause for the displacements. The lines enumerated under (1) are known to rise to the highest level of any in the solar spectrum, and even the other lines due to these elements, as well as those of titanium, vanadium, and scandium, are known from chromospheric observations to lie at an appreciably higher average level than those of iron, for example. The cause producing the observed displacements accordingly must be inferred to be most effective at the lower levels.

The point referred to under (3), that the elements of very high atomic weight show small shifts, is of decided interest, especially in view of the fact that laboratory results seem to indicate that pressure-shifts increase in general with the atomic weight of the element. A very simple explanation, however, presents itself. The lines of these elements are greatly weakened at the limb, in some cases being almost obliterated. It is known that the vapor giving rise to these lines

lies in a thin layer close to the solar photosphere. At the sun's limb the light from this low-lying layer is probably considerably scattered in the course of the long path traversed through the overlying gases, as well as actually cut off in large part by the higher portions of the photosphere, which, seen in projection at the limb, act as a shield, shutting off the view of all below their own level. A similar explanation would account in a simple way for the disappearance of the wings from most of the heavy winged lines at the sun's limb. In such a case it is evident that if a part of the layer of gas producing, for example, a lanthanum line, is concealed from view, the path of the light from the remainder might be no longer, and, in fact, could be shorter, than at the center of the sun. For an equal path we should expect practically no displacement, and this is essentially what is found for the average of the lines of lanthanum and cerium which have been measured.

The displacements of the lines in the cyanogen fluting have been discussed previously. In addition it should be stated that some of the lines in the ultra-violet fluting are probably blended with the lines of elements which show marked displacements, and that this may account for the abnormally large values given by certain of the lines. This region is so extraordinarily rich in lines that even with very high dispersion cases of this kind are almost certain to occur. The same is probably true of the line λ 4207.566 ascribed by Rowland to *CN*. There can be little doubt that this line is a blend, its width being much too great for a single line of its intensity.

The greater number of negative displacements shown in the list are without doubt to be regarded as due simply to errors of measurement. The titanium line at λ 4527.490, however, is unquestionably an actual case of a negative displacement. Measures on six plates have given for this line

$$-0.005, \ -0.006, \ -0.007, \ -0.003, \ -0.003, \ -0.003 \text{ Ångström.}$$

In the arc spectrum it is a line of medium intensity, and, so far as I know, is abnormal in but one respect. Under the influence of a magnetic field it is separated into seven components, of which four are produced by vibrations in a plane perpendicular to the lines of force, and the other three by vibrations parallel to the lines of force. In

the spectrum of a spark under pressure its behavior is entirely normal, the displacement being toward the red and about equal numerically to the shifts of the other lines in its vicinity. At present there seems to be no adequate explanation for the behavior of this line. If magnetic effects influenced the spectrum at the center or the limb of the sun to any considerable extent, a reasonable conclusion would be that, owing to a difference of polarization, different components might be involved in the two regions, and so an apparent shift of the line might be introduced. Visual and photographic experiments with a Nicol prism and rhomb indicate, however, that if any such magnetic effects are present they are too minute to be detected with the dispersion employed.

The conclusion given under (6), that the lines most strengthened at the limb usually show small displacements, is based rather on the means of groups of lines than upon separate cases, since there are numerous individual exceptions to it. On the whole, however, the tendency seems to be well marked. Among iron lines especially noticeable cases are λ 4482.3 and λ 6497.128, which are greatly strengthened and show very small shifts. The lines λ 4232.887, λ 4258.477, λ 4291.630, λ 4461.618, and λ 4489.911 are all instances of iron lines which are decidedly strengthened and which show moderate shifts. The whole question is greatly complicated by the general widening of the lines and the difficulty of estimating actual increases of intensity, as well as by the fact that we have little knowledge of the displacements of most of these lines under pressure in the laboratory. It seems probable from comparisons of the strengthened and the weakened lines at the limb with those in sun-spots, that the principal changes in intensity (especially for the strengthened lines) are to be explained on the basis of temperature. Since the lowest temperature is probably to be found in the higher portions of the sun's atmosphere, the lines showing marked strengthening are to be regarded as high-level lines. For such lines the increase in the length of path in the upper layers would be of great relative importance, and might result in an increase of intensity. For the same reason we should expect their displacements to be small. The behavior of known high-level lines, such as $H\alpha$, the D lines of sodium, and the b lines of magnesium, all of which lose their wings at the

limb and are decidedly strengthened, lends considerable weight to this hypothesis.

Reference should also be made at this point to the apparent tendency on the part of the strong winged lines of iron to give smaller displacements than would be expected. This effect is by no means proven, but there is sufficient evidence to furnish a considerable presumption in its favor. If present it is probably due to the fact that, as stated in the discussion of (3), a portion of the iron vapor, which lies lowest in the solar atmosphere and which produces the wings on these lines at the center of the sun, is concealed from view at the limb. This gas is under the greatest pressure and its obscuration tends to reduce the total amount of displacement at the limb.

The results found for the enhanced lines form perhaps the single most important product of this investigation. The fact that these lines as a class give the largest displacements of any in the spectrum has proved the more unexpected as it is directly opposed to the conclusion of Halm,[1] derived through measures of the enhanced line of iron at λ 6516.311. The result of Halm's measures on this line was a shift of -0.002 Ångström, an amount which he considered practically negligible. My own observations, based on nine measures, give $+0.013$ Ångström, one of the largest displacements found in this part of the spectrum. I am quite unable to account for this discrepancy. My results for the other enhanced lines in this region, however, confirm the larger value. Thus we have:

λ	Δ
6238.598	+0.012
6247.774	+0.014
6456.603	+0.013
6417.133	+0.012

A total of 85 enhanced lines is included in the results of these measures. Of these, 45 are due to titanium and 27 to iron, with the remainder divided among chromium, vanadium, and other elements. In each case the mean displacement for a group of these lines averages higher than that for an equal number of arc lines of the same element. Toward the less refrangible end of the spectrum, however, the difference becomes less marked, until in the red, in the

[1] *Astronomische Nachrichten*, **173**, 273, 1907.

case of iron at least, the values become essentially equal. More complete numerical data will be given in connection with the discussion of the variation of the displacements with wave-length, but a brief analysis of the region λ 4400–λ 4600, which is very rich in enhanced lines, will perhaps be of interest at this point. For this region we find as the average displacements:

12 arc lines of *Ti*	+0.004
11 enhanced lines of *Ti*	+0.007
12 arc lines of *Fe*	+0.007
9 enhanced lines of *Fe*	+0.010

It is noteworthy that the enhanced lines of titanium differ from the enhanced lines of iron by just the same amount as the arc lines of the two elements.

There appears to be a general tendency for the displacements of the enhanced lines to increase with the degree of enhancement in the spark spectrum. This effect is hardly more than roughly indicated, but it probably deserves a word of comment. It seems to hold especially for the enhanced lines which either are not seen at all, or appear as mere traces in the arc spectrum. Thus, if we take Lockyer's list of enhanced lines in the iron spectrum between λ 3846 and λ 5316, we find that ten lines out of eighteen are given as zero or fainter in the arc spectrum. The mean displacement for these ten lines is +0.010 Ångström as against +0.008 Ångström for the other eight lines. An especially noteworthy case is λ 4385.548, which is given by Lockyer as a "trace" in the arc spectrum. This line shows a shift of +0.013 Ångström, the largest value obtained from any line to the violet of λ 5500. The titanium lines show a very similar behavior, except that the number of enhanced lines which are very faint in the arc spectrum is relatively considerably smaller than for iron. The whole question is a most interesting one, but more evidence is needed, both on the solar and on the laboratory side, to make adequate discussion possible. Reference should, however, be made to the fact that there seems to be a well-defined relationship between the amount of enhancement of these lines in the spark spectrum and the amount of their weakening in the spectrum of sun-spots.[1] There can be little doubt that both of these effects

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 40; *Astrophysical Journal*, **30**, 86, 1909.

are intimately connected with the origin of these lines in the solar spectrum.

The level occupied by the enhanced lines in the solar atmosphere has been a subject of considerable discussion. The fact that they appear in the "flash" and chromospheric spectrum with an intensity out of proportion to the intensity of the dark lines which correspond to them in the general solar spectrum, is at present hardly questioned seriously. Quite a different interpretation of this result is given by different observers, however. Thus Lockyer and Fowler regard the enhanced lines as chiefly restricted to the higher levels of the chromosphere, while Evershed and some others conclude that there is no essential difference in their relative intensities throughout the entire depth of the chromospheric layer. A most interesting theory is advanced by Evershed to account for the differences of relative intensity of the enhanced lines in the dark-line solar spectrum and in the "flash" spectrum.[1] In accordance with his view, the enhanced lines find their origin chiefly in the jets of intensely hot gas which are found ascending in a radial direction all over the surface of the sun. These regions would correspond more closely to the conditions present in the electric spark, while the cooler descending gases would better represent those present in the arc. At the sun's limb, accordingly, the light from the radiations characteristic of the very hot gases (the enhanced lines) would traverse the long path through the other cooler gases with little diminution of intensity. The cooler gases would, however, tend to neutralize that portion of the more intense spectrum of the hot gases which is common to the temperatures of both. Of the spectrum at the center of the sun Evershed writes:

> The relatively cool gases would obviously determine the character of the absorption spectrum of the disk, and the only effect of the hotter eruptions, supposing them to be too small to be individually distinguishable in the spectroscope, would be to produce a faint emission line of about the same intensity as the background of continuous spectrum, and tending to diminish the intensity and width of all the dark lines, particularly the enhanced spark lines.

The latter part of this statement is not wholly clear to the writer. It would seem that if the radiations characteristic of the enhanced lines (for the moment limiting these to such as do not appear at all

[1] J. Evershed, "Solar Eclipse of 1900, May 28.—General Discussion of Spectroscopic Results," *Philosophical Transactions*, A, **201**, 457.

in the arc spectrum) can be emitted only by the hotter ascending gases, the corresponding absorption lines found on the disk of the sun can be produced only by these gases as well. In other words, we should ascribe the presence of the dark enhanced lines of the solar spectrum almost entirely to these ascending jets of very hot gas.

An interesting point of evidence bearing on this question is furnished by the observations of Fox on the spectrum of one of the dark interstices or "pores" between the granulations on the surface of the sun.[1] He found a slight but unmistakable difference in the character of the spectrum in the direction of that observed in sun-spots. That is, the spectrum of the pore indicates a region of cooler temperature than does the general solar spectrum. Since the latter is made up of the mingled light from many pores and granulations, it is clear that the spectrum of the granulations must be characteristic of regions of higher temperature in order to give the average found in the ordinary solar spectrum.' This is what Evershed's theory requires.

The application of this discussion to the results found from measurements of the displacements of the enhanced lines is of great interest. If the enhanced lines are due almost exclusively to the ascending streams of intensely hot gas (the granulations), while the other dark lines in the spectrum are due both to the pores and the granulations, it is clear that the enhanced lines, if compared with the corresponding lines in the spectrum of the spark (neglecting pressure-shifts), would show a slight displacement to the violet, while the other lines would show either no displacement, or a very minute displacement toward the red, owing to the preponderance of the effect of the pores. Even for the enhanced lines the shift would be difficult of measurement in any direct manner. Through measurement of the center and the limb, however, double the amount is obtained, and this in a purely differential way. Accordingly, on this basis, a simple explanation is afforded of the larger displacements shown by the enhanced lines. It is due to the fact that these lines are affected relatively more by the ascending currents at the center of the sun than are the other lines in the spectrum. Furthermore, it is evident that such of the enhanced lines as appear in the

[1] From a paper read at the 1909 meeting of the American Astronomical and Astrophysical Society, but, so far as I know, not yet published.

arc spectrum can probably be produced to some extent by the pores. For these lines, accordingly, we should expect the displacements to be less, that is, for the lines as a whole the displacements will be the greater, the greater the amount of enhancement. That such is apparently the case, we have already seen. In addition it should be noted that the fact that the difference of displacement between the enhanced and the arc lines is the same for iron and titanium indicates that the cause producing the larger displacements is of such a character as to affect different elements equally. This would argue for an external cause rather than for one inherent in the nature of the enhanced lines themselves.

The amount of the motion in the solar granulations which would account for the difference between the enhanced lines and the arc lines may be found readily in the same way as in the earlier case of the cyanogen lines. The average difference between the displacements of the enhanced and the arc lines of iron and titanium is about 0.003 Ångström, and the few scattered lines of other elements give approximately the same value. An upward motion in the solar granulations of about 0.12 km a second, an amount sufficient to produce a displacement of 0.0015 Ångström, would account for the results found. This is, of course, a mean value from enhanced lines of all sorts, including such as appear to some extent in the arc spectrum. It is probable that the granulations move somewhat faster than this, since the lines which are most enhanced show differences of 0.004 or 0.005 Ångström from the arc values.

Another possible explanation of the larger displacements given by the enhanced lines must not, however, be overlooked. This is the possibility that the enhanced lines may show larger shifts under pressure in the laboratory. Practically all of the investigations up to this time on the effect of pressure have been carried out with the use of the electric arc, and so these lines have not been employed. An investigation of the spark under pressure was begun by Mr. Gale in the Pasadena Laboratory in March 1909, but unfortunately was interrupted by an accident to the observer. It is hoped that it may be continued during the coming year.

We now may pass to the consideration of pressure as a means of accounting for the displacements found at the sun's limb. A

brief summary of the points bearing on this question will be of value.

1. The displacements are largest for the elements which lie at the lowest level in the sun's atmosphere. This is exclusive of such elements as lanthanum and cerium, the behavior of which has already been discussed in full.

2. The displacements increase with the wave-length of the lines observed.

3. The lines of the cyanogen flutings are but very slightly shifted, and these shifts are satisfactorily accounted for by small motions in the line of sight at the center of the sun.

4. Direct comparisons of the two limbs and the center of the sun with the spectrum from an electric arc have shown that the displacements cannot be due to currents in the reversing layer moving parallel to the surface of the sun.

5. The lines measured, with the exception of certain very high-level lines, such as those of hydrogen, calcium, sodium, and magnesium, are widened on the red edge, the violet edge retaining its normal position. This result is in agreement with that found by Buisson and Fabry.

These different considerations are all strongly in favor of pressure as the agent effective in producing the displacements observed. A direct comparison with laboratory results, however, is made most difficult by the question of the level of the various lines involved. This is shown clearly by the following simple calculation. Let us assume four layers of gas each 500 km thick, one above the other, over the surface of the sun. If the radius of the sun is taken as 740,000 km the length of path traversed by the light at the sun's limb would be

For the lowest layer	27,100 km
For the second layer	11,300
For the third layer	8,700
For the fourth layer	7,300

In practice, of course, no photographs have been taken exactly at the sun's limb on account of the disturbing effect of the chromospheric spectrum, but at a small distance inside. Accordingly these quantities would be greatly modified. If we assume as an average for the plates a distance inside the limb of 0.2 mm, which corresponds to about 1760 km on the sun's radius, we find

For the lowest layer	6,800 km
For the second layer	6,080
For the third layer	5,560
For the fourth layer	5,150

Accordingly, for a gas which rises to a height of 1000 km above the sun's surface the ratio of the lengths of path in the lower and the upper layers is 1.12 to 1, while it is 1 to 1 at the center of the sun. Since the density of the gas increases very rapidly toward the surface of the sun, the magnitude of this effect would be increased by dividing the stratum of gas into thinner layers and considering their action separately. Moreover, in the case of elements which rise to a considerable height in the sun's atmosphere, the density of the gas in the lower strata is much less than in the case of low-lying elements. This is well illustrated by the fact that the lines of titanium show almost no wings, even when of equal intensity to some of the strongly winged lines of iron. Accordingly, for such an element the effect of an increase in the length of path in the lower strata of the gas will be much less than for gases confined to a lower level. Hence we should expect to find the lines of iron showing comparatively large displacements, while those of titanium would show considerably smaller values.

Since the matter of level appears to be the most significant factor in determining the amount of displacement for any given line, and since the different lines, even of the same element, in the solar spectrum, show wide differences of level, it is clear that any agreement with laboratory pressure-shifts can at best be only approximate. Some comparison of the sort should, however, be made.

In the case of iron, if we take the values given by Humphreys for the lines in the list,[1] and divide them into two sets according as the shift for a pressure of 42 atmospheres is less or greater than 0.1 Ångström, we obtain the following comparisons:

Number of Lines	Average Pressure-Shift	Displacement at Limb
25	+0.065	+0.007
15	+0.107	+0.009

While the displacements at the limb are larger for the lines showing

[1] *Jahrbuch der Radioaktivität und Elektronik*, **5**, 324, 1908.

the larger pressure-shifts, the ratios are widely different. A considerable part of this is due, however, to three lines in the list near λ 4900, which show enormous displacements under pressure. The omission of these lines would reduce the average pressure-shift to +0.148 Ångström, while the displacement at the limb would remain the same. It seems probable that in the sun these three lines are of comparatively high level. The line λ 4494.738 agrees very well with the laboratory results. At a pressure of 42 atmospheres it shows the large displacement of +0.200 Ångström. The displacement at the limb is +0.010 Ångström, which is also very large for this region of the spectrum. On the other hand the line λ 4447.892, which is displaced +0.180 Ångström in the laboratory, shows but +0.005 Ångström at the limb.

In the case of titanium the results are very similar, but the laboratory material is much more limited. A comparison with the values of Humphreys gives the following results:

Number of Lines	Average Pressure-Shift	Displacement at Limb
6	+0.048	+0.003
6	+0.120	+0.005

The material available for the other elements is so fragmentary as to make comparisons of little value. Attention should, however, be called to the group of nickel lines at about λ 4650. These show very large displacements under pressure in the laboratory. At the limb they give an average displacement of +0.009, which is also exceptionally large for this region of the spectrum.

In conclusion it may be said that the comparison of the laboratory with the solar displacements is on the whole in favor of the view that pressure produces the observed shifts. Though the agreement is by no means complete, the average values are uniformly in the same direction. Moreover, the discordances are almost always in the direction of too small values for the solar displacements. This is precisely what would be expected. In the laboratory the entire mass of vapor producing the spectrum lines is under the same pressure. In the sun, on the other hand, there is a pressure gradually increasing downward throughout the strata whose absorption produces the dark

lines. The result is that the lines are widened toward the red, the violet edges retaining their normal positions, and the measures made on the centers of the line can give only an average value.

In a recent discussion of the application of anomalous dispersion to solar phenomena,[1] Julius has ascribed the displacements found at the sun's limb to this cause. According to his point of view the photospheric light is anomalously refracted in the vicinity of the absorption lines produced by the metallic vapors, and, since in general the density-gradient decreases outward, the widening will be upon the red side of the lines producing the observed displacements. The fact that the sodium lines D_1 and D_2 are not displaced, although they show the largest amount of anomalous dispersion of any which have been investigated for this effect, is rather strongly opposed to this view. The same is true of H, K, and λ 4227 of calcium, all of which show strong anomalous dispersion but are not displaced at the limb. Since these lines, however, might be considered as somewhat exceptional, I have made a comparison with a large number of lines investigated by Geisler.[2] The results are summarized below.

Element	Anomalous Dispersion				Displacement at Limb
	Strong	Moderate	Weak	Very Weak	
Aluminium	..	2	..	..	+0.002 Å
Barium	2	..	..	..	+0.011
	..	1	..	..	+0.004
	..	..	..	1	+0.006
Calcium	3	..	..	..	+0.001
	..	..	4	..	+0.007
	..	..	..	18	+0.007
Chromium	4	..	..	..	+0.006
	..	..	3	..	+0.008
	..	..	..	2	+0.006
Cobalt	..	..	..	1	+0.006
Iron	..	..	2	..	+0.005
	..	..	..	7	+0.007
Magnesium	..	..	3	..	0.000
	..	..	..	2	+0.001
Manganese	1	..	..	..	+0.007
Sodium	2	..	..	..	+0.001
Strontium	1	..	..	..	+0.001
Zinc	3	..	..	..	+0.007

[1] To be published in *Memorie della Società degli Spettroscopisti Italiani.*

[2] *Zeitschrift für wissenschaftliche Photographie,* **7**, 89, 1909.

There seems to be no clear relationship between these results. The values for calcium, sodium, and strontium would appear to indicate that lines showing the greatest anomalous dispersion give the smallest displacements. The results for manganese and barium, however, appear to contradict this. Without much more positive evidence it is difficult to see how any relationship between anomalous dispersion and these results can be considered as established.

	Calcium			Nickel		
λ	Δ	Δ'	No. Lines	Δ	Δ'	No. Lines
3700–4200	....	....	..	+0.0050	+0.0030	3
4200–4700	+0.0056	+0.0034	11	0.0077	0.0052	4
4700–5200	0.0055	0.0035	2	0.0075	0.0048	8
5200–5700	0.0067	0.0042	9	0.0100	0.0070	1
5700–6200	0.0085	0.0053	6	+0.0113	+0.0082	6
>6200	+0.0072	+0.0040	6	....	....	..

	Iron			Enhanced Iron		
λ	Δ	Δ'	No. Lines	Δ	Δ'	No. Lines
3700–4200	+0.0060	+0.0040	18	+0.0070	+0.0050	4
4200–4700	0.0065	0.0044	28	0.0104	0.0079	11
4700–5200	0.0083	0.0058	24	0.0090	0.0067	3
5200–5700	0.0095	0.0067	21	0.0120	0.0090	1
5700–6200	0.0116	0.0086	14	0.0117	0.0087	3
>6200	+0.0116	+0.0084	13	+0.0128	+0.0098	5

	Titanium			Enhanced Titanium		
λ	Δ	Δ'	No. Lines	Δ	Δ'	No. Lines
3700–4200	+0.0037	+0.0016	9	+0.0051	+0.0031	13
4200–4700	0.0049	0.0025	17	0.0070	0.0048	22
4700–5200	0.0051	0.0027	7	0.0068	0.0043	4
5200–5700	0.0075	0.0047	4	+0.0097	+0.0070	3
5700–6200	0.0065	0.0035	2	....	....	..
>6200	+0.0075	+0.0045	2	....	....	..

One of the most important questions relating to the displacements at the sun's limb is that of their variation with wave-length. The range of spectrum covered by the photographs is so great that an effect of this kind should be very marked. Moreover, it is made of especial interest by the fact that the proportionality between pressure displacements and wave-length cannot yet be considered as fully established for laboratory results. Thus in 1908 Humphreys writes:[1]

[1] *Jahrbuch der Radioaktivitat und Elektronik*, **5**, 324, 1908.

Im allgemeinen scheint die Druckverschiebung von Spektrallinien mit der Wellenlänge zuzunehmen; aber wahrscheinlich gilt dies nur von den Linien derselben Serie. Jedenfalls ist es nicht zutreffend in Falle von Eisen, Nickel und anderen Elementen, deren Linien zu vielen Serien oder zu keiner zu gehören scheinen.

A simple inspection of the tables shows that the displacements increase toward the red end of the spectrum. In order to furnish a numerical comparison, however, I have collected the results for the principal elements in the following table, taking the means for the lines included within each 500 Ångström units of spectrum. Both Δ and Δ' are given.

Except in the case of iron the number of lines in some of the 500 Ångström intervals is so small, especially toward longer wave-lengths, as to make the influence of individual lines larger than it should be. Better results will probably be obtained by taking means over 1000 Ångströms. If Δ alone is considered we obtain the following values. The figures in parentheses after the displacements indicate the number of lines.

	λ 3700—λ 4700	λ 4700—λ 5700	> λ 5700
Ca	+0.0056 (11)	+0.0065 (11)	+0.0078 (12)
Fe	0.0063 (46)	0.0089 (45)	0.0116 (27)
Enh. *Fe*	0.0095 (15)	0.0098 (4)	0.0124 (8)
Ni	0.0065 (7)	0.0078 (9)	0.0113 (6)
Ti	0.0045 (26)	0.0060 (11)	+0.0070 (4)
Enh. *Ti*	+0.0063 (35)	+0.0080 (7)	......

An interesting difference is shown by the elements in this table. The displacements for calcium and titanium, for the latter element both for the arc and the enhanced lines, vary almost exactly in direct proportion to the wave-length. In fact the largest deviation from this relationship for these three sets of lines is 0.0003 Ångström. For the enhanced lines of iron the results are doubtful, but an approximately linear relationship seems to hold. For nickel, however, and the arc lines of iron, the displacements increase more rapidly than in direct proportion to wave-length. Thus for iron, where the direct proportion would require a ratio for the three values of 1:1.24:1.48, the ratio actually found is 1:1.41:1.84. Since the values for iron are of very high weight on account of the large number of lines employed, this result is, I think, to be regarded as genuine.

A most simple explanation of this difference of behavior is found in the scattering of light in the solar atmosphere. I have already referred to the great difference in the relative intensity of the spectrum of the center and the limb of the sun in different parts of the spectrum, the ratio of about 1 to 5 in the red increasing to about 1 to 15 in the ultra-violet. There can be little doubt that this difference is due to the relatively greater scattering of the ultra-violet light from the lower strata of the solar atmosphere in the long path traversed at the sun's limb. If such is the case we should expect the light from the gases of elements which are confined to the lower strata to be relatively more weakened than that from gases which extend throughout a considerable range of level. Moreover, since the vapors of iron and nickel lie at a considerably lower level than those of calcium and titanium, the contribution of the lowest strata to the formation of the absorption lines is relatively greater in their case. As these strata are under the greatest pressure this relative weakening of the light in the ultra-violet will show itself by reduced displacements in this part of the spectrum. Toward longer wave-lengths, on the other hand, the light from the lower strata will pass more freely, and displacements will increase rapidly. In the case of titanium and calcium the effect of these lower strata is relatively much less, and the influence of the increased scattering upon the displacements comparatively small.

There is but one other matter in the discussion of these results to which I wish to call attention. A considerable number of lines are included in this list, particularly in the less refrangible part of the spectrum, which are found as doublets, or in some cases triplets, in the spectrum of sun-spots. The range of displacement for these lines is considerable. Thus the *Fe* line at λ 6499.168 has a displacement of +0.008 Ångström, while another *Fe* line at λ 6213.644 has +0.014 Ångström. Both of these lines are doublets in the spot spectrum and show approximately the same separation. If we assume, as seems altogether probable from this investigation, that difference of displacement in general indicates difference of level, we must conclude that in sun-spots the distribution of the gases is very different from that in the reversing layer, or that the magnetic field extends with practically uniform strength throughout the range of level indicated by these lines.

I wish to express my appreciation to several members of the Computing Division, particularly Miss Lasby, Miss Waterman, and Miss Wickham, for assistance in the progress of this research. A large part of the measures of the plates included in the list are due to Miss Lasby.

MOUNT WILSON SOLAR OBSERVATORY
November 1909

INTENSITY RELATIONS IN THE HYDROGEN SPECTRUM

By P. G. NUTTING and ORIN TUGMAN

The various lines of the hydrogen spectrum are known to vary in relative intensity as the intensity of excitation and gas density are varied. We have undertaken a quantitative study of these effects by comparing the intensities of α, β, and γ and certain parts of the primary spectrum when produced under varied conditions, with their intensities under fixed standard conditions.

The gist of the method was the use of two similar tubes, one of which was held constant as a reference standard while conditions in the other were varied. Both tubes were operated with small 5000- or 10,000-volt transformers, the current in each being controlled by variable resistances in the 120-volt side. A polarization spectrophotometer was employed throughout. The current through each tube passed through an alternating current precision milliammeter in all cases except when a condenser was used.

The tubes were connected together and to the pump, manometer, and hydrogen supply by all glass connections, so arranged that either or both tubes could be quickly refilled with pure dry hydrogen. Contamination with mercury was carefully guarded against. The tubes were always reduced to a non-conducting vacuum before refilling. A special form of oil manometer, previous'y described,[1] gave pressures down to 0.2 mm with sufficient accuracy without the use of a McLeod gauge. Gauge and pump oil and cock-grease (previously vacuum boiled) gave no evidence of contamination in the spectral tubes after the first few hours' exposure to a vacuum. The hydrogen employed was prepared electrolytically from alkaline solution and kept over phosphorus pentoxide until used.

After some preliminary work on the effects of temperature, age of tube, impurities, etc., we adopted an open circuit programme, the current passing through the tubes only when a double key was pressed. Thus the tubes remained at or near room temperature and were spared

[1] *Bulletin of the Bureau of Standards*, **4**, 514, 1907.

unnecessary use. The greatest difficulty in working with hydrogen is the rapid deterioration of the walls of the tube under the action of the discharge. This necessitated replacing the tubes or their capillary portions after about an hour's use. Our results were obtained chiefly with large tubes having capillaries 2.6 mm in diameter and 7 cm long, viewed side-on at the middle. For these tubes 1 mm pressure and 15 milliamperes current was taken as the normal. Later, through the remarkable skill of Mr. Sperling, we were supplied with an end-on tube whose capillary portion was of porcelain. This tube carried as high as 500 m.a. and showed but slight deterioration after a week's service.

Readings were taken at six different wave-lengths, three in the secondary, α 656, β 486, and γ 434, and three in the primary referred to as a, b, and c, the approximate wave-lengths of which are 611, 545, and 460 $\mu\mu$. Both ocular and collimator slits were open about 6 $\mu\mu$.

VARIATION OF LINE-INTENSITY WITH CURRENT

The data taken with heavy currents, using the tube with porcelain capillary, are given below. The range of current was from 20 to 440 milliamperes. The porcelain capillary was 3.0 mm in diameter and 30 mm long, and was viewed end-on. All values for light-

Current Milliamperes	Line-Intensity ÷ Intensity at $i=20$			
	S_α	S_β	$S\gamma$	$Pabc$
20	1.0	1.0	1.0	1.0
40 (4 sets)	2.6	2.2	2.0	1.7
50	3.9	2.6	2.9	2.3
75	5.4	3.9	2.7	3.3
120 (3)	14.2	10.2	5.5	4.4
200 (2)	20.5	17.0	6.5	7.1
250 (5)	29.2	18.1	10.7	8.2
300 (3)	48.8	18.2	16.3	8.9
370	57.0	26.5	14.0	11.5
380 (2)	59.5	24.0	12.0	10.7
400 (2)	....	....	....	13.0
440 (2)	66.0	26.5	....	11.1

intensity are in terms of the intensity at 20 m.a. A zero reading ($i=20$) was taken just before and just after the reading for each higher current and if the zero showed any change the reading was rejected. Both tubes were filled with fresh gas at 1 mm pressure for

each set of observations. The primary spectrum showed no selective (wave-length) effect as great as 2 per cent. even at the highest currents, so the three readings for the three spectral regions are grouped together in the column headed *Pabc*.

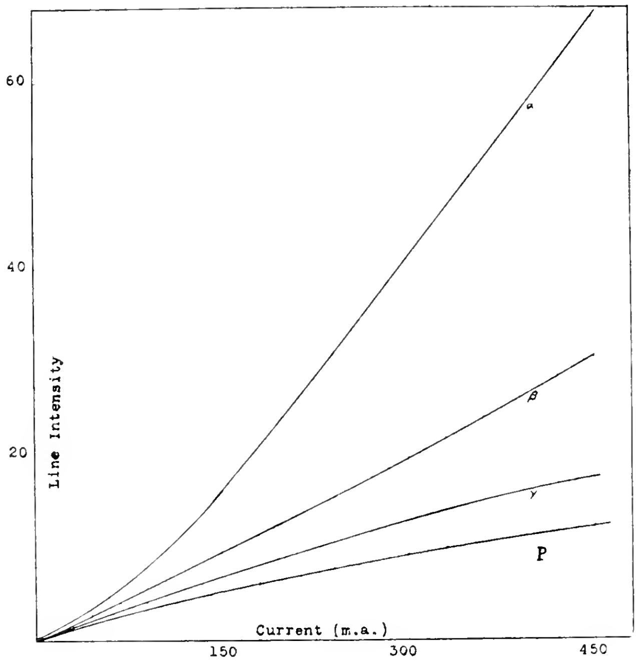


FIG. 1

A least-square reduction for the intensity P of the primary spectrum gives

$$P=P_{20}\left[1.042\frac{i}{20}-.0432\left(\frac{i}{20}\right)^2+.00098\left(\frac{i}{20}\right)^3\right],$$

while for the three lines of the secondary spectrum

$$S_\alpha=P^{1.07}\ ;\quad S_\beta=P^{1.35}\ ;\quad S_\gamma=P^{1.14}$$

for all currents. Hence,

$$S_\alpha^{0.599}=S_\beta^{0.729}=S_\gamma^{0.876}=P,$$

all referred to intensity at 20 m.a. as a unit.

In the relation

$$S=P^m,$$

m as a function of λ may be closely represented by

$$m=681\left(\frac{1}{251}-\frac{1}{\lambda}\right).$$

The uncertainty in the computed values of m is about 2 per cent. The deviations of m from the mean indicate that it is entirely independent of the current.

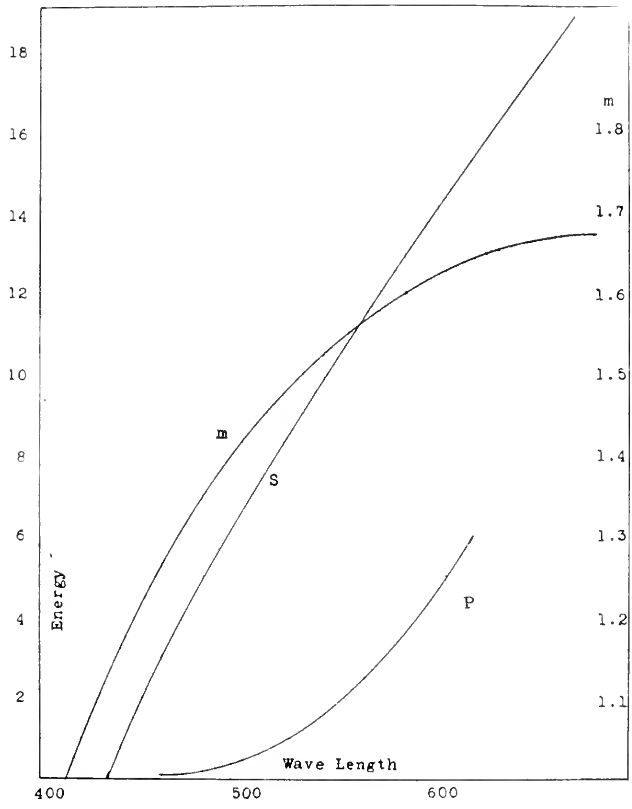


FIG. 2

These data are shown graphically in Figs. 1 and 2. The computed curves for α, β, and γ fit the data as well as any that could be drawn. The curve for m indicates that the fifth secondary line (λ 397) would vary at the same rate ($m=1$) as the primary spectrum, while lines farther out toward the head of the series would increase less rapidly than the primary with increase in current.

The most probable values of the spectral intensity taken from the adjusted curves are:

i	P	α	β	γ
20	1.00	1.00	1.00	1.00
50	2.1	3.45	2.72	2.33
100	3.8	9.3	6.1	4.6
150	5.4	16.7	9.7	6.8
200	6.8	24.5	13.3	8.9
250	8.2	33.5	17.1	11.0
300	9.4	42.0	20.5	12.9
350	10.4	49.8	23.5	14.4
400	11.5	59.1	27.0	16.2
450	12.3	66.1	29.5	17.4

At half the normal current $i=10$, taking $P=0.60$, the intensities in the secondary would have been $\alpha=0.43$, $\beta=0.50$, $\gamma=0.56$, had these lines followed the same law as at high currents. The greatest observed differences between α and γ were only about 0.05 and varied in sign, so that we must conclude that there is no selective effect at these current-densities.

Finally the energy emission at standard current (20 m.a.) was obtained by comparison with a carbon filament glow-lamp whose energy-curve at a given voltage had been determined by Dr. Coblentz. The uncertainty in the data given is perhaps 5 per cent.

Line	λ	Lamp E	Hyd. E
α	656	197	184 ($E_\gamma=1$)
β	486	24	58 ($E_\gamma=1$)
γ	434	7.0	1
a	611	130	59.7 ($E_c=1$)
b	515	38	8.3 ($E_c=1$)
c	460	14	1.0

Our data for small current-density was obtained with large glass tubes with capillary portions 2.6 mm in diameter and 7 cm long, of good quality, medium thickness, ordinary tubing. This was viewed from the side of the middle where it was reddest. The ends of the capillary are much whiter than the middle, indicating a larger proportion of primary spectrum. This end effect and the end-on effect were investigated separately. Readings are relative to those for

15 m.a. current. The data below were taken with six different tubes, either new or provided with fresh capillary, all of the same bore filled with gas at 1 mm pressure. Readings were taken at all six wave-lengths in each case. As neither primary nor secondary spectrum showed any selective effect, the results for each spectrum are combined.

Current.........	2	3	5	7	9	12	15
Primary.........	0.163	.241	.34	.49	.65	.81	1.0
Secondary.......	0.043	.094	.17	.29	.42	.65	1.0

Writing as before $S=P^m$, $m=1.73\pm.02$, independent of the current as with large currents.

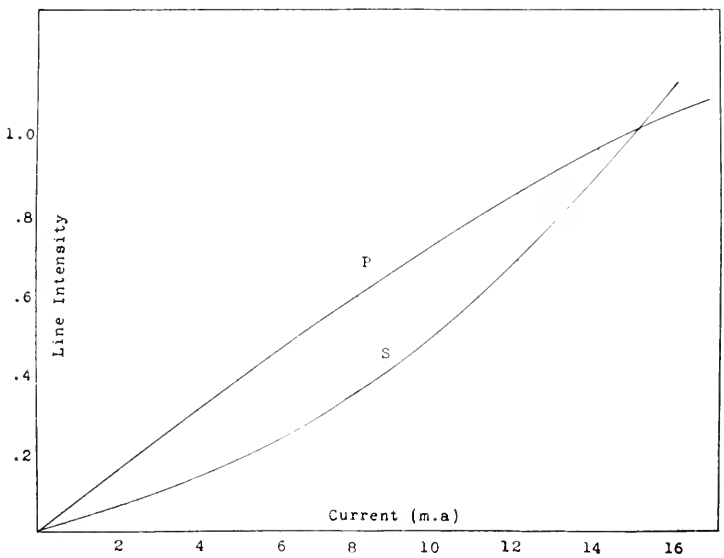


FIG. 3

This lack of selective effect at low current was at first attributed to lack of saturation in the thin column viewed from the side, but careful determinations at 10 m.a. with the end-on porcelain capillary tube showed that there was no selective effect in that case either. The nature of the secondary radiation must then change in the neighborhood of the current 20 m.a. and a little further study of the con-

duction of the gas and the theory of its radiation showed this to be plausible.

The potential-gradient in the 2.6 mm capillary was determined by cutting out about 10 cm of the capillary. This determination was made but twice, but results were in good agreement.

VOLTS PER CM IN H

M.A.	Pressure (mm)				
i	0.5	1.0	2.0	3.0	8.0
2	44.5	60.2	97.0	125	191
5	37.8	56.1	90.4	107	150
9	33.7	52.1	81.1	93	128
12	34.2	50.5	74.6	84	122
15	35.8	49.5	67.4	78	120

It may be noted that at 1 mm pressure there is a minimum gradient near 15 or 20 m.a., so that above and below this point different conditions of ionization obtain.

GAS-DENSITY AND OTHER EFFECTS

The effect of pressure on line-intensity at constant current was studied with glass tubes similar to those last described. From several hundred observations the following mean values were adopted. No selective effect was apparent in either primary or secondary, but these differ considerably from each other. All values are in terms of those at 1 mm pressure as unity.

Pressure	Secondary	Primary
0.25 mm	0.55	...
0.50	.78	0.80
0.75	.97	.97
1.00	1.00	1.00
1.5	.90	.92
2.0	.78	.85
3	.60	.72
4	.47	.61
5	.38	.53
6	.32	.48
7	.28	.44
8	.25	.41
10	.20	.38

The current was 15 m.a. (tubes in series) and the reference tube

filled at 1 mm pressure in each case. These results are shown graphically in Fig. 4.

With both tubes filled with hydrogen at 0.3, 0.5, 2, and 8 mm pressure and a current of 15 m.a. through the reference tube, readings were taken with variable current through the observation tube. These sets of readings differed very little from those taken at 1 mm pressure.

Temperature of wall.—When the capillary part of a tube was heated with a flame to about 200°, α fell off 20 to 30 per cent., while

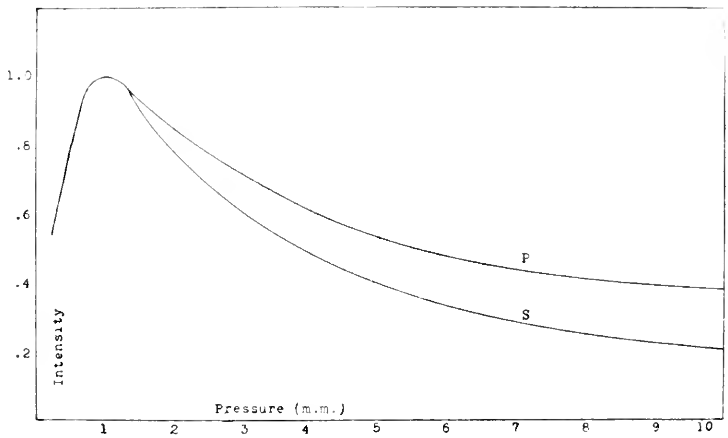


FIG. 4

β and γ and the whole of the primary were not affected by as much as 3 per cent. The tube recovers as it cools and the effect was repeated on several tubes. Since the current itself warms the tube 20° to 100°, very erroneous results may be obtained unless the tubes are maintained at constant temperatures.

Effect of use.—A tube of hydrogen after half an hour of continuous operation shows a marked decrease in intensity of all lines of both spectra. After an hour's run, a purple tinge may be detected in the capillary, and if this be cut out of the tube and viewed end-on, a layer of glass less than 0.2 mm thick, next to the inner wall, will be seen to be highly colored. This coloring is not affected by moderate heating but is completely removed by heating to the softening point of the glass; the effect increases steadily with continued use. Refilling

a tube with fresh gas at the same pressure produces no measurable effect on spectral intensity.

This effect on the inner wall might be due either to negative electron bombardment or to ionization by the extreme ultra-violet waves in which the hydrogen spectrum is so rich. We are inclined to the latter view. The effect is greater for hydrogen than any other gas. A helium tube has been run a hundred hours without appreciable deterioration at a current that would have produced a 10 per cent. change in a hydrogen tube in half an hour.

Oxygen with hydrogen.—Both reference tube and working tube were filled with hydrogen at 2 mm pressure, then zero readings were taken, the reference tube closed off, and an additional 2 mm of oxygen admitted to the working tube. Then with tubes in series (15 m.a. through both) it was found that the oxygen had cut down α, β, and γ about 30 per cent. each, and a, b, and c considerably more than that amount, and a more than c so that the red primary barely showed. The net effect of the oxygen then is to make the secondary spectrum much more prominent. It is easy to see why this spectrum has so often been attributed to water-vapor. This is a rich field for further study; we have touched upon it only to guard against possible error.

In the *cathode glow* and *striations* the relation of primary to secondary varies from that in the ordinary anode column. The primary is relatively brighter (50 per cent.) in the brighter parts of the striations where recombination is supposed to be particularly active. In an ordinary tube with disk electrodes and plenty of room for the cathode glow, the secondary is slightly more prominent in that glow than in the capillary; but in other tubes with cylindrical electrodes set close to the walls, the cathode's glow is nearly white and strongly favors the primary.

At *atmospheric pressure* a zinc spark (2 mm) in hydrogen gave hydrogen spectra not very different nor in different proportion from those in the comparison tube. The increased width of the lines, however, made a more exact comparison difficult.

A tube with *fine capillary* (about 0.3 mm) when adjusted to the same current-density as the reference tube gave spectra so nearly like those of the reference tube that no certain differences could be detected.

End-on effect.—To test whether we were dealing with saturated radiation in using the light from the side of a 2.6 mm capillary, we compared it with an end-on tube with a capillary portion 3.2×32 mm. This gave for relative intensities:

	α	β	γ	a	b	c
$\frac{\text{End-on}}{\text{Side-on}}$	1	1.35	1.58	4.37	4.37	4.39

These are relative to α, since there was an indeterminate constant factor involved. This apparently great departure from saturation we found to be due to a difference in the electrical conditions as the current leaves a capillary. We later obtained similar readings (not quite as high) from one end of a capillary viewed from the *side*.

Spark, capacity, and inductance.—The disruptive discharge was studied in considerable detail with various tubes containing gas at 1, 5, and 10 mm pressure. Two identical tubes were adjusted to 15 m.a. current, but on independent circuits, and then the discharge in one of them made disruptive, the reference tube remaining at 1 mm and 15 m.a. in each case. Some typical results are given below, the three primary readings being combined, as they remained equal throughout to within the uncertainty of measurement.

Pressure, 1 mm	α	β	γ	P
Spark : No Spark	0.59	0.56	0.60	0.73
Spark+C : No Spark	0.191	0.118	0.063	0.095
S+C+I : S+C	0.22	0.13	0.110	0.191
5 : 1 mm				
Spark : No Spark	1.0	0.80	0.74	0.90
S+C : No Spark	1.0	0.35	0.23	0.25
S+C+I : S+C	0.76	0.86	1.22	1.31
10 : 1 mm				
Spark : No Spark	2.30	1.20	0.90	1.10
S+C : No Spark	5.88	2.55	1.00	0.87
S+C+I : S+C	0.50	0.60	1.00	1.50
Porcelain 1 : 1 mm 200 m.a.				
S+C : No S	0.79	0.64	0.52	0.46

These results show the same selective effects as were obtained with heavy steady currents. In the first case for example, the effect of capacity was to give α, β, and γ the ratio 3:2:1 while P lies between β and γ. These agree well with the values for a current of 200 to 250 m.a. in the porcelain tube. On the other hand in the last case where a heavy current (200 m.a.) was used with the condenser, the

effect was much less marked, about equivalent to raising the current to 300 m.a. The apparent weakening of the discharge was due to the longer interval in which no current is passing. The effect of inductance appears to be merely to neutralize part of the capacity effect. The secondary lines α, β, and γ are affected by both capacity and inductance in the order named, α most and β least.

THEORY

Before going into the theory of radiation from conducting gases, it may present our results in a clearer light to compare them with corresponding results for radiating solids. For many substances it is well known that the radiation in the visible spectrum may be closely represented as a function of wave-length and temperature by the Wien-Paschen function

$$E=c_1\lambda^{-n}e^{-c_2/\lambda T}.$$

For a perfect radiator $n=4.96$ or 5, for platinum about 6, and for tungsten about 7, while c_2 is in the neighborhood of 15,000 when wave-lengths are expressed in microns (μ) and temperatures in Centigrade degrees absolute.

The relative radiation, using that at a fixed temperature T_0 as a standard, would be

$$\left(\frac{E}{E_0}\right)_\lambda=e^{-\frac{c_2}{\lambda}\left(\frac{1}{T}-\frac{1}{T_0}\right)},$$

while the relative intensities of the radiation of two different wave-lengths would be

$$\left(\frac{E_1}{E_2}\right)_T=\left(\frac{\lambda_1}{\lambda_2}\right)^{-n}e^{-\frac{c_2}{T}\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right)},$$

If hydrogen obeyed this radiation law our results (p. 67) would give

	n	c_2/T
For the primary spectrum..	36.1	26,700
For the secondary spectrum	159	89,600

values farther from those of a perfect radiator than for any solid yet studied, but yet of the proper sign and not impossible values.

We found further

$$\frac{S}{S_0}=\left(\frac{P}{P_0}\right)^m \text{ where } m=c\left(\frac{1}{\lambda_0}-\frac{1}{\lambda}\right),$$

$$=e^{m\log P/P_0}$$

which again is in accord with known radiation laws, provided P/P_0 is expressible as a function of free internal energy alone.

Consider a column of conducting gas consisting of neutral atoms, positive ions, and negative electrons. When a steady current is flowing, the total radiation from the gas will be equal to the power-supply, gradient times current (X/i). This transformation of energy is undoubtedly accomplished by the charged particles taking energy from the current by falling down the gradient and then giving it up by impact to be radiated away or to cause fresh ionization. The problem is to determine the amounts of energy involved in the various kinds of collision under different conditions of current and gas density.

With 50 volts per cm in hydrogen at 1 mm pressure the negative electrons (correcting the path for small size and high speed by the factor $4\sqrt{2}$) move with a mean velocity over 10,000 times as great as the neutral atoms and 100 times as great as the positive ions. Hence the energy of collision is determined chiefly by the velocity of the negative electrons, and we have to consider four types of collision of these electrons with:

1. Neutral atoms producing perturbations only,
2. Neutral atoms producing an additional electron,
3. Positive ions resulting in recombination,
4. Positive ions producing perturbations only.

Let ν be the average number of times an electron hits either a neutral atom or a positive ion in a second. If N is the number of both of these and n the number of the latter in a cubic centimeter, then $\nu(N-n)/N$ is the collision frequency of an electron with whole atoms and $\nu n/N$ with positive ions. These frequencies multiplied by N is the number of electron collisions in each cc of gas per second.

Each collision of an electron with an atom will produce ionization or not, according to how it strikes and the velocity with which it strikes. To obtain the collision frequency of each of the above four classes we must then integrate to or from certain critical velocities and introduce an arbitrary factor depending upon how the electron strikes. If we assume a hard smooth spherical atom we must resolve the motion of the electron normal to the surface. If, however, the atom be rough or soft, the motion would have its full effect. In the first case a would involve a surface integral, in the second a is unity. As we are

here concerned only with the relation of radiation to current and pressure, we need not go farther into the matter.

Of n paths, those lying between s and $s+ds$ are

$$dn=\frac{n}{s_0}e^{-s/s_0}dS$$

s_0 being the mean path,
hence,

$$\int_0^{s_c} dn=n(1-e^{-s_c/s_0})$$

$$\int_{s_c}^{\infty} dn=ne^{-s_c/s_0}.$$

The frequencies of collision of the four classes are then

$$\nu_1=a_1\nu n\frac{N-n}{N}(e^{-s_1/s_0}-e^{-s_2/s_0})$$

$$\nu_2=a_2\nu n\frac{N-n}{N}e^{-s_2/s_0}$$

$$\nu_3=a_3\nu n\frac{n}{N}(1-e^{-s_3/s_0})$$

$$\nu_4=a_4\nu n\frac{n}{N}e^{-s_3/s_0}.$$

The critical paths S_1, S_2, and S_3 correspond with the critical velocities u_1, u_2, and u_3; u_3 being the limiting velocity beyond which recombination will not occur, u_2, a velocity below which ionization will not occur, and u_1, a velocity below which radiation will not be excited.

For steady current $(i=\Sigma neu)$ the rates of recombination and ionization must be equal, hence $\nu_2=\nu_3$ and hence the total ionization is determined in terms of current-density.

The energy of collision in each of the four classes of impact will be the integral of $\frac{1}{2}mu^2dn$ between the same limits as before. Remembering that $u^2=2Xe/mS$ and that $evn=i/s_0$, we have in the four cases:

$$E_1=Xia_1\frac{N-n}{N}\left[\left(1+\frac{s_1}{s_0}\right)e^{-s_1/s_0}-\left(1+\frac{s_2}{s_0}\right)e^{-s_2/s_0}\right]$$

$$E_2=Xia_2\frac{N-n}{N}\left(1+\frac{s_2}{s_0}\right)e^{-s_2/s_0}$$

$$E_3=Xia_3\frac{n}{N}\left[1-\left(1+\frac{s_3}{s_0}\right)e^{-s_3/s_0}\right]$$

$$E_4=Xia_4\frac{n}{N}\left(1+\frac{s_3}{s_0}\right)e^{-s_3/s_0}.$$

This is the energy imparted to the radiators in a manner capable

of producing radiation. If this energy cannot easily be transformed into heat energy, the energy received will be the energy radiated as light; experiment must tell us which forms of radiators are most active.

In each of the four cases the predominant term is Xi, the product of gradient and current, i. e., the rate of total energy-supply. The final exponential term is constant except when a variation in density varies the mean path S_0. With increase in the current, X decreases slightly to a minimum and then increases. n would be proportional to i but for the slight variation in X and is very small in comparison with N even for large currents.

The collision results in a whole atom in the first and third cases, in a positive ion in the second and fourth. If now the primary spectrum comes from the whole atom and the secondary from the positive ion, as the great majority of experimental evidence indicates, we are still unable to say whether the chief energy-supply is due to recombination, ionization, or from being "shot up." But if further the secondary is nearly proportional to the square of the primary, as we observed, then most of the spectral radiation must come from the first and fourth types of collision, that is, where atoms and positive ions are shot through by electrons without resulting ionization or recombination.

In the second case (ionization) it is plausible that the energy of impact should be quite used up in tearing the extra electron loose from the atom, but in the third case, it is not easy to see why recombination should not produce radiation.

The effect of increased pressure is to diminish the free path, S_0, and hence to increase the intensity of the primary spectrum (current remaining constant) and decrease the secondary as observed. Accompanying the increase in pressure, there is a slight increase in X and in n acting as a slight compensation for the decrease in s_0.

It is easy to see why there should be a selective effect in the secondary spectrum for heavy but not for weak currents. Since the velocity and energy of impact depend only upon the potential-gradient, an increased current will result only in an increased number of impacts until the current becomes so large that the gradient increases with it. Our results then indicate that the slower modes of vibration take up the larger share of the increased energy of excitation.

Bureau of Standards
November 30, 1909

MEASUREMENTS OF WAVE-LENGTHS OF STANDARD IRON LINES

By P. EVERSHEIM

In this paper I give the measurements of standard wave-lengths in the spectrum of iron. The paper will appear *in extenso* in a forthcoming number of the *Annalen der Physik*.

WAVE-LENGTHS OF STANDARD IRON LINES

Eversheim	Fabry and Buisson	Differences E. − F. & B.	Mean Error	Eversheim	Fabry and Buisson	Differences E. − F. & B.	Mean Error
4282.408	.407	+ .001	0005	5266.569	....	+ .001	.....
4315.089	.089	.000	.0005	5302.316	.316	.000	.0005
4352.741	.741	.000	.0005	5324.196	.195	+ .001	.0005
4375.934	.935	− .001	.0006	5371.493	.498	− .005	.0008
4427.313	.314	− .001	.0006	5405.780	.780	.000	.0010
4466.557	.554	+ .003	.0005	5434.524	.530	− .006	.0005
4494.571	.572	− .001	.0004	5455.611	.616	− .005	.0008
4528.622	....	......	.0004	5497.523	.521	+ .002	.0006
4547.853	.854	− .001	.0006	5506.785	.783	+ .002	.0007
4592.658	.658	.000	.0005	5569.636	.632	+ .004	.0006
4602.948	.944	+ .004	.0004	5586.773	.770	+ .003	.0004
4647.441	.437	+ .004	.0004	5615.662	.658	+ .004	.0008
4691.419	....	......	.0006	5658.838	.835	+ .003	.0010
4707.292	.287	+ .005	.0007	5763.013	.013	.000	.0008
4736.785*	.785	.000	.0010	5826.294 *Ba*	....	......	.0003
4736.787	....	+ .002	.....	5857.759 *Ni*	.759	.000	.00010
4754.049 *Mn*	.046	+ .003	.0007	5892.881 *Ni*	.881	.000	.0005
4789.658	.657	+ .001	.0010	5971.715 *Ba*	....	......	.0006
4823.523	.521	+ .002	.0006	5997.102 *Ba*	....	......	.0005
4859.758	.756	+ .002	.0006	6065.493	.493	.000	.0007
4878.224	.226	− .002	.0008	6108.121 *Ni*	....	......	.0004
4903.327	.324	+ .003	.0004	6191.568	.569	− .001	.0005
4919.007	.006	+ .001	.0003	6230.736	.732	+ .004	.0005
4966.105	.104	+ .001	.0007	6318.028	.029	− .001	.0007
5001.885	.880	+ .005	.0008	6335.342	.343	− .001	.00010
5012.074	.072	+ .002	.0006	6393.613	.612	+ .001	.0006
5049.827	.827	.000	.0006	6430.862	.859	+ .003	.0006
5083.346	.343	+ .003	.0008	6494.994	.994	.000	.0007
5110.414	.415	− .001	.0008	6546.252	....	......	.00012
5167.491	.492	− .001	.0006	6592.931	....	......	.0008
5191.473	....	......	.0010	6678.008	....	......	.00010
5232.958	.958	.000	.0005	6750.162	....	......	.00010
5266.566†	.568	+ .002	.0020	6945.223	....	......	.00025
5266.569*	....	+ .001	.....				

* With grating, referred to $\lambda = 4707.287$ and $\lambda = 4789.657$.

† With grating, referred to $\lambda = 5232.958$ and $\lambda = 5302.316$.

I employed the method of MM. Fabry and Buisson, as published in their paper, "Mesures de longueurs d'onde pour l'établissement d'un système de repères spectroscopiques."[1] Where feasible I measured the lines for which they had published the wave-lengths. The three places of decimals of their determinations are given in the second column, while the differences of our values[2] are found in the third column. The fourth column contains the mean probable errors of my measurements in units of the fourth place of decimals of the Ångström unit.[3]

[1] *Journal de Physique* (4), **7**, 169, 1908; *Astrophysical Journal*, **28**, 169, 1908.

[2] Occurring in the third decimal.

[3] The wave-lengths below λ 4282 will be published as soon as the measurements are finished.

Bonn, Physical Institute of the University
October 18, 1909

THE ANALYSIS OF THE PRINCIPAL MERCURY LINES BY A DIFFRACTION GRATING AND A COMPARISON WITH THE RESULTS OBTAINED BY OTHER METHODS

By HENRY G. GALE and HARVEY B. LEMON

In 1892 Professor Michelson[1] discovered that many spectral lines are accompanied by satellites. The structure of the lines was deduced from the form of the visibility-curves, and the limitations of the method were fully recognized.

In 1899 Fabry and Perot[2] announced similar results, obtained with their form of the interferometer. Since the invention of the echelon,[3] further investigations in this field have been made by numerous observers.[4] In 1904 Barnes[5] invented an ingenious form of interferometer and analyzed a number of bright lines. Lummer and Gehrcke,[6] in 1903, published results obtained with their interference plate, and this work has been extended by Gehrcke and von Baeyer.[7]

There has been a certain amount of concordance in the results obtained by various observers, and not a little lack of agreement, caused in some instances by the use of different sources, as in the well-known case of the green cadmium line,[8] and in others, no doubt, by false lines due to instrumental imperfections.

[1] *Phil. Mag.* (5), **34**, 280, 1892.

[2] *Ann. de Chimie et de Physique* (7), **16**, 115, 1899.

[3] A. A. Michelson, *Astrophysical Journal*, **8**, 37, 1898.

[4] Gray and Stewart, *Proc. Royal Soc.*, **72**, 16, 1904; R. A. Houston, *Phil. Mag.* (6), **7**, 456, 1904; L. Janicki, *Annalen der Physik*, **19**, 36, 1906; **29**, 833, 1909; B. Galitzin, *Bull. de l'Acad. Impériale des Sciences de St. Pétersbourg*, 150, 1907.

[5] James Barnes, *Astrophysical Journal*, **19**, 190, 1904; *Phil. Mag.* (6), **7**, 485, 1904.

[6] Lummer and Gehrcke, *Annalen der Physik*, **10**, 457, 1903.

[7] Gehrcke and von Baeyer, *Annalen der Physik*, **20**, 267, 1906; O. von Baeyer, *Astrophysical Journal*, **25**, 267, 1907; *Verh. der Deutsch. Phys. Gesell.*, **9**, 84, 1907; *ibid.*, **10**, 733, 1908; and *Phys. Zeit.*, **9** 831, 1908.

[8] M. Hamy, *C. R.*, **130**, 489, 1900; **130**, 700, 1900; Fabry and Perot, *C. R.*, **130**, 654 (note), 1900; *Astrophysical Journal*, **16**, 36, 1902; Louis Bell, *ibid.*, **15**, 157, 1902; **18**, 192, 1902; J. Hartman, *ibid.*, **18**, 187, 1903; C. Fabry, *Astrophysical Journal*, **19**, 116, 1904.

In the spring of 1909 a comparison of the structure of the green mercury line, as given by three different echelons, with that given by a grating recently ruled by Professor Michelson was made by one of us.[1] The constants of the echelons were only roughly determined, but the photographs taken showed conclusively that the grating gave results comparable with those of the echelons. The ease of manipulation of the grating and the absence of ambiguity due to conflicting adjacent orders made its use a very obvious advantage.

Through the kindness of Professor Michelson we have been able to photograph the principal lines of mercury with one of the gratings ruled by him. The grating used has a ruled surface $6\frac{1}{2}$ inches by $2\frac{7}{8}$ inches (16.5 cm by 7.3 cm). It was mounted in the Littrow form with a Brashear lens of 20 ft. (6.1 m) focal length. This particular grating is exceptionally bright in the higher orders on one side, and satisfactory photographs could be obtained in the fourth order in from five minutes to forty minutes, depending on the line sought.

It is well known that in certain cases the relative intensities of the satellites is different with different sources, and we have therefore thought it wise to use a commercial Cooper-Hewitt lamp, since this is a very satisfactory source and one easily available for comparison by others.

Figs. 1 to 4 show diagrammatically the results of various investigators. Tables I to IV give the corresponding values of the wave-lengths. The letters accompanying the diagrams and tables have the following significance: M., Michelson;[2] F. and P., Fabry and Perot;[3] v. B., von Baeyer;[4] J., Janicki;[5] G., Galitzin;[6] S., Stansfield;[7] and Gr., the grating results obtained by us. Intensities are

[1] Paper read by Mr. Lemon before the American Physical Society at the November meeting, 1909, at the University of Illinois.

[2] A. A. Michelson, *Phil. Mag.* (5), **34**, 280, 1892.

[3] Fabry and Perot, *Ann. de Chimie et de Physique* (7), **16**, 115, 1899. For their last values for the green line see Zeeman, *Astrophysical Journal*, **15**, 218, 1902.

[4] O. von Baeyer, *Verh. der Deutsch. Physik. Gesell.*, **10**, 733, 1908; also *Phys. Zeit.*, **9**, 831, 1908.

[5] L. Janicki, *Annalen der Physik* (4), **19**, 36, 1906; *ibid.*, **29**, 833, 1909.

[6] B. Galitzin, *Bull. de l'Acad. Impériale des Sciences de St. Pétersbourg*, 159, 1907.

[7] H. Stansfield, *Phil. Mag.* (6), **18**, 371, 1909.

assigned in the tables in accordance with the methods of the various authors.

TABLE I. λ 5790

M.	F. and P.	v. B.	J.	G.	Gr.
+.24 $\frac{1}{10}$	−.130 very weak	+.228 3	+.230 $\frac{1}{7}$	+.228 3	+.229 2
+.13 $\frac{1}{10}$		+.133 2	+.168 $\frac{1}{10}$	+.169 6	+.135 2
−.12 $\frac{1}{5}$		−.122 1	+.132 $\frac{1}{7}$	+.132 2	−.119 1
		−.180 weak	+.084 $\frac{1}{9}$	+.086 5	−.184 3
			−.119 $\frac{1}{5}$	−.121 1	−.931 4
			−.187 $\frac{1}{10}$	−.190 4	−.998 3
			−.251 $\frac{1}{10}$		

TABLE II. λ 5769

M.	F. and P.	v. B.	J.	G.	Gr.
±.043 $\frac{1}{5}$	+.048	+.044 1	+.048 2	+.042 2	+.040 1
		−.048 2	−.052 2	−.049 1	−.044 2
			−.114 3		

TABLE III. λ 5461

M.	F. and P.	v. B.	J.	G.	S.	Gr.
+.13 $\frac{1}{10}$	+.136 $\frac{1}{6}$	+.211 6	+.133 $\frac{1}{5}$	+.129 4	+.216 2	+.217 4
+.10 $\frac{1}{4}$	+.082 $\frac{1}{4}$	+.125 3	+.088 $\frac{1}{3}$	+.085 3	+.131 4	+.130 3
(+.01)	+.008 $\frac{1}{2}$	+.082 1	−.066 $\frac{1}{7}$	−.047 6	+.087 1	+.083 2
−.07 $\frac{1}{10}$	−.052 $\frac{1}{5}$	−.024 1	−.099 $\frac{1}{10}$	−.068 2	−.068 3	−.054 3
(−.23)	−.076 $\frac{1}{7}$	−.049 5	−.232 $\frac{1}{5}$	−.099 5	−.097 3	−.094 3
	−.224 $\frac{1}{3}$	−.068 3		−.236 1	−.232 1	−.233 1
		−.101 4				
		−.237 2				

TABLE IV. λ 4358

M.	v. B.	J.	G.	G.′	Gr.
±.17 $\frac{1}{10}$	+.185 3	+.121 $\frac{2}{3}$	(+.131) 2′	+.194	+.194 2
(±.019)	+.114 5	+.105 $\frac{2}{3}$	+.126 2	+.118	+.118 3
	+.044 5	+.043 $\frac{1}{4}$	(+.118) 2″	+.053	+.040 2
	+.028 6	+.020 $\frac{1}{5}$	+.053 3	+.027	−.088 4
	+.019 5	−.023 1	+.027 4	−.092	−.155 1
	−.017 1	−.052 $\frac{1}{7}$	−.092 1	−.155	
	−.045 7	−.097 $\frac{2}{3}$			
	−.093 4	−.112 $\frac{2}{3}$			
	−.107 4				
	−.159 2				

The yellow lines, λ 5790 and λ 5769, are shown in Figs. 1 and 2, respectively, the green line, λ 5461, in Fig. 3, and the violet line, λ 4358, in Fig. 4. Figs. 5 to 8, Plate VI, are made from our photographs, enlarged about eighteen diameters laterally, and given vertical motion to smooth out the grain of the plates. Some of the detail of the original plates is of course lost. The faint satellite of λ 5790

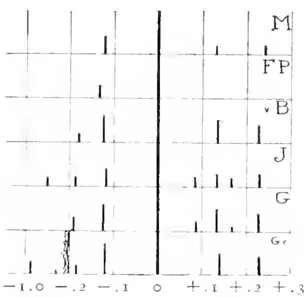


FIG. 1

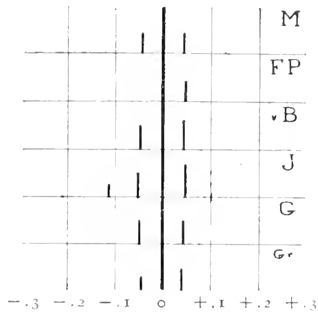


FIG. 2

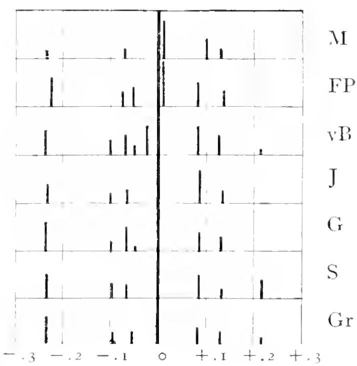


FIG. 3

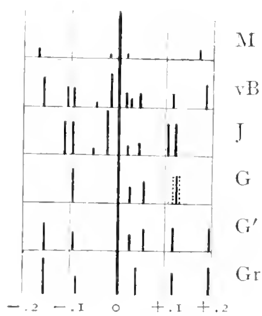


FIG. 4

at −.931 shows plainly on the original negative. The main line of λ 5769 is narrow and sharp and its satellites are broad and fuzzy. The satellites of λ 5461 at −.054 and −.094 are clearly resolved on our original negatives. Visually the satellities at +.083 and +.130 are separated by approximately three times the width of the lines in the fourth order. The satellite at +.217 is definite and certain with the Cooper-Hewitt source. It appears possible that it is a double in the small arc, the weaker component being at +.217 and a slightly

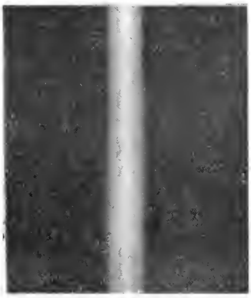
stronger one at about +.170. We are not prepared to make a final statement on this point at present. The effect is magnified in the reproduction, probably on account of the trailing of a dust particle across the plate during the vertical motion. In the case of λ 4358 the satellite at +.040 is clearly separated from the main line on some of our best plates, and shows plainly as a region of diminished brightness on all of the fourth- and fifth-order plates, but in the reproduction it appears to overlap and merge into the main line.

Seed's Panchromatic and Cramer's Spectrum plates were used for the yellow, Cramer's Medium Isochromatic for the green, and Seed's "27" for the violet. Our plates were measured on a Zeiss comparator, and each value is the mean of a number of separate determinations. All should be correct to within a few thousandths of an Ångström unit.

Aside from the very satisfactory results obtained with the grating the most interesting feature of the work was the verification, to such a remarkable degree, of the results obtained by Professor Michelson in 1892. It should be remembered that those results were absolutely the first in this field, that they were obtained by a method requiring such a high degree of personal skill that no one since has used it with success, and that the whole investigation was, in a sense, incidental to the more important task of determining the number of wave-lengths in the meter.

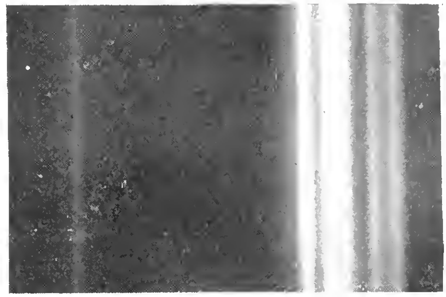
It does not seem to be generally understood that it is possible to determine whether a satellite lies on the red or violet side of a line with the Michelson interferometer. Although Professor Michelson made no attempt to do this, he pointed out very clearly in the original article how it may be done. The fringes at a minimum will lag behind a perfectly definite fractional part of a fringe if the satellite is on the violet side of the main line, and they will be a definite fractional part of a fringe ahead at a minimum if the satellite is on the red side. If the visibility-curve indicates satellites when there is no such shift at a minimum, there must be two satellites, equally distant from the main line, one on each side. By comparing the fringes with those of a line which has no satellites, like the red line λ 6438 of cadmium, the so-called phase-curve may be plotted, i. e., a curve in which the differences of path of the interfering beams are

PLATE VI



Scale: −0.1 0 +0.1

FIG. 5.—λ 5760

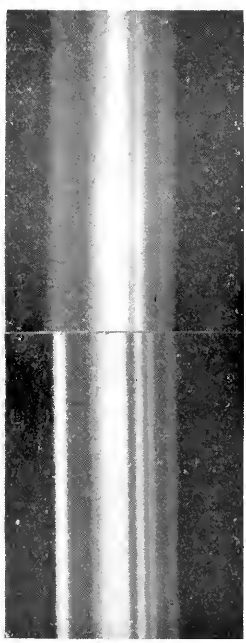


Scale: −1.0 −0.2 0 +0.2

FIG. 6.—λ 5790

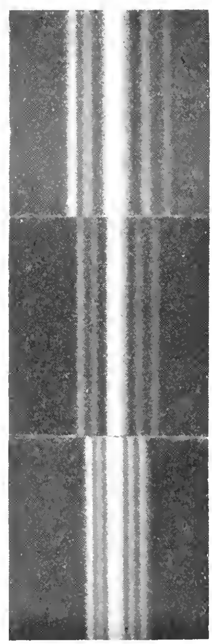
Small Arc

Cooper-
Hewitt



Scale: −0.2 0 +0.2

FIG. 7.—λ 5461



5th order

4th order

3rd order

Scale: −0.2 0 +0.2
3rd order

FIG. 8.—λ 4358

abscissae and the shift of the observed fringes due to satellites are ordinates. The visibility-curve and the phase-curve together give a definite solution, without ambiguity.

Although Professor Michelson's value for the distance of the satellite of λ 5769 has often been quoted, it does not seem to have occurred to anyone to re-examine the visibility-curve. The visibility-curves of λ 5769 and λ 5790 are reproduced in Fig. 9. A glance is sufficient to show that the minima of λ 5769 are a little more than twice as far apart as those due to the principal satellite of λ 5790. The satellite of λ 5769 must therefore be a little less than half as far from its main line as the principal satellite of λ 5790. A recomputation gives its distance as .043 Å. U. instead of .019 Å. U. as originally given. It seems odd that none of those who have quoted this latter value has noted the discrepancy.

It may have escaped the notice of some of the recent workers in this field that there is a faint line −.998 Å. U. from λ 5790. A faint satellite accompanies it at −.931. The constant of the echelons used by Janicki and Galitzin is about .543 at this wave-length, and unless light from these lines were excluded they would appear at +.088 and +.155. We have detected a line of intensity about 1/100 of that of the main line at +.085. On account of its great faintness, however, we regard it as possible that the line measured by Janicki and Galitzin as at +.085 may be due to the line at −.998, which would appear at +.088. We have been unable to detect any line near +.168 and regard it as possible that the line given by them there may be due to the faint line at −.931, which would appear to be at +.155. The discrepancy in wave-length is rather large but the line is exceedingly faint and hard to measure accurately.

Professor Michelson's visibility-curve for λ 5461 is reproduced in Fig. 10. He indicates a satellite very close to the main line on each side. He might equally well have chosen to regard them as being both on the same side. We have chosen this interpretation and have computed from the visibility-curve the distance of this satellite from the main line. The rise in the visibility-curve at about 315 indicates clearly that the main line is a close double. The calculation indicates a satellite at .010 which agrees with the observations of Fabry and Perot, who give a satellite at +.008, and of Barnes, who gives one

at $+.01$. We have added this satellite to the diagram, Fig. 3, placing it on the positive side. Workers in this field seem to be pretty well agreed that the main line is a close double, resolvable only with very high powers, and when the vapor-density is very low. Incidentally it may be pointed out that the visibility-curves as obtained with the

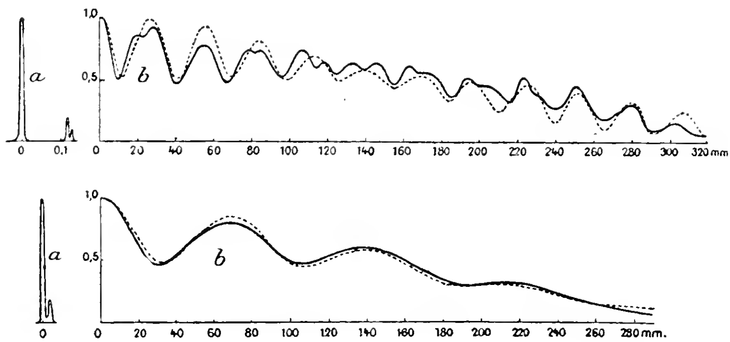


FIG. 9

Michelson interferometer form the only means of resolving lines which actually merge together, or of getting the distribution of light within a line. This, it will be recalled, was done in the case of H_α, λ 6563, long before the line was admitted to be double by many spectroscopists. It should also be borne in mind that an apparent doubling of a line may be due to a reversal. It should however as a

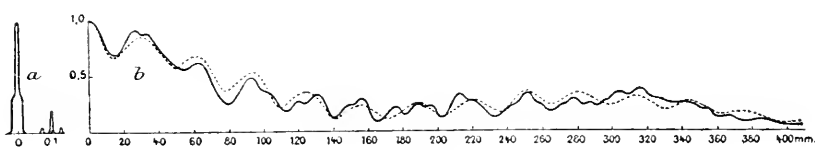


FIG. 10

rule be a simple matter to test whether a line is a true double or a reversed line. In the former case the separation should be most marked at low vapor-densities. In the latter case the apparent separation should increase with the vapor-density.

In the case of Fig. 10 it will be noticed that there are faint minima at roughly 112, 123, 140, 152, 164, 180, 190, 200, etc., which are not present in the dotted curve. These minima indicate that in the tube

used by Professor Michelson there was a faint satellite at a distance a little more than twice that of the main satellite. A calculation from the curve gives its distance as .23 Å. U., and it is undoubtedly the same as the line observed by us at −.233 Å. U. We have therefore added it also to the diagram, Fig. 3. Plate VI, Fig. 7, shows the variation of the relative intensity of this satellite when the source is a small mercury arc, instead of the Cooper-Hewitt lamp. In the Cooper-Hewitt lamp (the lower spectrum) this satellite is the principal one, while in the small mercury arc there are several other satellites of equal or of greater strength. The intensity of this satellite seems to be especially sensitive to a change in conditions, and in the tubes from which the visibility-curves were plotted it was undoubtedly one of the fainter satellites, and was therefore ignored.

The violet line λ 4358 is not easy to observe visually, but the results obtained in 1892 by Professor Michelson are again noteworthy. He indicates a satellite at .17, "and two fainter ones near the main line." If we put half of the main satellite on each side and the two fainter ones as indicated, we get the result indicated in Fig. 4. The considerable number of lines found by some observers may be the result of reversals as it is well known that in general the tendency to reverse increases toward the violet. It should also be borne in mind that great care must be taken to have the lines in sharp focus on the photographic plate or strange results will be obtained with this line as with others. Possibly also some of the optical parts used may have been sufficiently accurate for the yellow and green regions but not in this region of shorter wave-length and greater refrangibility. An instrument like the Lummer-Gehrcke plate which reflects the light would be especially sensitive to an error in the surfaces and an error which might be negligible in the green and yellow might produce serious results in the violet. The very excellent agreement of our results with those of von Baeyer for the longer wave-lengths, and the relatively poor agreement at λ 4358 suggest some possible source of error. It should be borne in mind however that the satellites may have a very different appearance in different forms of tubes. In the small arc referred to above, the five satellites of λ 4358 are all of nearly equal intensity.

The line at λ 4358 also brings out the greatest disadvantage of the echelon and interference plate, ambiguity due to overlapping orders. Galitzin describes his satellite at $+.126$ as very strong, and says further that it "erscheint zuweilen doppelt und bestehend aus zwei nahen Linien." On some of his plates he measured the position of these two lines, getting their wave-lengths as $+.118$ and $+.131$. We have found no line at $+.126$, but have a line at $+.118$. Galitzin gives his constant as .286 at this wave-length, and the line at $+.131$ might equally well have been taken as at $-.155$, in exact agreement with our line at $-.155$. He also gives the line at $-.092$ as very strong and in regard to it remarks, "Bei dem trabanten B_1, kann man ebenfalls eine Verdoppelung vermuthen, aber die Erscheinung ist sehr undeutlich," etc. We have a satellite at $-.088$ and another at $+.194$ which is exactly the wave-length which would be assigned to this satellite if it were regarded as on the positive side. We therefore regard Galitzin's line at $-.092$ as made up of our two lines, $-.088$ and $+.194$. Furthermore, the mean of his two lines at $+.053$ and $+.027$ is $+.040$, exactly the position of one of our satellites. We have therefore rearranged Galitzin's satellites under G′, Fig. 4 and Table IV, and the agreement with our results is excellent.

We do not feel that our results furnish the last word as to the resolution of the principal lines of mercury with gratings. When larger gratings become available, and a longer focus lens for the Littrow mounting, and when a mounting designed especially for steadiness and constancy of temperature is used, it may be possible to resolve into doubles some of the lines reported by us as single. Thus there seems to be a triplet on the violet side of λ 5461. This was indicated quite clearly in the echelon results obtained by Mr. Lemon, referred to above. Our line at $-.054$ might well be a blend of the lines given by Galitzin and von Baeyer at $-.048$ and $-.068$. The group has a fuzzy appearance with the grating and it is difficult to resolve the lines clearly on photographs. We feel quite confident, however, that all the lines reported by us are real, and that, aside from the possible resolution of some lines into doubles, there are no others in the Cooper-Hewitt source unless their intensity is of the order of 1/100 of that of the main lines.

In conclusion we desire to express our sincere thanks to Professor Michelson for the use of the splendid grating, and of his private laboratory in which all the photographs were taken.

RYERSON PHYSICAL LABORATORY
THE UNIVERSITY OF CHICAGO
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Minor Contributions and Notes

THE SIZE OF METEORS

In the November number of this *Journal* (p. 318), Professor Fabry criticizes my statement, published in the June number, as to the probable size of meteors. There are several reasons for the difference in our results, but the chief one depends on the fact that he assumes the intrinsic brightness of a meteor to be that of the cup of an electric arc, while I have assumed it to equal the light given out by the carbons in a horizontal direction. This light varies in different parts of the carbons, but I assumed an illuminated area of one-half inch in diameter, or practically one square centimeter (see *Harvard Annals*, **41**, 141) Assuming a horizontal illumination of 250 candle-power, the ratio of the intrinsic brilliancies that we have adopted stands at about 80 to 1.

In my investigation I chose the horizontal brilliancy because the most direct method of comparing a meteor with an arc light seemed to be to measure the latter photometrically from a known distance, expressing its brightness directly in stellar magnitudes. Three arc lights situated at a distance of about 2.5 km were carefully measured, and were found to give unexpectedly uniform results. This was twelve years ago, when the open arcs were used. It was concluded that at that distance their brightness was that of a star of magnitude −1.

That the temperature of a meteor is not very different from that of the carbons is shown by the fact that when about equally brilliant their colors are approximately the same, some meteors being more blue, and some more yellow, than the artificial source. However, just as in photographing an electric arc, while what we see on the ground glass are the luminous carbons, yet what we photograph is chiefly the blue arc between them; so with meteors, when we photograph their spectra during flight, what we find is a luminous gas, not an incandescent solid. This is very different from the case of an extremely hot body like the sun, where the light from the incandescent

gases is concealed by the glare from the incandescent solid or liquid particles.

No accurate determination of the size of a falling star is possible by any means, but if we reduce the estimated luminous area of the incandescent carbons to ten square millimeters, or one-tenth the value formerly adopted, the corresponding diameter of a falling star of the third magnitude would still be 5 or 6 centimeters, which would imply a mass to be measured in hundreds of grams, instead of milligrams, as usually stated.

The minimum diameter of a third-magnitude meteor as computed by Professor Fabry should be 5.42 mm, and not 25.4 as printed, evidently through a typographical error.[1] This result is based on his determination that a 0-magnitude star is equivalent to 2.1×10^{-6} candle-power at one meter distance. In the *Harvard Annals*, **61**, 69, it is shown from comparisons of *Arcturus* with a distant standard lamp, that a 0-magnitude star is equivalent to one candle-power at a distance of 526 meters. From this we deduce that a 0-magnitude star is equivalent to 3.61×10^{-6} candle-power, a result nearly twice as large as that given by Professor Fabry. This would lead to a diameter for a meteorite of the third magnitude of 7.0 mm. A fireball of the -2 magnitude would have a diameter of 7 cm. To suppose that such an insignificant body could emit a luminous atmosphere, such as sometimes occurs, measuring 1.5 km in diameter, and 15 to 50 km in length, is manifestly absurd.[2]

WILLIAM H. PICKERING

[1] This typographical error is corrected on p. 400, December 1909.—EDS.

[2] C. C. Trowbridge, *Astrophysical Journal*, **26**, 95, 1907.

Reviews

Der Bau des Fixsternsystems. Von Dr. Hermann Kobold. Braunschweig: F. Vieweg und Sohn, 1906. 8vo, pp. 256; Figs. 22. M. 4.

As a compendium of the results of researches made into the realms of the cosmogony of our visible stellar universe, this book is excellent.

The subject of the probable structure of the visible universe has always interested thinkers, whether they be astronomers or not, and many have been the conjectures, based on speculation, as to the ultimate destination of the stars and the laws governing their motions.

With the advent of astronomy of precision, with the discovery of proper motion, with the employment of spectrum analysis, fields hitherto closed have opened to the modern astronomer, so that speculation is giving way to scientific fact. But the leanness of material and its large probable errors still leave a wide range of interpretation of the deduced results, so that we must not be surprised if the views of various investigators differ to an alarming extent.

Kobold's *Bau des Fixsternsystems* is an attempt to gather and classify the results attained by various investigators, giving each its worth.

In Parts I and II the author gives brief summaries of the instruments and methods by which we have arrived at our conclusions as to the composition, disposition, and motion of the stars.

With Part III begins the exposition of the results of various investigators on the varied questions involved in the investigation of our universe, with comments and investigations by the author.

Thus the works of Kapteyn, Schoenfeld, Bakhuyzen, Ristenpart, Plassmann, the excellent works of Seeliger, and many others too numerous to detail, are all set forth, making it an excellent reference book. Add to this the frequent references to various publications in connection with every branch of the subject, and the book becomes the more valuable.

There are many illustrative tables and diagrams throughout the text, and at the end a table of large parallaxes, and another of proper motions greater than $0\overset{''}{.}5$.

As to the views adopted by the author, and his conclusions drawn therefrom, they may well be doubted in the light of more recent work upon the subject, especially as to the more or less random motion of the stars.

Benjamin Boss

A Treatise on Spherical Astronomy. By SIR ROBERT BALL. Cambridge: The University Press, 1908; New York: G. P. Putnam's Sons. Pp. xii+506. Price $3.75.

"By spherical astronomy I mean that part of Mathematical Astronomy which lies between the vast domain of Dynamical Astronomy on the one hand and the multitudinous details of Practical Astronomy on the other." Such is the author's definition of his subject-matter and it is coupled with the statement that his treatise is prepared for the use of the student familiar with the ordinary processes of plane and spherical trigonometry and having some knowledge of analytic geometry and the infinitesimal calculus. The reader thus defined will usually be a university undergraduate and the book is obviously addressed to the English university student. Before him there is spread with mathematical elegance a larger variety of subject-matter than is to be found in any similar treatise within our ken, e. g., the measurement of time, transformation of spherical co-ordinates, projection of maps, refraction, parallax, precession, aberration, eclipses, solar, lunar, and planetary phenomena, with excursions into celestial mechanics, the method of least squares, and the theory of instruments; the whole supplemented by a noteworthy collection of exercises and problems largely chosen from the mathematical tripos and other university examinations.

It is obviously unfair to judge such a work by the criteria applicable to the treatises of Chauvenet, Brünnow, or Newcomb, and yet it should not be left unsaid that, from the standpoint of the astronomer who desires a book of reference, or for the student who seeks to lay the foundation of practice in the arts of the astronomer, the book appears to the reviewer inadequate through the omission of matter too important to be ignored even in an elementary treatise. In this respect a single chapter must suffice in illustration of characteristic limitations of the work. In the twenty-nine pages devoted to atmospheric refraction, while much space is given to matter of little more than historic interest, such as the derivation of the refraction formulae of Cassini, Simpson, and Bradley, there is no suggestion of the incomparably more important modern work of Gyldén and Radau. Bessel's work finds recognition in one line of a footnote and tables of the refraction are wholly ignored save for brief reference to two minor adaptations, of British origin.

Per contra, from the standpoint of the student of mathematics, there is much to be commended and the experienced astronomer will find novelty and interest in the methods of approach to familiar problems. Probably the most conspicuous illustration of these is to be found in the chapter devoted to the "Generalized Instrument," in which the author constructs

a mathematical theory of a hypothetical instrument, of which the transit, the equatorial, altazimuth, almucantar, etc., are special cases whose respective theories result from the general theory by the introduction of suitable limitations. As the *Oxford Note-Book* puts it:

> The time is come, Sir Robert said,
> To make a great combine
> Of all the various instruments
> Down which a star may shine.
> The following simple formula
> Will bring them into line.

Admirable as the chapter is from the standpoint of analysis and valuable for its breadth of view, it presents points of analogy to a generalized confession of faith of which all creeds should be special cases, its theoretical comprehensiveness and practical limitations making it objectionable to the partisans of each. Thus most observers with a transit instrument would consider inadequate for practical uses a theory which ignores the spirit level as an auxiliary apparatus and suggests only nadir observations as a means of determining the inclination of the axis.

The book will be seen at its best if considered as a textbook of a special kind of applied mathematics in which the student who seeks concrete applications of pure theory will find material excellently suited to his purpose, in which the student of education may obtain interesting glimpses of English university practice, and in which the future expositor of spherical astronomy will find much that is worthy of his consideration.

G. C. Comstock

The Story of the Comets. By George F. Chambers. Oxford: The Clarendon Press, 1909. 8vo., pp. 256; Figs. 106. 6*s.* net.

Chambers' *Handbook of Astronomy* is a considerable work, treating of the sun, moon, planets, and comets. The present book comprises the portion of the *Handbook* which was devoted to comets, which is now enlarged and brought up to date under the above title, with the added phrase, "simply told for general readers."

Chambers' books on astronomy are always interesting. They have been, for a great many years, a storehouse to which the public, and also the astronomers, could apply for general information on the subject. Today there are many other sources of general information of this kind, but Chambers' books are still of much value for reference, especially where

the information sought belongs to the older astronomy. It is therefore a pleasure to see the present book under its new form, and modernized. There is much improvement in this volume over the previous work. We still find in it, however, many of our old friends, the wood engravings of former days. These pictures will always have a special charm about them, notwithstanding that they seldom look very much like the objects they represent. Some of these woodcuts are classical and have been borrowed freely for use in many other works.

Some photographs of recent comets have been introduced, but there is not so varied and complete a selection as should be in a work of this kind. Some of these photographs have been unintentionally falsified in the reproduction; especially unfortunate in this respect is Fig. 22, Plate VII, where the entire comet seems to be enveloped in an outer casing, or sheath, of nebulosity, which is fairly definite, and which no one could possibly distinguish from a real cometary phenomenon. Fig. 23 on the same plate suffers somewhat from a similar defect, which, however, is not so misleading. Fig. 99 of Plate XXVI is falsified outright by an attempt of the engraver to get a black sky by cutting away the sky about the image. This gives a very incorrect impression of the comet, which is bizarre enough without the ill-treatment it has received. Fig. 19 of Plate IV is defective in the same manner—the engraver having removed all the sky and stars, leaving what he thought might be the comet. The result gives the comet a rather gay appearance, which, however, is not nearly so startling as the original. The two photographs of Plate VIII are good reproductions. All these photographs, though they are not so credited, were made by the present reviewer, which will explain the disappointment in the unsatisfactory reproduction of all but the last two. By some mistake, the exposure time of one of the photographs is wrongly given. Fig. 22, Plate VII, should read September 30^d 20^h 22^m G. M. T. instead of 17^h 16^m.

There are two good reproductions, Plates V and VI, which are from excellent photographs by Mr. P. Morris. A serious omission in these two plates, however, is the exact time of exposure. The dates alone are given. Especially is this omission unfortunate in the photograph of October 15, 1908, where the exact time is of the utmost importance. In the days of hand drawings of comets, this omission would not have been serious, but today, when such illustrations are the actual photographs themselves, it is very important that the exact time of exposure be given, whether the picture appears in a popular work or not.

The histories, with fairly good descriptions, of all the great comets are gone into quite thoroughly. In this respect there is an extensive account

of Halley's comet which is important at this time when this celebrated object is once more visible in our skies.

A new chapter has been introduced which is of great interest from a literary standpoint. It is devoted to "Comets in History and Poetry." The chapter on "Comets in the Spectroscope" is important also, for it gives somewhat of a historical account of the various efforts (successful and otherwise) to investigate the nature of these bodies from a spectroscopic standpoint. In this chapter Mr. Chambers justly emphasizes the importance of the objective prism for cometary work where motion in the line of sight is not considered.

The title of Plate XIX[a], "The Comet of 1862 (iii)," is certainly misleading to the lay reader, who will see in the drawings, which represent a telescopic view of a small portion only of the phenomena connected with the nucleus, a representation of the comet as a whole. A less misleading title would be "Phenomena of the Nucleus and Jets in the Head of the Comet of 1862 (iii)." The use of the small Roman numeral, instead of the larger one, designating a comet by the order of its perihelion passage, as "the Comet of 1861 (i)" instead of "Comet I 1861," is to be regretted.

In speaking of the improvements in this book, one of the best of the changes that have been made, and one that all fair-minded persons will heartily applaud, is in the statement concerning the observations of a double comet which appeared in 1860, and was observed only by Liais in Brazil. In the edition of 1889, p. 409, the statement is made that—

> It is to be regretted that this object remained visible for so short a time . . . . and that our knowledge of it depends on the authority of but one observer, and he a Frenchman.

In the present volume, p. 16, the obnoxious phrase, "and he a Frenchman," is omitted.

The book is to be commended to those, whether astronomers or general readers, who take any interest in these wonderful bodies, the comets.

E. E. B.

An Atlas of Absorption Spectra. By C. E. Kenneth Mees. New York: Longmans, Green & Co., 1909. Pp. 74.

The absorption of the various dyes used in the manufacture of light-filters for photographic purposes has been investigated in the research laboratory of Wratten and Wainwright under the direction of Mees, and the results published in a very useful little atlas of octavo size. The twelve

pages of text describe the material and apparatus used, and give an index of the dyes and filters. Then follow 43 pages containing 170 halftone cuts showing spectral intensity-curves of the dyes. Each cut carries an approximate scale of wave-lengths. There are also given the curves for 70 stock filters, including 7 so-called "Monochromat" filters which transmit 300 to 600 Ångströms.

This atlas supplements that of Uhler and Wood, since it gives absorption spectra as far into the red as λ 7800, but nothing in what is usually considered the "infra-red," though such an extension is mentioned in the text. By the aid of this atlas one may select a commercial filter suitable for almost any photographic purpose.

J. A. P.

Cours d'Astronomie. Seconde Partie: Astronomie Pratique. Par H. ANDOYER. Paris: A. Hermann et fils, 1909. Pp. 304, with 60 diagrams. Fr. 10.

Following as it does the author's previous work on theoretical astronomy, the present volume completes a textbook for the technical student of astronomy which is quite usable. The author does not attempt to write a complete treatise on the subject but, in common with most writers of textbooks, confines himself to introducing the reader to general methods, leaving him prepared to follow whatever special studies he may desire.

The work is divided into three books of seven chapters, with a short supplementary chapter treating briefly of the determination of an elliptic or parabolic orbit. The first book deals with principles of computing and the method of least squares. This book contains 63 pages. Book II has 109 pages, and treats of instruments, classified as accessory, principal, and miscellaneous. The last book contains 90 pages and deals with observations: first those made for deriving the fundamental constants of astronomy, such as precession and nutation, then those made for determining the observer's position on land and sea. In each case the necessary formulae are derived and the final equations are given in convenient form.

The practical astronomer in looking through this recent work notices with regret the absence of a chapter on the determination of the place of a celestial body from photographic plates, half a page alone being accorded to this subject. The photographic method is now coming into such widespread use in precise astrometry that the value of the volume would have been enhanced appreciably had a score or two of pages been devoted to

deriving the formulae necessary to both rigorous and approximate reductions of stellar photographs.

The book is well printed, and the diagrams are good. It is to be hoped that there will be a call for a second edition of the first part, so that it may have the same typographical appearance: it may be recalled that it was reproduced by the zinc process from a manuscript copy.

O. J. L.

THE

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APPLICATION DE LA MÉTHODE INTERFÉRENTIELLE À LA MESURE DE TRÈS PETITS DÉPLACEMENTS DE RAIES. COMPARAISON DU SPECTRE SOLAIRE AVEC LE SPECTRE D'ARC DU FER. COMPARAISON DU CENTRE ET DU BORD DU SOLEIL

PAR CH. FABRY ET H. BUISSON

Un grand nombre de problèmes d'astrophysique conduisent à étudier de petits changements ou de petites différences de longueurs d'onde. La méthode ordinairement employée consiste à faire des mesures micrométriques de positions de raies dans un spectre très dispersé. Cela ne va pas toujours sans difficultés; un certain nombre de causes d'erreur sont à éviter: déplacements ou déformations de l'appareil, dissymétries d'éclairement, variations de température. Dans certains cas, on a été amené à employer un artifice délicat pour surmonter ces difficultés: comparaison de la raie variable avec des raies telluriques très voisines; cela n'est évidemment applicable que dans des cas très particuliers et l'on peut avoir un léger doute sur la fixité des repères lorsque la hauteur du soleil varie.

Les méthodes interférentielles permettent de mesurer de petites variations de la longueur d'onde d'une radiation en valeur absolue, sans faire intervenir aucun repère. La chose invariable est une dimension matérielle (épaisseur d'un étalon) dont il est facile de maintenir et de contrôler la constance. La précision et l'invariabilité de l'appareil dispersif n'interviennent pas: cet appareil ne sert qu'à séparer les radiations.

Nous avons fait quelques applications de ces méthodes. Nous nous proposons, dans ce mémoire, d'en faire la théorie, de décrire les appareils employés, et d'exposer les résultats obtenus.

I. MÉTHODE ET APPAREILS

Le dispositif employé se compose de deux parties: l'appareil interférentiel, et l'appareil dispersif qui sépare les interférences dues aux diverses radiations.

L'appareil interférentiel est un système de deux surfaces argentées maintenues au parallélisme et à distance invariable pendant les expériences. Le plus souvent c'était un *étalon* de la forme construite par Jobin.[1] Il fallait le mettre à l'abri des variations brusques de température, en particulier de celles qui peuvent être produites par la présence des observateurs; il a suffi pour cela de le placer dans une boite en carton munie des ouvertures convenables pour laisser passer la lumière.[2] Nous avons employé des épaisseurs de 2. 5, 5, et 10 mm. Exceptionnellement, pour d'autres épaisseurs, nous avons employé l'interféromètre.[3]

Chaque radiation monochromatique qui traverse la lame d'air donne un système d'anneaux à l'infini; un objectif de 26 cm de foyer en projette l'image dans son plan focal.

Un spectroscope sans astigmatisme placé ensuite sépare les aspects relatifs à chaque radiation. Quand le spectre est à raies très nombreuses, la dispersion de ce spectroscope doit être grande pour bien séparer les diverses raies. Nous avons presque toujours employé un spectroscope du type autocollimateur à réseau plan de Rowland (568 traits par mm, 8×5 cm de partie striée), avec objectif de 3.10 m de foyer. On fait défiler le spectre par simple rotation du réseau, au moyen de cordons que l'observateur a sous la main. Ce type de spectroscope est très commode et très peu encombrant: il tient tout entier sur une poutre de 3.50 m de long et 25 cm de large. Un inconvénient provient de la lumière réfléchie par l'objectif, qui donne un fond lumineux dans le champ. Avec l'objectif achromatique que

[1] *Astrophysical Journal*, **15**, 81, 1902.

[2] Les étalons dont nous nous sommes servis étaient en acier. L'emploi d'étalons en invar aurait eu de grands avantages.

[3] *Astrophysical Journal*, **13**, 265, 1901.

nous employons, cette lumière réfléchie se partage en trois faisceaux dont l'un donne une image réelle de la fente et les deux autres des images virtuelles. L'image réelle, qui se trouve à 60 cm en avant de l'objectif, est interceptée par un fil métallique de 1 mm de diamètre. La lumière de l'une des images virtuelles, située prés de l'objectif et en arrière, est arrêtée par un petit écran de 15×6 mm collé sur l'objectif. Enfin, en inclinant très légérement l'objectif, on rejette en dehors du champ le troisième faisceau, qui est très peu divergent. On arrive ainsi à s'affranchir de la lumière réfléchie, sans diminuer sensiblement la surface utilisée du réseau.

Nous utilisons ordinairement le troisième spectre, ce qui donne une dispersion de 0.58 mm par ångström.

Dans certains cas, pour avoir plus de lumière, nous avons employé un spectroscope autocollimateur à prismes, avec objectif de 1 mètre de distance focale, et deux prismes de flint.[1] Dans la région 4300 un ångström occupe 0.15 mm.

Dans tous les cas, la fente du spectroscope est dans le plan focal de l'objectif qui projette les anneaux, suivant un diamètre de ceux-ci. L'arrangement est celui qui nous a servi pour la mesure des longueurs d'onde des repères fondamentaux du spectre.[2]

La surface utilisée de l'appareil interférentiel est très petite: elle est limitée par un écran percé d'une ouverture, dont les dimensions n'excèdent pas quelques millimètres. La distance de cette ouverture à l'objectif qui projette les anneaux est telle qu'une image réelle en soit projetée, à travers la fente, sur le réseau. De cette manière, toute la lumière qui a traversé l'ouverture et la fente est finalement utilisée par le spectroscope.

Théorie des interférences produites par les raies noires d'un spectre.—Dans le cas d'un spectre à raies brillantes, la théorie du phénomène ne présente aucune difficulté: chaque image monochromatique de la fente est réduite à un certain nombre de points brillants, intersections de la fente avec les anneaux d'interférence produits par la radiation correspondante. La largeur maxima de la fente n'est limitée que par la condition que les images des différentes raies n'empiètent pas.

[1] *Journal de Physique* (4), **3**, 204, 1904.

[2] *Astrophysical Journal*, **28**, 169, 1908.

Dans le cas du spectre solaire, on a un spectre continu avec des raies noires, et les mesures doivent porter sur celles-ci. On peut alors, dans des conditions convenables, obtenir l'aspect complémentaire de celui qu'on a avec des raies brillantes, comme si l'on avait des interférences produites par les raies noires, c'est-à-dire par des radiations absentes dans le spectre. Ce phénomène, en apparence paradoxal, s'explique par les considérations suivantes.

A travers l'appareil disposé comme il a été indiqué ci-dessus, faisons passer une lumière donnant un spectre rigoureusement continu. Supposons le spectroscope à fente infiniment fine, et ayant un pouvoir de définition infini, de telle manière qu'à chaque point du champ corresponde une radiation rigoureusement définie. Dans le spectre, la lumière se répartit alors en lignes brillantes formant des cannelures légèrement courbes: d'un point à un autre de la fente, la différence de marche varie, elle est maximum au point où se projette le centre des anneaux, et décroît de part et d'autre comme le carré de la distance à ce point. A mesure que l'épaisseur de l'appareil interférentiel augumente, les franges deviennent plus serrées. Ces cannelures ont l'aspect ordinaire aux franges des lames argentées: ce sont des lignes brillantes dont la largeur est faible par rapport à celle des espaces noirs qui les séparent; cet effet est d'autant plus marqué que le pouvoir réflecteur est plus élevé. En somme, pour chaque radiation, c'est-à-dire pour chaque ligne verticale du spectre, les interférences ramassent la lumière en certains points.

Si l'on élargit la fente, les lignes brillantes qui forment les cannelures s'élargissent d'une quantité égale à la largeur de la fente, et lorsque chaque bande brillante rejoint la bande voisine les cannelures disparaissent. L'aspect est devenu celui d'un simple spectre continu, comme si l'appareil interférentiel était enlevé; toutefois, la constitution de ce spectre est très différente de celle d'un spectre continu avec fente large. Dans ce dernier cas, chaque radiation monochromatique se répartit uniformément sur un rectangle, image de la fente, et en chaque point on a un mélange de radiations. Au contraire, dans le spectre dont l'aspect est devenu continu par disparition des cannelures, chaque radiation occupe seulement des lignes horizontales dont la longueur est égale à la largeur de la fente. Les radiations voisines s'échelonnent en hauteur, avec un très petit décalage horizontal

correspondant à la dispersion du spectroscope. Les petits traits de lumière monochromatique remplissent tout le champ lorsque la fente a la largeur voulue, et l'œil, qui n'est pas un appareil spectroscopique, ne distingue pas le spectre ainsi obtenu d'un spectre continu ordinaire.

Pour une largeur de la fente plus grande que celle-là, les cannelures reparaissent, chaque trait empiétant sur le suivant.

D'autre part, pour un pouvoir de définition limité, les cannelures ont sensiblement leur aspect théorique lorsqu'elles sont peu serrées (différence de marche faible); si la différence de marche va en augmentant, les cannelures perdent d'abord leur aspect de bandes brillantes fines, s'estompent, et finissent par disparaître lorsque leur intervalle tombe au dessous du pouvoir de définition du spectroscope. On supposera que l'on n'arrive pas à ce cas.

Supposons maintenant une raie noire dans le spectre. Elle est forcément de largeur finie, c'est-à-dire que toutes les radiations comprises entre deux limites déterminées sont absentes, ou d'intensité négligeable. Avec une fente étroite, on aura les cannelures, lignes brillantes fines, coupées par la raie noire. Celle-ci ne se manifeste qu'aux points où elle rencontre une cannelure brillante, par une interruption de la cannelure. Cet aspect n'est pas commode pour les mesures, et d'ailleurs la luminosité est faible. Elargissons la fente. Cela revient à juxtaposer des aspects analogues au précédent, mais déplacés dans le sens horizontal. Les cannelures élargies sont alors coupées par des traits noirs horizontaux. On a alors dans le spectre deux espèces d'intervalles sombres: les intervalles entre les cannelures brillantes, et les traits noirs relatifs aux radiations absentes; ce sont ces derniers qui sont intéressants pour les mesures; il y a intérêt à faire disparaître les premiers. C'est ce qui a lieu lorsque la fente a la largeur qui fait disparaître les cannelures. Il ne reste plus alors que les traits noirs, dont la largeur dans le sens horizontal est égale à la largeur de la fente, alignés verticalement sur l'image de la raie, et qui se détachent sur un fond uniforme.

Ce que l'on a dit plus haut sur la constitution du spectre continu dont on a fait disparaître les cannelures fait d'ailleurs voir immédiatement ce dernier résultat: les diverses radiations sont séparées en hauteur, et l'on obtient des traits noirs correspondant à celles qui manquent.

L'épaisseur, dans le sens vertical, des traits noirs correspondant à une raie, dépend de la largeur de celle-ci. On obtient ainsi des rectangles noirs; lorsque l'épaisseur de chacun est devenue assez grande pour qu'il rejoigne le rectangle voisin, les interférences cessent d'être visibles, et la raie est uniformément noire dans le sens de sa hauteur. Cela arrive lorsque l'ordre d'interférence varie d'une unité dans la largeur de la raie noire. La largeur de la raie, dans un spectre parfaitement pur, est alors égale à la distance de deux cannelures. Avec un appareil interférential d'épaisseur e, la largeur maxima $d\lambda$ d'une raie pouvant donner des interférences est donnée par

$$\frac{d\lambda}{\lambda}=\frac{\lambda}{2e}=\frac{1}{p}$$

p étant l'ordre d'interférence. Inversement, pour une raie donnée, les interférences cesseront d'être observables lorsque l'épaisseur de l'appareil interférentiel dépassera la limite e donnée par la même équation.

Dans tout ce qui précède, on a raisonné comme si le pouvoir réflecteur des lames argentées était égal à 1, ce qui produit des cannelures infiniment nettes. En réalité il n'en est pas ainsi; des cannelures sont légèrement estompées. Il en résulte une limite du pouvoir de définition interférentiel, tout-à-fait analogue à la limite du pouvoir de résolution spectroscopique, et qui fait que les raies de largeur inférieure à une certaine limite ne sont pas visibles pour une différence de marche donnée. Il en résulte que, pour l'observation d'une raie donnée, l'épaisseur de l'appareil interférentiel ne doit pas descendre au dessous d'une certaine limite, qui peut être évaluée à un dixième de la limite supérieure.

Calcul et observations.—Supposons que l'on ait à étudier de petits déplacements d'une raie; soient λ et λ' les deux valeurs successives de la longueur d'onde. Si l'on mesure les diamètres d'un même anneau correspondant à ces deux valeurs, on peut calculer la variation de longueur d'onde $\lambda'-\lambda$. Soit en effet l'anneau d'ordre P, a et a' ses diamètres angulaires successifs. L'ordre d'interférence au centre est, dans le premier cas,

$$p=P\left(1+\frac{a^2}{8}\right)=\frac{2e}{\lambda},$$

en appelant e la distance des deux surfaces argentées.

De même, avec la radiation λ'

$$p' = P\left(1 + \frac{\alpha'^2}{8}\right) = \frac{2e}{\lambda'} .$$

On en déduit facilement:

$$\lambda' - \lambda = \lambda \frac{\alpha^2 - \alpha'^2}{8} .$$

α et α' sont les diamètres angulaires exprimés en radians. Si l'on mesure les diamètres linéaires d et d' d'images réelles des anneaux projetées au moyen d'un objectif de distance focale f, on aura:

$$\lambda' - \lambda = \lambda \frac{d^2 - d'^2}{8f^2} .$$

La seule détermination qui doive être faite avec précision est celle de la différence des diamètres d'anneaux.

Aucune des corrections qui interviennent dans les mesures absolues (changement de phase par réflexion, dispersion de l'air) n'est à considérer.

Les observations peuvent être faites visuellement ou par photographie.

Dans les observations visuelles, on mesure directement les diamètres angulaires: l'appareil interférentiel peut subir de petites rotations autour d'un axe horizontal perpendiculaire au faisceau lumineux, ce qui déplace le centre des anneaux sur la fente. On peut ainsi amener successivement les deux extrémités du diamètre d'un anneau sur un fil horizontal fixe placé dans le plan du spectre. L'angle dont il faut faire tourner l'appareil interférentiel est égal au diamètre angulaire de l'anneau. On le mesure en visant, avec une lunette fixe, dans un miroir lié à l'appareil interférentiel, l'image d'une échelle divisée.

Dans le cas d'observations photographiques, on mesure sur le cliché les diamètres linéaires des anneaux au moyen d'un comparateur.

Dans la comparaison du spectre solaire avec celui d'une source terrestre, il faut tenir compte du déplacement de l'observateur par rapport au soleil.

II. COMPARAISON DES LONGUEURS D'ONDE DES SPECTRES DE L'ARC ET DU SOLEIL

La figure 1 représente le dispositif employé pour l'utilisation du faisceau solaire.

A est un héliostat polaire, muni d'un miroir plan de 20 cm de diamètre. Le faisceau, renvoyé suivant l'axe du monde, tombe sur l'objectif B de 3 m de foyer, et traverse un tunnel percé dans la muraille de la salle. Il se réfléchit sur le miroir C qui le renvoie verticalement, et enfin sur le prisme à réflexion totale D qui le dirige horizontalement; en F on obtient l'image réelle du soleil, qui a 28 mm de diamètre. Ce dispositif est très commode dans les conditions où nous nous trouvons: il tient très peu de place; en projection horizontale, il n'y a qu'une très petite distance entre l'héliostat et l'image du soleil. Les miroirs A et C sont de bonne qualité; le second, qui reçoit un faisceau déja notablement rétréci n'a que 8 cm de diamètre. La troisième réflexion se fait sur un prisme à réflexion totale; celui-ci ne gâte pas l'image, qui se forme très près de lui. Cette dernière réflexion permet de renvoyer la lumière dans n'importe quelle direction du plan horizontal. L'image du soleil peut subir de petits déplacements dans son plan par de petites variations d'orientation du miroir C.

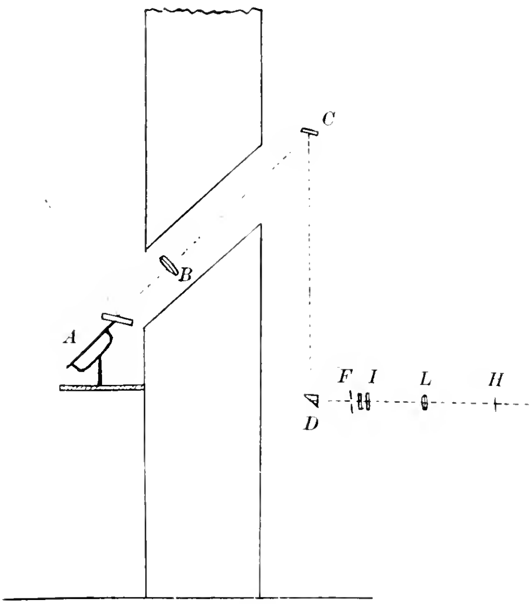


FIG. 1

Dans le plan F, où vient se former l'image du soleil, est un écran percé d'une ouverture qui limite la région utilisée du soleil. Immédiatement après est placé l'appareil interférentiel I. L'image des anneaux est projetée sur la fente H du spectroscope au moyen de la lentille L.

Echauffement de l'appareil interférentiel dû au faisceau solaire.— Nous avons rencontré une difficulté due à l'échauffement, par le

faisceau solaire, des lames de verre de l'appareil interférentiel: dès que ce faisceau traverse l'appareil, les surfaces se déforment. La lame d'air n'est plus limitée par des surfaces planes, et son épaisseur dans la partie centrale subit une diminution notable. Nous sommes arrivés à nous affranchir complètement de cette difficulté par l'emploi d'un certain nombre de précautions:

1. Des lames de quartz se déforment infiniment moins que des lames de verre, parce qu'étant beaucoup moins absorbantes pour l'infra-rouge, elles ne s'échauffent presque pas.

2. On peut diminuer beaucoup l'échauffement des lames de verre en absorbant l'infra-rouge. La substance qui nous a donné le meilleur résultat est une solution de sulfate de cuivre à 4 pour 100 sous une épaisseur de 16 mm. L'énergie totale de la radiation solaire est réduite au sixième de sa valeur, et dans le spectre visible l'absorption n'est sensible que pour les parties extrêmes du spectre.

3. Il est rationnel de ne laisser passer à travers l'appareil interférentiel que la lumière réellement utilisée par la suite. Or, sur la fente du spectroscope se forme une image presque nette de l'objectif *B* (fig. 1) placé à 3 m de la lentille de court foyer qui projette les anneaux sur la fente. On peut donc, sans inconvénient, diaphragmer beaucoup l'objectif *B* dans le sens de la largeur. Nous laissons libre seulement une ouverture rectangulaire de 1 cm de large sur 12 de long. Dans le plan de la fente du spectroscope, il reste une bande lumineuse étroite à bords un peu flous, dont la fente occupe le milieu.

Arc.—On s'est borné à étudier le spectre du fer. Il est produit dans les mêmes conditions que celles employées pour obtenir notre atlas de ce spectre:[1] arc entre tiges de fer de 7 mm de diamètre, alimenté en courant continu sous tension de 220 volts, avec des intensités de courant variables selon les cas.

Dans la suite de cette étude (voir plus loin) nous avons été amenés à employer l'arc dans le vide. Nos premières expériences ont été faites avec un appareil improvisé de la manière suivante: deux tiges de fer verticales sont placées dans un ballon de verre; l'une est mastiquée dans une tubulure à la partie inférieure; l'autre peut coulisser dans une seconde tubulure placée à la partie supérieure, pour l'al-

[1] *Annales de la Faculté des sciences de Marseille*, **17**, 111. Hermann, éditeur, Paris 1909.

lumage et le réglage de l'arc. Une troisième ouverture permet de faire le vide; un col horizontal, fermé par une glace laisse passer la lumière. On évite l'échauffement par une circulation d'eau sur les parois du ballon, ou en l'immergeant complètement dans l'eau.

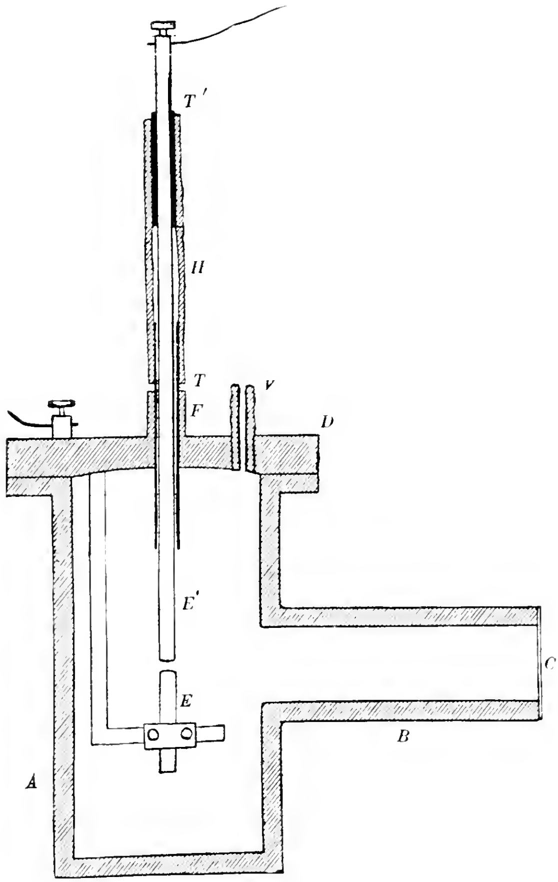


FIG. 2

Nous avons depuis construit un appareil plus commode (fig. 2). Le cylindre de fonte *A* porte un col horizontal *B* fermé par une lame de verre ou de quartz *C* qui laisse passer la lumière. Le couvercle rodé *D* porte une ouverture *V* reliée à la pompe. L'une des électrodes *E* est fixée au couvercle par l'intermédiaire d'une pince qui permet de la mettre en place. L'autre électrode *E'* peut glisser verticalement:

elle est mastiquée dans le tube de verre T' et passe librement à travers le tube de verre T mastiqué dans la tubulure F du couvercle. Les deux tubes de verre TT' sont reliés par le tube de caoutchouc H qui assure l'étanchéité, et laisse un jeu suffisant pour l'allumage et le réglage de l'arc.

A basse pression, l'arc est stable si les électrodes sont couvertes d'une goutte d'oxyde fondu, qui se forme spontanément lorsque l'arc est produit à l'air libre. Nous avons opéré sous une pression de quelques millimètres. Dans ces conditions, l'arc est beaucoup moins lumineux qu'à la pression atmosphérique. Nous employons un courant de 8 ampères.

Mode opératoire.—L'arc est placé latéralement par rapport au faisceau solaire. Son image est projetée sur la petite ouverture qui est devant l'appareil interférentiel, au moyen d'une lentille et par réflexion sur un prisme à réflexion totale. L'appareil reçoit la lumière de l'arc ou celle du soleil selon que le prisme est mis en place ou enlevé.

L'ouverture du diaphragme qui limite le faisceau est un rectangle de 8 mm sur 5; elle est telle que son image couvre complètement le réseau. L'image du soleil est centrée sur l'ouverture, de façon à éliminer toute influence de la rotation solaire.

Dans le cas de mesures visuelles, on fait une série de pointés alternativement avec les deux sources. Pour les mesures photographiques, on fait deux poses avec une des sources, séparées par une pose avec l'autre. Si l'appareil interférentiel n'a subi aucune variation, la première et la dernière pose doivent donner des diamètres d'anneaux identiques. En réalité, la différence entre les deux valeurs de l'ordre d'interférence au centre ne dépasse pas 0.02. On prend la moyenne des résultats des mesures obtenues sur les deux poses extrêmes, pour la comparer avec le résultat de la mesure sur la pose intermédiaire.

Comparaison du spectre solaire avec celui de l'arc dans l'air.—Il a été soupçonné depuis longtemps qu'il existe des différences entre les longueurs d'onde dans l'arc et le soleil. Ces différences se sont montrées dans le travail de Rowland. Une série de mesures comparatives a été faite par Jewell.[1] D'une façon générale, les longueurs d'onde dans le spectre solaire se trouvent un peu plus grandes que dans

[1] *Astrophysical Journal*, **3**, 89, 1896.

le spectre de l'arc, mais avec de nombreuses exceptions. L'accroissement de longueur d'onde lorsqu'on passe de l'arc au soleil, peut être expliqué par la pression de la couche renversante, et la valeur du déplacement peut donner la valeur de cette pression. Mais l'existence de déplacements de sens inverse laisse un doute sur l'explication et sur les résultats.

Dans nos mesures, nous ne nous sommes pas attachés à étudier le plus grand nombre de raies possible. D'autre part, nous avons éliminé toutes les fortes raies, qui sont trop larges pour des mesures précises.

Nous avons étudié par photographie la région 4530-4060; c'est à cette région que se rapporte le travail de Duffield[1] sur le déplacement des raies par la pression. Visuellement, nous avons étudié la région verte, et en outre quelques raies dans le jaune et le rouge.

Les nombres du tableau suivant sont des moyennes de résultats obtenus, le plus souvent, avec des épaisseurs différentes d'appareil interférentiel: nous avons employé des épaisseurs de 2.5 et 5 mm.

Pour un certain nombre de raies, ce tableau renferme (3^{me} colonne) une valeur de la différence soleil—arc *déduite des mesures absolues.* Les nombres de cette colonne ont été obtenus en comparant les valeurs absolues mesurées dans les deux spectres au moyen d'expériences complètement indépendantes: (1) dans le spectre solaire, valeurs absolues obtenues en 1900 par Perot et Fabry;[2] (2) dans le spectre de l'arc, valeurs absolues déterminées par Fabry et Buisson[3] en 1908, dont la plupart ont été publiées, et quelques-unes mesurées plus tard sur les mêmes clichés. Il y a une concordance très satisfaisante entre les différences directement mesurées et celles obtenues de cette manière indirecte. Cela constitue une vérification intéressante de l'exactitude des diverses séries de mesures.

Quelques-unes de nos raies ont été aussi étudiées par Jewell: la quatrième colonne donne ses résultats, qui sont en accord satisfaisant avec les nôtres.

Dans ce tableau, comme dans tous les suivants, les différences de longueurs d'onde sont exprimées en millièmes d'ångström.

[1] *Philosophical Transactions*, A, **208**, 111, 1908.

[2] *Astrophysical Journal*, **15**, 73, 261, 1902.

[3] *Ibid.* **28**, 169, 1908.

TABLEAU I

Longueurs d'onde (système internat.)	$\lambda_{\odot} - \lambda_{arc}$	Mesures absolues	Jewell	Groupe*	Longueurs d'onde (système internat.)	$\lambda_{\odot} - \lambda_{arc}$	Mesures absolues	Jewell	Groupe*
4062.45	+ 4		±0	S†	4859.75	− 7			DR
4118.55	+ 8			S	4871.34	−15			DR
4127.61	+ 2			S	4919.00	− 8			DR
4134.68	+ 5			S	5123.73	+10	+12		S
4153.92	−25			DR‡	5171.61	+25	+21		
4154.51	+ 3			S	5266.57	± 0			DR
4154.82	− 6			DR	5269.55	+12			S
4158.80	+ 8			S	5281.80	− 8			DR
4175.65	+ 4			S	5283.63	+ 5			S
4181.76	+ 2			S	5302.31	− 6			DR
4187.04	− 3			DR	5324.19	± 0			DR
4191.44	− 3			DR	5339.95	−11			DR
4202.04	+18		+ 6		5341.03	+18			S
4216.19	+11			S	5364.88	+30			DV
4222.22	−10		−14	DR	5365.41	± 0			DR
4227.44	−29			DR	5367.48	+30	+27		DV
4233.61	− 8			DR	5369.98	+22			DV
4235.95	−19			DR	5383.38	+23			DV
4250.12	−14			DR	5393.18	− 8			DR
4250.78	+23			DV§	5397.15	+13			S
4271.16	−18			DR	5405.78	+15			S
4282.41	+ 7			S	5410.92	+26			DV
4337.05	+ 7			S	5415.22	+27			DV
4352.74	+ 4		+ 7	S	5424.09	+30			DV
4369.77	+ 6		+ 4	S	5434.53	+ 7	+12		S
4375.93	+12		+10	S	5497.52	+ 5	+13		S
4422.57	+ 4			S	5501.47	+ 9			S
4427.31	+ 4			S	5506.78	+ 8	+ 9		S
4430.61	+ 6			S	5586.77	+ 3	+ 6		S
4442.34	+12			S	5763.01	−14	−11		DR
4443.19	+ 7			S	5934.68	−25	−19		DR
4447.72	+ 7		+ 6	S	5987.08	+19	+15		
4461.65	+ 9			S	6065.49	+10	+11		S
4466.55	+14			S	6230.73	+12	+11		S
4531.15	+ 5			S	6393.61	+ 4			S
4786.83	+ 6			S	6408.03	−20	−22		DR
4789.66	+ 8			S					

* La 5me colonne donne l'indication des groupes auxquels appartiennent les diverses raies (voir plus loin).

† S désigne les raies à élargissement symétrique.

‡ DR désigne les raies qui s'élargissent vers le rouge.

§ DV désigne les raies qui s'élargissent vers le violet.

En résumé, l'existence des différences entre le soleil et l'arc est incontestable. Les déplacements sont très différents d'une raie à une autre. Dans la majorité des cas, quand on passe de l'arc au soleil, on a un accroissement de longueur d'onde de quelques millièmes d'ångström, mais d'assez nombreuses raies se comportent autre-

ment: les unes donnent un déplacement de même sens, mais beaucoup plus grand, s'élevant jusqu'à 0.030 ångström; les autres donnent un déplacement très notable en sens inverse. Le déplacement par la pression ne suffit évidemment pas à expliquer ces résultats. D'autres causes agissent certainement, et si l'on veut pouvoir calculer la pression de la couche renversante il faut d'abord déterminer quelles sont les autres causes de déplacement. Nous les avons trouvées dans l'étude de l'élargissement des raies d'émission.

Elargissement des raies.—Aucune raie de l'arc n'est infiniment fine, et la largeur dépend de diverses circonstances. En particulier, les raies s'élargissent quand on augmente l'intensité du courant ou la pression de l'atmosphère ambiante. Les raies les plus fines sont obtenues en produisant l'arc dans le vide. L'observation des limites d'interférences donne une mesure de la largeur des raies. Dans le vide, toutes les raies de l'arc au fer ont à peu près la même largeur, d'environ 0.03 ångström.

Si l'on passe à l'arc dans l'air, on trouve les raies notablement élargies et cet élargissement s'accentue avec l'intensité du courant. Mais les diverses raies se comportent de façons très différentes quant au mode d'élargissement. Pour beaucoup de raies, l'élargissement est symétrique par rapport à la position de la raie fine, et il n'est pas très grand. Ce sont des raies relativement fines dans l'arc à la pression atmosphérique, du moins lorsque le courant n'est pas trop intense; leur largeur est alors d'environ 0.06 ångström. Quand on passe du vide à l'air on ne trouve, comme déplacement, que le petit accroissement de longueur d'onde dû à l'augmentation de pression (0.002 à 0.003 angström). Lorsque ces raies peuvent se renverser dans le spectre de l'arc, la raie d'absorption occupe le milieu de la raie d'émission. Elles constituent le groupe des *raies à élargissement symétrique.*

Pour d'autres raies, l'élargissement, quelle qu'en soit la cause, se fait d'une manière dissymétrique, et il est plus grand que pour les raies du groupe précédent; cet effet est déja notable à la pression atmosphérique, surtout lorsque le courant est intense. Pour certaines raies l'élargissement est assez grand, dans ces conditions, pour que leur aspect les distingue nettement dans un spectre de réseau, alors que, dans l'arc dans le vide, elles ne présentent rien de particulier.

Il est évident que cet élargissement dissymétrique produit un

déplacement de la raie dans le sens où l'élargissement est le plus grand; la position apparente de ces raies dépend donc de leur largeur, et par suite des conditions dans lesquelles on les produit. Lorsqu'on passe du vide à la pression atmosphérique, il y a non seulement le petit déplacement dû à la pression, mais encore l'effet de l'élargissement dissymétrique, qui est beaucoup plus grand et masque complètement le premier. D'autre part, dans l'arc à la pression atmosphérique, ces raies ne sont pas parfaitement fixes: elles subissent un déplacement apparent à mesure que l'intensité du courant augmente, et dans certains cas le déplacement est assez grand pour être constaté visuellement dans un spectre de réseau.

Les raies à élargissement dissymétrique forment deux groupes: (1) raies à élargissement vers le rouge; (2) raies à élargissement vers le violet.

Il est bien remarquable que certaines raies aient ainsi une tendance déterminée à s'élargir d'un seul côté, et cela sous l'action de causes en apparence aussi différentes que l'augmentation de pression et l'accroissement d'intensité de courant.

Pour un certain nombre de raies, nous avons mesuré par interférences, suivant la méthode décrite ci-dessus, la variation de longueur d'onde lorsqu'on passe de l'arc dans le vide à l'arc dans l'air; dans ce dernier cas, l'intensité du courant était faible (3 ampères environ), pour être dans les meilleures conditions de finesse des raies. Pour les raies à élargissement dissymétrique, les déplacements observés auraient été beaucoup plus grands si l'intensité du courant avait été plus forte.

Le tableau II (p. 112) donne le résultat de ces mesures; les raies sont réparties d'après le groupe auquel elles appartiennent.

Notre classification des raies a une certaine ressemblance avec celle que Duffield a déduite de ses expériences sur le déplacement par la pression: toutes les raies à élargissement vers le rouge appartiennent au groupe III de Duffield, qui est caractérisé par un grand déplacement sous l'action de la pression. Les raies à élargissement vers le violet ne se trouvent pas parmi celles que l'on a étudiées au point de vue de l'action de la pression; ces raies subissent, pour la variation de pression de 1 atmosphère lorsqu'on passe du vide à la pression atmosphérique, un déplacement apparent vers le violet;

il serait très intéressant de savoir comment elles se comportent aux pressions élevées.

Explication des anomalies observées dans la comparaison du spectre solaire avec celui de l'arc.—Dans le spectre solaire, ces différences d'aspect ne se manifestent aucunement. On n'aperçoit aucune dissymétrie dans l'absorption; les raies diffèrent uniquement par leur intensité, ou, ce qui revient au même, par leur largeur. Certaines raies qui, dans l'arc à la pression atmosphérique, sont nettement diffuses et se distinguent des autres au premier coup d'œil, ne présentent rien de particulier dans le spectre solaire. On peut citer, comme exemple

TABLEAU II

Longueurs d'onde (système internat.)	$\lambda_{\text{arc air}} - \lambda_{\text{arc vide}}$	$\lambda_{\odot} - \lambda_{\text{arc air}}$	Groupe
4181.76	+ 2	+ 2	Raies à élargissement symétrique
4315.09	+ 4		
5434.53	+ 1	+ 7	
4187.04	+11	− 3	Raies à élargissement vers le rouge
4191.44	+10	− 3	
4227.44	+20	−29	
4233.61	+12	− 8	
4235.95	+11	−19	
4250.12	+13	−14	
4859.75	+17	− 7	
4871.34	+10	−15	
5415.22	−15	+27	Raies à élargissement vers le violet
5424.09	−17	+30	

frappant de ce fait, les raies 5410.92, 5415.22, 5424.09. Il semble donc que la cause qui agit pour produire l'élargissement dissymétrique des raies d'émission ne se fait pas sentir sur les raies d'absorption. La raie d'absorption doit donc correspondre à la position qu'occupe la raie d'émission lorsqu'elle est rendue fine. Dans l'arc au fer, les raies à élargissement dissymétrique ne se renversent généralement pas.

Si l'on compare le spectre d'absorption du soleil avec le spectre d'émission de l'arc à la pression atmosphérique, on obtient la somme de deux effets: déplacement par la pression, et élargissement dissymétrique de certaines raies. L'existence de cette dernière cause explique complètement les résultats, en apparence incohérents, que l'on obtient en comparant le soleil à l'arc dans l'air. En effet:

Toutes les raies du groupe à élargissement symétrique donnent lorsqu'on passe de l'arc au soleil, un léger accroissement de longueur d'onde.

Toutes les raies du groupe à élargissement vers le rouge donnent une diminution de longueur d'onde.

Toutes les raies du groupe à élargissement vers le violet donnent une forte augmentation de longueur d'onde.

Ces résultats sont visibles, soit sur le tableau II où l'on a indiqué dans la 3^me^ colonne la valeur du déplacement en passant de l'arc au soleil, soit sur le ableau I, où l'on a indiqué dans la 5^me^ colonne le groupe auquel appartient chaque raie: *S* désigne les raies du groupe symétrique, *DV* les raies qui s'élargissent vers le violet, et *DR* celles qui s'élargissent vers le rouge.

Si, au lieu de partir de l'arc dans l'air, on compare directement le spectre solaire avec celui de l'arc dans le vide, l'élargissement dissymétrique de la raie d'émission est supprimé, ou du moins très atténué; les anomalies résultant des dissymétries disparaissent et toutes les différences sont de même signe. Le tableau III donne, pour quelques raies, le résultat de ces comparaisons.

TABLEAU III

Longueur d'onde (système internat.)	$\lambda_{\odot} - \lambda_{\text{arc vide}}$
4181.76	+ 8
4187.04	+10
4191.44	+ 5
4222.22	+ 5
4227.44	+ 6
4233.61	+12
4235.95	+ 3
4250.12	+11

On pourrait calculer indirectement ces mêmes différences en partant des variations constatées lorsqu'on compare successivement l'arc dans l'air à l'arc dans le vide et au soleil. Mais, comme pour les raies à élargissement dissymétrique l'arc dans l'air ne donne pas des raies parfaitement fixes, ce calcul indirect ne peut donner, pour ces raies, une bien grande précision.

Pression de la couche renversante.—L'influence de l'élargissement dissymétrique des raies étant éliminée, les différences qui subsistent entre les longueurs d'onde de l'arc et du soleil sont attribuables à la

pression de la couche renversante, et peuvent servir à calculer cette pression. Il faut pour cela connaître le déplacement des diverses raies par la pression.

Pour les raies qui s'élargissent d'une manière dissymétrique, ce coefficient de pression, déduit de mesures faites sur le spectre d'émission, ne paraît pas avoir de sens précis. En particulier, Duffield trouve, pour les raies de son groupe III, un coefficient de pression très élevé. Mais ces raies ne se renversent pas; aux pressions élevées elles sont fortement élargies, dissymétriquement du côté du rouge; les pointés faits sur une pareille raie ne donnent aucune indication sur la position de la raie d'absorption, seule intéressante au point de vue de l'étude du spectre solaire. En fait, lorsqu'on passe de l'arc dans le vide au soleil, le déplacement de ces raies est de même ordre de grandeur que pour celles qui s'élargissent symétriquement.

On utilisera donc, pour le calcul de la pression, les raies du premier groupe. Comme les valeurs des déplacements ne sont connues qu'avec une précision relative faible, aussi bien le déplacement de l'arc au soleil que le déplacement des raies d'émission par la pression, il est rationnel d'opérer sur des moyennes.

Sur 22 raies entre 4000 et 4500 le déplacement, quand on passe de l'arc à la pression atmosphérique au soleil est en moyenne de 0.0062 ångström; pour ces mêmes raies, le déplacement moyen dû à la pression[1] est de 0.00145 ångström par atmosphère. Il en résulte, pour la pression de la couche renversante, 4.5 atmosphères au dessus de la pression atmosphérique.

Sur 10 raies entre 5100 et 5500 la différence moyenne entre le soleil et l'arc est 0.0103, et le déplacement moyen dû à la pression est de 0.0024 par atmosphère, ce qui donne une pression de 4.5 atmosphères au dessus de la pression atmosphérique.

Ces deux résultats bien concordants conduisent à cette conclusion que, dans la région de l'atmosphère solaire où se produit l'absorption par la vapeur de fer, la pression est de 5 à 6 atmosphères.

III. LARGEUR DES RAIES DU SPECTRE SOLAIRE

Les raies du spectre solaire ne sont pas seulement caractérisées par leurs longueurs d'onde. Pour chacune d'elles, les tables, celle de

[1] D'après les observations de Duffield (*loc. cit.*) et de Humphreys (*Astrophysical Journal*, **26**, 18, 1907).

Rowland par exemple, donnent un chiffre dans une colonne intitulée *intensité;* cette valeur, dont la définition est un peu vague, caractérise, en quelque sorte, la visibilité de la raie. Il paraît a priori assez difficile d'assigner un sens précis à cette comparaison de deux raies lorsqu'elles sont placées dans des régions très éloignées du spectre. On peut essayer d'obtenir une donnée numérique plus précise en mesurant la largeur de chaque raie.

L'examen direct du spectre solaire ne peut donner que des indications imparfaites sur la largeur des raies fines, dont la largeur ne dépasse pas beaucoup le pouvoir de définition du réseau employé. On peut obtenir cette largeur en produisant des interférences avec des différences de marche croissantes et cherchant la limite de visibilité.

Pour faire cette étude, nous avons employé le dispositif qui vient d'être décrit. On utilisait seulement la lumière du centre du disque solaire, en limitant l'ouverture de l'écran à un cercle de 2 mm de diamètre. L'image du soleil ayant 28 mm, l'élargissement parasite dû à l'effet Doppler-Fizeau est pratiquement éliminé. On opère avec des différences de marche croissantes, en employant l'interféromètre. Nous nous sommes limités à la région 4400, et à des observations photographiques. On a fait une série de poses photographiques, avec des différences de marche croissantes de 10 à 30 mm. Soit, pour une raie, Δ la différence de marche (double de l'épaisseur de l'appareil interférentiel) pour laquelle les interférences cessent d'être visibles. La largeur $d\lambda$ de la raie est alors donnée par l'équation

$$d\lambda = \frac{\lambda^2}{\Delta}$$

On trouve ainsi que, pour une même valeur de l'intensité donnée par Rowland, la largeur est très sensiblement constante. On peut donc faire une table donnant la largeur de la raie en fonction de l'intensité de Rowland. Le tableau IV (p. 116) donne, pour chaque intensité, la valeur de la différence de marche limite Δ, et la largeur $d\lambda$ exprimée en ångström.

Par la manière même dont on les a obtenues, les valeurs de la largeur pourraient être erronées par excès: le calcul suppose qu'aucune autre cause que la largeur des raies n'intervient pour fixer la limite d'interférence; les imperfections de l'appareil interférentiel et du

pectroscope peuvent y contribuer. Toutefois, nos mesures fixent l'ordre de grandeur des largeurs de raies, largeurs qui ne paraissent pas avoir été mesurées jusqu'ici.

TABLEAU IV

Intensité	Différence de marche limite	Largeur
1............	28 mm	0.07 ångström
2............	23	0.085
3............	19	0.10
4............	17	0.115
5............	15	0.13
6............	14	0.14
8............	12	0.16

IV. COMPARAISON DES SPECTRES DU CENTRE ET DU BORD DU SOLEIL

Les spectres du centre et d'un point du bord du soleil présentent une notable différence dans la position des raies, phénomène purement cinématique dû à l'effet Doppler-Fizeau produit par la rotation de l'astre. Cette différence, qui dépend de la position du point sur le bord, s'annule quand il est au pôle; on peut aussi s'en affranchir en utilisant deux points diamétralement opposés. Ayant éliminé cette variation, on peut comparer les spectres à un point de vue purement physique. On trouve ainsi que les deux spectres diffèrent par plusieurs caractères.

1. Il y a un changement d'aspect pour certaines raies. Pour les fortes raies, la pénombre est affaiblie dans le spectre du bord. Parmi les autres raies, quelques-unes sont renforcées ou affaiblies au bord du d sque, et le changement est en général de même sens que dans le spectre des taches.[1]

2. Halm a annoncé[2] que certaines raies subissent, du centre au bord, un léger accroissement de longueur d'onde; ses mesures ont été faites sur deux raies rouges du fer, en comparant leurs positions à celles de deux raies telluriques voisines. Il a trouvé un déplacement de 0.012 ångström.

La méthode employée par Halm n'est applicable qu'à quelques raies exceptionnelles, très voisines de raies telluriques. Notre méthode

[1] G. E. Hale and W. S. Adams, *Astrophysical Journal*, **25**, 300, 1907.

[2] *Astronomische Nachrichten*, **173**, 273, 1907.

interférentielle a l'avantage de permettre la mesure des déplacements d'une raie sans la rapporter à aucune raie voisine.

L'image du soleil tombe sur l'écran percé d'une ouverture circulaire de 2 mm de diamètre. En agissant légèrement sur le miroir C (fig. 1) on peut amener sur cette ouverture telle région que l'on veut du disque solaire.

L'influence de la rotation solaire s'élimine en prenant successivement les deux extrémités d'un diamètre de l'image solaire, et faisant la moyenne des résultats ainsi obtenus. Il y a évidemment avantage à prendre le diamètre polaire; les résultats des mesures faites sur les deux extrémités doivent alors être identiques, ce qui fournit une vérification.

Nous avons opéré par photographie: on produit successivement sur la même plaque les spectres avec interférences provenant d'abord d'un pôle, puis du centre, et enfin de l'autre pôle. Nous avons utilisé plusieurs clichés, obtenus avec des différences de marche de 5 mm et 10 mm. A cause de la petite ouverture nécessaire pour limiter une région définie du disque solaire, et d'autre part à cause du faible éclat du bord, le spectre est assez peu lumineux. Nous avons employé le spectroscope à prismes dont on a parlé plus haut. Les mesures étant assez délicates, nous n'avons pas fait une étude complète de toutes les raies solaires, et nous avons préféré étudier avec soin les déplacements d'un petit nombre de raies de la région 4400. Les mesures ont porté sur 14 raies de différents métaux, d'intensité faible ou modérée (de 2 à 6 dans l'échelle de Rowland).

Le tableau suivant (p. 118) donne, pour chacune des raies étudiées, l'intensité et la substance d'après Rowland, et la différence entre la longueur d'onde au bord et au centre, exprimée en millièmes d'ångström.

Ces résultats sont d'accord avec celui de Halm. Lorsqu'on passe du centre au bord, il y a un petit accroissement de longueur d'onde, qui, pour les raies étudiées, varie de 0.003 à 0.007 ångström. Exceptionnellement, les deux raies du vanadium ne montrent aucun déplacement.

Elargissement des raies dans le spectre du bord du disque solaire.— L'accroissement de longueur d'onde n'est pas seule modification que subissent les raies en passant du centre au bord du disque. Elles

subissent en outre un élargissement, qui se manifeste par une diminution de netteté des interférences, diminution déja visible sur les clichés ayant servi aux mesures de déplacements (différence de marche 10 mm). Nous avons étudié spécialement ce phénomène en faisant une série de photographies du spectre avec des différences de marche progressivement croissantes.

On trouve ainsi que, dans le spectre du bord, chaque raie est un peu élargie; cet élargissement paraît un peu variable d'une raie à l'autre; il est moyenne, dans la région 4400, de 0.010 ångström.

On voit que pour la plupart des raies on trouve, en passant du centre au bord du disque: (1) un déplacement vers le rouge de 0.005 ångström; (2) un élargissement de 0.010 ångström.

TABLEAU V

Longueurs d'onde (système internat.)	Intensité	Substance	λ bord $-$ λ centre
4346.56	2	*Fe*	5 millièmes d'ångström
4348.94	2	*Fe*	4
4351.05	3	*Cr*	5
4375.93	6	*Fe*	4
4379.23	4	*Va*	0
4400.38	3	*Sc*	3
4406.64	2	*Va*	0
4422.57	3	*Fe*, *Y*	3
4435.68	4	*Ca*	4
4461.64	4	*Fe*	4
4468.49	5	*Ti*	7
4485.67	3	*Fe*	7
4496.85	3	*Cr*	6
4534.78	4	*Ti*	5

Ces deux résultats peuvent se résumer en un seul énoncé: la seule modification que subit la raie est un déplacement de son bord rouge, s'élevant à 0.010 ångström, l'autre bord ne variant pas. Exceptionnellement, et c'est le cas des deux raies du vanadium, l'élargissement est symétrique.

Le fait que de nouvelles radiations sont absorbées, uniquement sur le bord rouge de la raie, peut être attribué à une absorption par les couches profondes de l'atmosphère solaire, où la pression est plus élevée: au centre du disque cette absorption ne produirait qu'un effet insignifiant, tandis qu'au bord, où toutes les couches sont traversées obliquement par la lumière, l'effet en pourrait être sensible, sans préjudice de l'absorption par les couches à pression modérée, produi-

sant la partie fixe de la raie. Un accroissement de pression de 7 atmosphères suffit à expliquer le changement observé, ce qui correspond à douze atmosphères environ pour la pression de la couche profonde.

Il faut remarquer que l'image solaire de 28 mm dont nous disposons est bien petite pour utiliser une région définie avec une ouverture de 2 mm, que l'on ne pouvait pas réduire sans diminuer par trop l'intensité de la lumière. On n'a pas seulement la lumière provenant rigoureusement du bord, et on aurait probablement des différences plus marquées si l'on pouvait opérer dans des conditions plus favorables.

D'ailleurs, notre but n'était pas d'épuiser la question; nous n'avions ni le temps ni le matériel nécessaire pour cela. Nous voulions seulement, après avoir élaboré une méthode, montrer les résultats qu'on peut en tirer, et faire voir combien l'étude approfondie des phénomènes au laboratoire est nécessaire pour l'interprétation des mesures brutes obtenues dans les observations astrophysiques.

Université de Marseille
Novembre 1909

ON A NEW MOUNTING FOR A CONCAVE GRATING

By ALBERT EAGLE

INTRODUCTORY

Notwithstanding the generally recognized pre-eminence of the concave grating for spectroscopic work of precision, this instrument has not yet taken the place it deserves in the everyday use of the ordinary spectroscopic laboratory. The reason for this is not hard to find. Not only is the Rowland mounting very expensive, but it requires an inordinate amount of room, frequently prohibitive, and moreover practically necessitates being used in a totally darkened room, which is often neither convenient nor possible.

It is with the object of drawing attention to the value of the concave grating not only for classical spectroscopic research, but also, when suitably mounted, for general use, that the present paper is undertaken.

An important need which is coming to be felt more and more by spectroscopists is that of examining faint spectra with instruments of high resolving power. For such work the loss of definition and resolution caused by a temperature-change in the grating during the long exposures necessary is very considerable, and may be fatal. How detrimental is a slight change of temperature of the grating during an exposure may be seen by noting that since the wave-length λ at any given point is proportional to the grating space b, we have $\frac{d\lambda}{\lambda}=\frac{db}{b}$. Now $\frac{db}{b}$ is equal to about 0.00002 for a change of 1° C. Hence if we have light of wave-length $\lambda=5000$ Å. U., $d\lambda=0.10$ Å. U. Now a good 4-inch (10 cm) grating with 15,000 lines to the inch (6000 per cm) can easily be made to resolve 0.13 Å. U. in this region in the first order, so that a temperature-change of 1° C. during the exposure will make it impossible to resolve lines less than 0.23 t.-m. apart, while in the third-order spectrum, the resolving power will be reduced to less than one-third. Moreover such a creep of the lines during an expo-

sure will make the accurate determination of wave-lengths from a superposed comparison spectrum impossible.

The seriousness of this temperature-change during long exposures may be seen from Mr. W. G. Duffield's paper on "The Effect of Pressure upon Arc Spectra,"[1] in which he states that he had to wait for several months before the temperature became steady enough to make his work possible. In the mounting to be described the temperature is kept steady by inclosing the grating in a heavily lagged box, which would be hardly possible with Rowland's form of mounting.

In view of the fact that a plane grating mounted as a Littrow spectrograph is coming to be regarded as the best and most convenient instrument for general use, it seems opportune to call attention to the fact that nearly all the advantages which this type of instrument possesses, together with others, may be obtained from a concave grating. This method of using a plane grating consists in placing a lens in front of it to render the incident light parallel, and rotating the grating until the diffracted light which is required returns along the line of incidence and is brought to a focus by the same lens. The advantages of this form of mounting are obvious: the space taken up is small, the instrument can be used in a lighted room, and, if it is desired to keep the temperature constant during long exposures, it is very easy to have the surrounding box lagged with some non-conductor of heat; or, if a still greater constancy of temperature is necessary, to place a thermostat inside.

This method of mounting gratings seems destined to find increasing favor among spectroscopists and some very large instruments on this principle have recently been constructed by Professor Hale on Mount Wilson in California. He secures constancy of temperature by making the axis of the instrument vertical, the slit and camera being at a convenient height above ground, while the grating is at the bottom of a well.

As against a concave grating mounted after Rowland, a plane grating used as above possesses the disadvantages of not giving a normal spectrum—though, as I shall show later, this is of slight conseqence—and also that such a spectrograph is of no use for the ultra-

[1] *Phil. Trans.*, A, **208**, 129, 1908.

violet, as the achromatic lens used in practice is always made of glass which will not transmit light of wave-length below about λ 3500.

The astigmatism possessed by a concave grating sometimes gives it an advantage over a plane grating. If, for instance, the spectrum of a very small source, such as a small spark, or a small region of a source, such as a sun-spot, be required with a plane grating it will be necessary to give width to enlargements made from the negative by the use of a cylindric lens or an up-and-down motion, either of which is liable to manufacture false lines out of specks, etc., on the plate. If a concave grating be employed and the whole of the slit be covered with a screen except that part on which the image of the required portion of the light source falls, width will be imparted to the spectrum by the astigmatism.

The method of mounting a concave grating about to be described consists in using it in a similar manner to that described above for plane gratings, save that no lens is employed. This method of using a concave grating was suggested independently, but the author has since found that it is not entirely new, Lord Rayleigh having informed him that he uses it in this manner himself, and possibly it has been so used by other spectroscopists and physicists. As, however, to my knowledge no actual mounting suitable for the continuous use of the grating in this manner has been published,[1] and as some of the points of the theory of the concave grating in this position do not appear to have received attenion, it seems worth while to describe the present mounting and the behavior of the grating when so used.

In order to secure the best definition with a concave grating it is necessary that the slit should be situated on a circle described on the radius of curvature of the grating as diameter, in which case all the spectral lines are focused on the same circle. The distance from the grating to the slit will accordingly vary considerably with the angle of incidence in the mounting suggested. Hence in using a grating in this manner it must be capable of considerable motion in the line of sight and also of rotation about a vertical axis. The only motion necessary for the camera face is one of rotation about a vertical axis through its center. It is clear that the angle of swing of the camera

[1] No reference to this method of using a concave grating is given in Kayser's *Handbuch der Spectroscopie.*

will need to be—theoretically—equal to the angle of incidence of the light on the grating.

DESCRIPTION OF MOUNTING

Fig. 1 represents a diagrammatic plan of the mounting. *G* is the grating-holder standing by means of leveling screws (not shown) on a worm-wheel *W* of 120 teeth. In order to prevent the grating-holder from falling over if accidentally knocked, it is fastened to the worm-wheel by means of a short length of spiral spring. The grating is kept up to the front of its holder by means of two light U-shaped springs which press against it from behind in the middle of the sides. A glass plate is slid down close in front of the grating when not in use to prevent tarnishing, and consequently it is not taken out of its holder. The worm-wheel *W* rests on a carriage *C* and may be rotated by means of a worm sliding on the long keywayed shaft *L*. The carriage *C* is mounted on rails *R* constituting a geometric slide, along which it is moved by means of the double-threaded screw *M* of half-inch (12.7 mm) pitch. Both this screw and the keywayed shaft *L* are connected through universal joints *U* with handles *H* and *K* each of which carries a divided head and a nut running on a screw of millimeter pitch and reading against a millimeter scale. The divided head

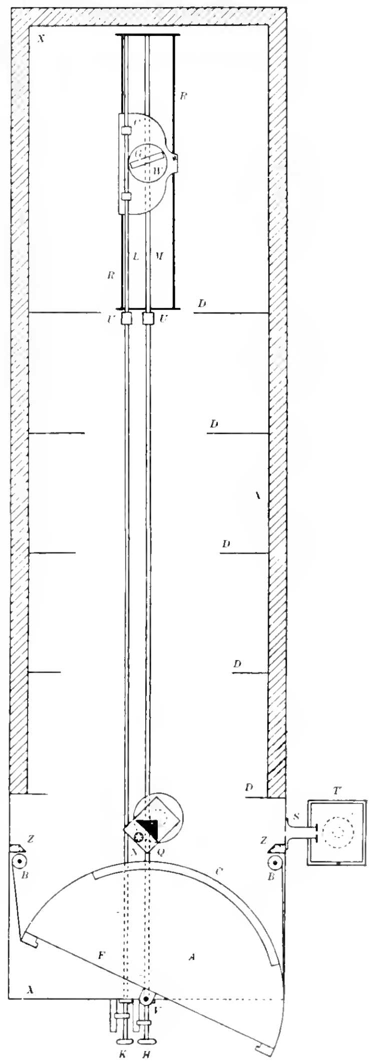


Fig. 1

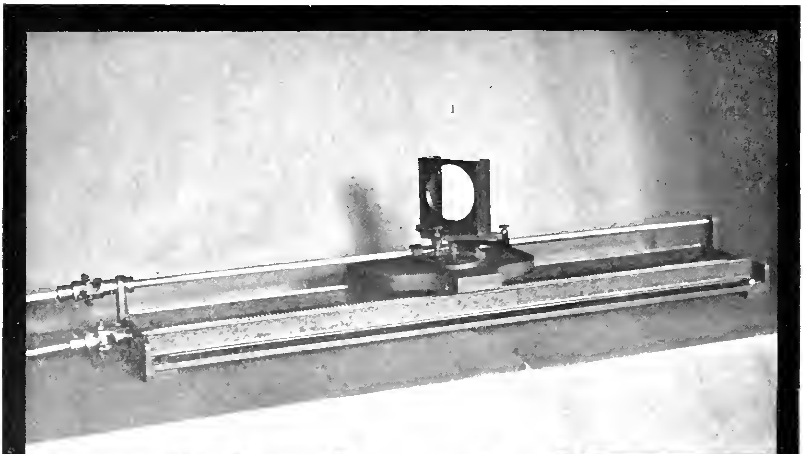
which serves to rotate the grating is graduated into 100 parts, while the other one is divided into 10 parts.

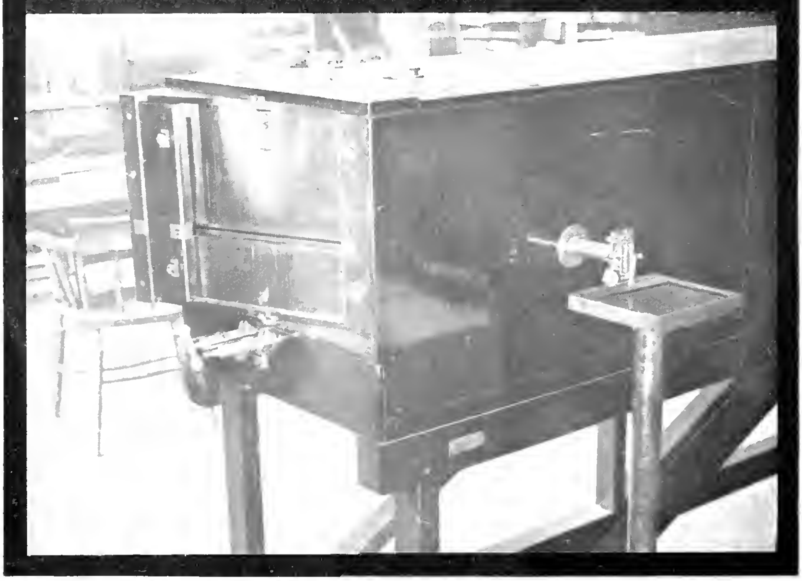
The surrounding box *X* is double-walled for nearly the whole of its length, the intervening space of $1\frac{1}{2}$ inches (37 mm) being packed with slag wool. The outside dimensions of the box are: length 11 feet 1 inch (337 cm); breadth 25 inches (63 cm); and height 22 inches (56 cm). A long lid in the top of the box just above the rails gives access to the grating. A series of five diaphragms *D* are spaced between the grating and the camera; these prevent the light which reaches the sides of the box from the other orders from fogging the photographic plates. The apertures in the diaphragms are 4 inches (10 cm) high and of varying width so as not to interfere with the passage of the light. *F* is the camera face, capable of turning about two strong pivots *V*. The lower one is attached to a platform 10 cm above the bottom of the box. Below this platform pass the two rods which serve to move the grating. These pivots should lie as nearly as possible in the surface of the photographic plate.

Attached to the edges of the camera face are light-tight blinds which are kept taut by means of spring rollers *B*. Strips of wood *Z* are placed behind the rollers in order to prevent light from entering. Three semicircular discs around which the blinds work are fastened on the inner side of the camera face, the upper and lower of these work in contact with the top of the box and the platform respectively, while the third is midway between them and just above the slot in the camera face. Sufficient height must be left in the camera face above the slot to allow the shutter of the dark slide to be opened after it is in place. The upper semicircular disc carries a flat millimeter scale around its edge which is visible through a small window in the top of the box. A bolt also passes through the top, by means of which, with a hand nut, the camera can be tightly clamped in position. Two removable cheeks on the camera face enable it to take dark slides holding plates either 18×3 inches (45×7.5 cm) or $12 \times 2\frac{1}{2}$ inches (30×6.3 cm). The dark slides have been constructed so as to hold the plates in the focal curve, especially thin plates about 1 mm thick being used which permit of the bending. The dark slides are supported in the camera by means of a catch at each end.

The slit is of Rowland's type and is placed in one side of the box

PLATE VII

1


2


1. Carriage for grating. 2. Camera end.

just below the central line of the grating. Light falls from the slit on a totally reflecting prism of quartz with faces $1\frac{1}{4}$ inches (3.2 cm) high and 1 inch (2.5 cm) broad. The distance of the slit from this prism should be nearly equal to the distance from the prism to the pivots on which the camera turns. The prism is cemented to a table carried on a pillar from the bottom of the box. By means of a leveling screw *N* reached by means of a hand-hole in the side of the box opposite to the slit, the table can be tilted about a horizontal axis parallel to the hypothenuse of the prism. The table also permits of being rotated about a vertical axis.

Plate VII, No. 1, is a photograph of the carriage and rails. The length of the rails is 3 feet 6 inches (106 cm), giving a travel of 2 feet 6 inches (76 cm). Plate VII, No. 2, shows a general view of the camera end of the instrument. It is supported at a convenient height above the floor by a rigid wooden girder resting on concrete piers built on foundations quite independent of the foundations of the building. Outside the slit is a small table, also shown at *T* in Fig. 1, supported independently by a column from the floor. On this can be placed a comparison shutter or any absorbing solution.

As the astigmatism except in the first order is too great to permit the employment of a comparison shutter outside the slit, a second comparison shutter is provided inside the instrument just in front of the photographic plate. This is operated by means of a small knob which can be moved up and down a slot near the top of the camera face and rested on different notches. The shutter is of thin sheet ebonite and made as light as possible. It contains a slot 1 inch (2.5 cm) wide which permits an unobstructed view of the spectrum, and also one a quarter of an inch (6.3 mm) wide which can be placed in three positions differing in height by $\frac{3}{16}$ of an inch (5 mm). The shutter is also capable of completely closing the slot in the camera face to keep out dust when not in use.

It is of course possible that the use of such a comparison shutter might introduce a slight shift between a spectrum and its comparison, but in half a dozen pairs of spectra which were taken as a test no such shift could be detected. If such a shift occurred it could be determined and allowed for by taking another plate in which the two spectra are directly superposed without the use of the comparison shutter.

THE MICROMETER EYEPIECE

The instrument has been provided with a micrometer eyepiece mounted on a board which fits in the camera face like the dark slide. In using this eyepiece the camera is set square on. By means of this, visual observations of particular lines may very conveniently be made, or the distance between close lines measured when it is not desired to take a photograph. By turning each handle while looking in the eyepiece the whole visible spectrum may be readily brought under review and examined. With this eyepiece observations and also measurements of the Zeeman effect may very conveniently be made.

It can easily be proved that the value of one division on the head of the micrometer eyepiece has a constant value in wave-lengths for any position in the spectrum. Calling i the angle of incidence and θ that of diffraction which is nearly equal to i, we have

$$Nm\lambda = \sin i + \sin \theta ,$$

where N is the number of rulings per cm and m is the order observed. Hence

$$Nmd\lambda = \cos \theta d\theta = \cos i d\theta .$$

If r be the distance from the eyepiece to the grating and ds be the apparent distance between two lines in the eyepiece, we have

$$d\theta = \frac{ds}{r} .$$

Now $r = R \cos i$, where R is the radius of curvature of the grating. Hence we obtain

$$ds = RNmd\lambda ,$$

showing that the scale in the eyepiece $\dfrac{ds}{d\lambda}$ is constant throughout a given order.

By the use of this eyepiece the difference of wave-length between close lines may be measured with a probable error of only two or three hundredths of an Ångström in the first order

Curves have been constructed from which the wave-length of a line in the eyepiece may be determined to an Ångström or two from the reading on the head which serves to rotate the grating. A second curve on the same diagram shows the position of the other head for which the spectrum is in focus.

ADJUSTMENTS AND FOCUSING

An arc lamp is set up at a distance of some feet from the slit and in such a position that the light entering the slit is horizontal and at right angles to the side of the box. A lens is then introduced to throw an image of the arc on the slit. It has been found convenient to have this image lens, which is an achromatic glass one of 2½ inches (6.3 cm) aperture and 12 inches (30 cm) focal length, mounted on rails parallel to the path of the light and the arc lamp on rails at right angles to this. For ultra-violet work a concave speculum metal mirror of 2¼ inches (5.7 cm) aperture and 9 inches (23 cm) focal length is employed, as a quartz lens does not produce an achromatic image. This is also mounted on rails, and in using it the arc is placed sufficiently out of the line of collimation to prevent the direct light from the arc which enters the slit from falling upon the grating.

The height of the lower edge of the slot in the camera face should be the same as that of the center of the grating, and the top of the quartz prism should be slightly below this so as to permit of an unobstructed view of the grating from the slot. Care must be taken when the rails and carriage are fixed in the box that the axis about which the grating rotates is vertical. This can be tested by means of a small spirit-level placed directly on the worm-wheel.

The position of the image-lens having been fixed, the totally reflecting quartz prism is adjusted by tilting and rotating about the vertical so that the reflected beam of light covers the grating symmetrically. The grating must now be leveled so that the spectrum is the right height in the field in all orders. This is readily done by first adjusting the two leveling screws behind the grating so that the spectra on each side of the normal are the same height in the eyepiece, and then adjusting the leveling screw in front till one of them is the correct height.

The instrument is focused for photography as follows. The handle K is turned till the required region is in the field of view. The grating is then advanced by means of the other handle till it is seen on observing the lines with an eyepiece that they are focused in approximately the right position in the center of the field. From the reading on K when the central image is brought into the center of the field of view, the inclination of the grating to the incident light can be found, the number of teeth on the worm-wheel being known. The camera

is then set to this inclination by means of its scale, the required scale-reading being given by adding the reading for no swing to the product of the inclination of the grating in radians into the radius of the scale. The final focusing must now be done by photography. A series of half a dozen photographs are taken on a single plate, the grating being advanced by $\frac{1}{20}$ of an inch (1.27 mm) between each successive exposure, the selected positions lying on each side of the approximate focus observed. The spectrum which shows the lines in the center of the field in best focus gives the position of the grating-carriage, while from the positions in which the ends of the plate are in focus the correction to be made to the swing can be worked out. It must be remembered that the focus moves in and out twice as fast as the grating. If δ is the difference in focus between the center of the plate and its ends, $2a$ its length, and r the radius of the scale recording the camera-swing, the correction to be made to the scale reading is $\frac{r\delta}{a}$.

The three position readings together with the range of wave-length obtained on the plate are then recorded, and to photograph any spectrum in this region in future, it is only necessary to set the three recorded readings on their respective scales, when the instrument will be in focus. A series of focusing plates as above are taken throughout the different orders and the results tabulated, from which in a few seconds the instrument can be set in adjustment for any desired region. Hence the instrument is hardly less convenient in use than the Rowland mounting, while the labor of taking the focusing plates is no more than that of originally setting the latter in adjustment.

ADVANTAGES OF THE PRESENT MOUNTING

In comparison with Rowland's mounting the one under consideration has the following advantages:

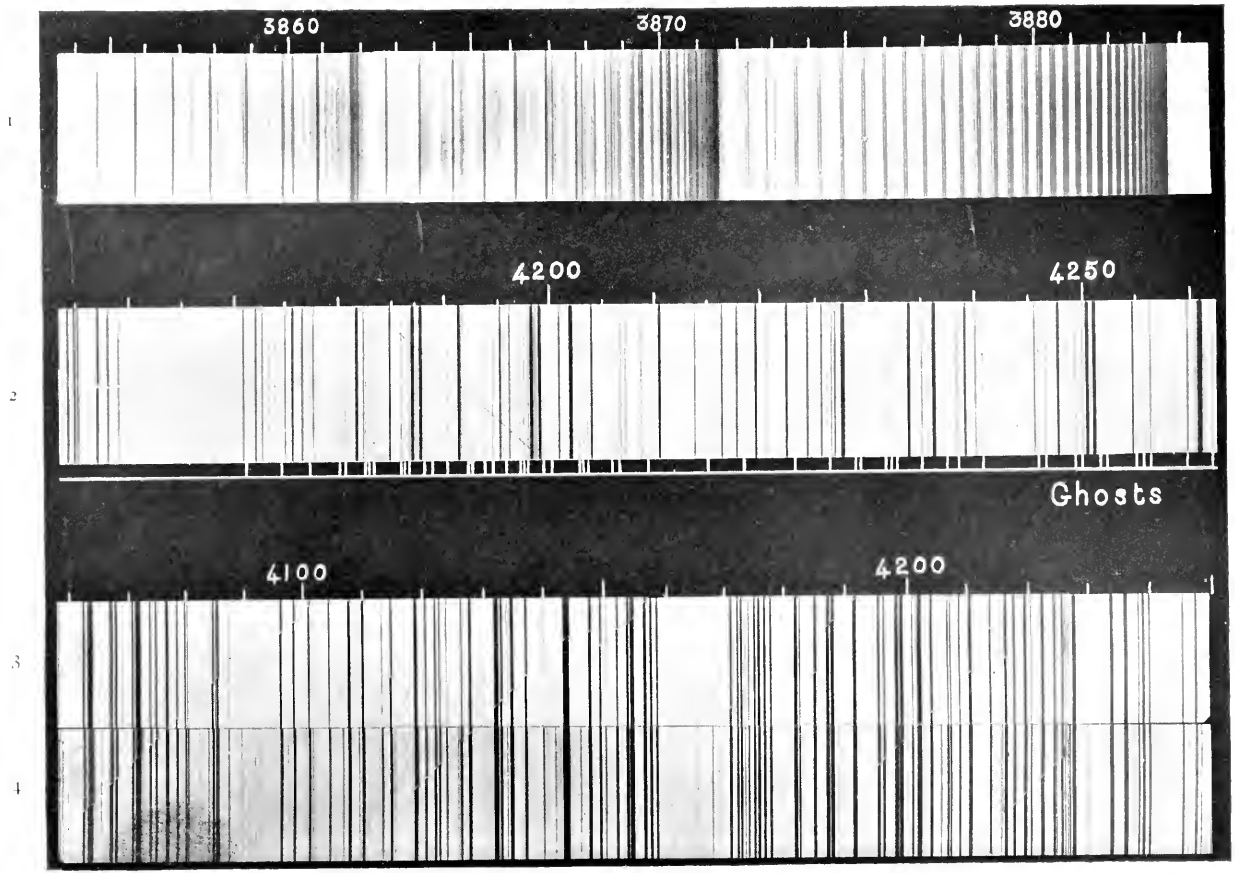
1. The very small space occupied by it.

2. No darkened room is necessary.

3. It is very much cheaper than the Rowland mounting. Even when well made with all desirable additions the cost would be only about half that of the latter.

4. Spectra on either side of the normal may be used with equal

PLATE VIII



1. Head of CN band at λ 3883. Fifth order.
2. Iron arc. Fifth order.
3. Iron arc. Fifth order. Single exposure.
4. The same. Quadruple exposure.

facility, a point of some value, as it may happen that the best third-order spectrum is on the opposite side to the best first-order spectrum.

5. Everything being on the same axis, great rigidity is obtained. External vibrations would tend to shake the instrument as a whole rather than one part with respect to another, and consequently such vibrations would not affect the definition.

6. A slightly increased dispersion is obtained, especially in the higher orders. Referring to the section on the micrometer eyepiece it will be seen that the scale of the spectrum in the eyepiece $\frac{ds}{d\lambda}=RNm$. When this is projected on a plate inclined at an angle i the scale is clearly

$$\frac{RNm}{\cos i}.$$

The scale in Rowland's mounting is RNm throughout. No increase in theoretical resolving power is hereby obtained however.

7. Higher orders can be obtained than is possible with Rowland's mounting. Using the previous notation, if light be incident at an angle i, the wave-length diffracted along the normal will be

$$\lambda=\frac{\sin i}{Nm},$$

while the wave-length of the light diffracted back along the line of incidence will be

$$\lambda=\frac{2\sin i}{Nm}.$$

Hence the same wave-length can be obtained in this case with an angle of incidence of 30° as with grazing incidence in the former case. The mounting described is capable of accommodating light up to an angle of incidence of 40°, the limit being imposed by the length of the slide. This would have been made longer if it had been thought that the spectra obtained would be bright enough to be of any value, as they were found to be.

The utility of these higher orders may be seen from the photographs in Plate VIII. No. 1 shows the head of the well-known cyanogen band at λ 3883 taken in the fifth order with an exposure of 40 minutes. An absorbing solution of iodine in carbon bisulphide in a glass cell was placed before the slit. On the original negative lines only 0.05 Ångström apart are distinctly resolved.

8. The steadiness of temperature secured—a point of vital necessity in long exposures. Fig. 2 shows the variation of temperature inside and outside the box during a day of 9 hours. The true temperature inside was probably much more constant than that indicated by the thermometer which was inserted in a hole in the lid. Plate VIII, No. 3, shows a portion of the iron spectrum taken in the first order with an exposure of 40 seconds, adjacent to it being the same spectrum obtained with four superposed exposures of 10 seconds each, made at intervals of one hour. No special precautions were taken and other work was carried on in the laboratory as usual.

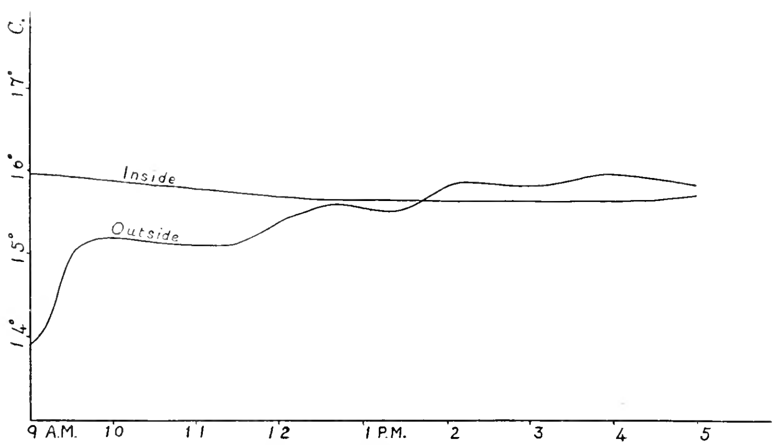


Fig. 2.—Temperature inside and outside of box

In a 28-hours exposure which was given on the hydrocarbon bands obtained from the flame of a Mecke burner, lines are resolved though only 0.15 Ångström apart. The resolving power for a short exposure would have been 0.11 Ångström (for a first-order spectrum), from which it can be calculated that the temperature must have remained effectively constant to 0.5° C. during the exposure, although the laboratory was not heated after 5 P. M. till 7 A. M. the next morning and the intervening night was frosty.

THE ASTIGMATISM OF A CONCAVE GRATING

One of the chief advantages of the present mounting is the great reduction in astigmatism which is effected, especially in the higher

orders which are thereby rendered much more brilliant. This reduction is so striking that it appears worth while to append proofs of the astigmatism in both cases, especially as no short proofs appear to have been given.

We will call the plane normal to the rulings at any point of the grating the principal plane of incidence at that point. If a pencil of light inclined at an angle α to this plane fall on the grating, it is clear that since there is no path-difference between rays falling on different portions of the same ruling all the diffracted pencils will also be inclined at an angle α to this plane, that is, they will lie on a cone of semi-vertical angle $\frac{\pi}{2}-\alpha$. The apparent angle β, between the incident pencil and the principal plane, when viewed in the direction of the intersection of this plane with the surface of the grating may easily be proved to be related to α by

$$\tan \beta = \frac{\tan \alpha}{\cos i},$$

where i is the angle of incidence projected on the principal plane. If α and β be small, we have $\alpha = \beta \cos i$.

Imagine a concave grating of radius R mounted in Rowland's manner and consider a pencil of light from a point at the center of the slit falling on the rulings at a height h above their center. We may regard this portion of the grating as part of a plane grating inclined at an angle $\frac{h}{R}$ to the vertical. In Fig. 3 let SA represent an incident ray and PA the trace of the principal plane of incidence at A, P being the center of curvature of the grating. Let AQ be the diffracted ray. The apparent inclination of the incident ray to the principal plane is

$$\beta = \frac{AC}{CS'} - \frac{AC}{CP}.$$

Now $CS' = GS \cos i = R \cos^2 i$, where i is the angle of incidence. Hence

$$\beta = \frac{h}{R}\left(\frac{1}{\cos^2 i} - 1\right) = \frac{h}{R} \tan^2 i .$$

α, the true inclination to the principal plane, is $\beta \cos i$. Hence

$$\alpha = \frac{h}{R} \tan i \sin i .$$

Since the diffracted ray is normal to the grating we have

$$\frac{QP}{AP}=a\ .$$

Hence $QP=aR=h\tan i\sin i$. If l be the total length of the rulings a point at the slit will therefore be drawn out into a line of length $l\tan i\sin i$ in the spectrum

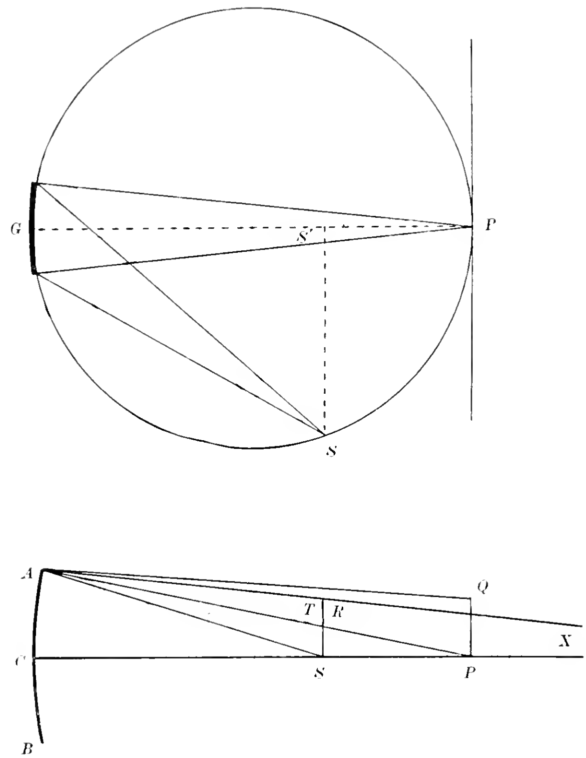


FIG. 3

Consider now the astigmation of the light diffracted back along the line of incidence. If AT be the diffracted ray, we shall have

$$R\hat{A}T=S\hat{A}R=\beta\ .$$

Hence

$$ST=2\beta\cdot GS=\frac{2h}{R}\tan^2 i\ R\cos^2 i\ ,$$

$$=2h\sin^2 i\ .$$

So the image of a point at the slit will be drawn out into a line of length $2l \sin^2 i$ in the spectrum.

Now in Rowland's mounting

$$\sin i = Nm\lambda$$

with the previous notation, hence the astigmatism is

$$\frac{N^2 m^2 \lambda^2 l}{\sqrt{1 - N^2 m^2 \lambda^2}},$$

while in the present mounting

$$2 \sin i = Nm\lambda,$$

and the astigmatism is consequently

$$\frac{N^2 m^2 \lambda^2 l}{2},$$

which, it is observed, is always less than half of the value in Rowland's mounting.

The following table gives a comparison of the angles of incidence and the astigmatism for the wave-length λ 5500 in the first five orders with a grating of 15,020 lines to the inch.

Order	Rowland's Mounting		Present Mounting	
	Angle of Incidence	Astigmatism	Angle of Incidence	Astigmatism
First	18° 59′	0.112l	9° 22′	0.053l
Second	40 35	0.557l	18 59	0.212l
Third	77 20	4.34l	29 12	0.476l
Fourth	Impossible	......	40 35	0.876l
Fifth	Impossible	......	54 23	1.321l
For $m\lambda = 16{,}911$	90 0	∞	30 0	0.5l

In view of the fact that the concave grating is sometimes discarded owing to the diminution of brightness caused by the astigmatism, it should be observed that so long as the length of slit illuminated exceeds the astigmatism, the brightness of the central portion of the spectrum is quite unaffected by it; but when it is less the brightness is directly proportional to the length of slit illuminated and inversely proportional to the astigmatism. Hence, as the above table shows, an enormous gain in brightness is secured in the higher orders with the present mounting as compared with Rowland's mounting. If the image-lens be arranged to magnify three times, it is generally possible to illuminate from 10 to 15 mm of the slit so that the whole of the first

two orders will be undiminished in brightness by the astigmatism. The brilliancy of one of the fourth-order spectra is such that at about λ 3800 a fairly heavy exposure of the iron arc may be obtained in three minutes.

As is well known, a stigmatic spectrum may be obtained with a concave grating if the screen instead of being close to the slit be placed at some distance in front of it. To find this position for the present mounting we observe that it is at the point X where AT produced intersects CP (Fig. 3).

Now

$$T\hat{X}S = A\hat{S}C - S\hat{A}X ,$$

or

$$\frac{TS}{SX} = \frac{AC}{GS} - \frac{TS}{GS} ;$$

that is,

$$\frac{2h \sin^2 i}{SX} = \frac{h}{R \cos i}(1 - 2 \sin^2 i) .$$

$$\therefore \quad SX = \frac{2R \sin^2 i \cos i}{1 - 2 \sin^2 i}$$

$$= 2 Ri^2 \text{ approximately}$$

when i is small.

In the present grating the spectrum can be rendered stigmatic at λ 5500 in the first order by placing the screen 6.7 inches (17 cm) in front of the slit. It is not advisable to use the method when the distance much exceeds this, owing to the loss of light. The method is occasionally of value, when for instance the spectrum of different regions of a source is required. The image of the source is then focused, not on the slit, but the required distance in front of it. These distances necessary to obtain a stigmatic spectrum are approximately the same as in Rowland's mounting.

In view of the rather prevalent idea that the concave grating must be used in Rowland's manner in order to obtain the best definition[1] it may be pointed out that theoretically the definition is slightly more perfect in the present case than in Rowland's, though since the path-errors involved are too small to appreciably affect the definition in either case the point is not of much practical importance. It can be

[1] "Rowland's mounting combines the best definition with a normal spectrum."—Schuster, *Optics*, p. 122.

shown[1] that if light be incident at an angle θ on a concave grating of truly spherical form, the path-error of the light which falls on the sides of the grating over that which falls on its center is

$$\frac{b^4}{8R^3}\sin\theta\tan\theta\,,$$

where $2b$ is the breadth of the ruled space and R is the radius of curvature. In Rowland's mounting there is no path-error in the diffracted light since this converges to the center of curvature, hence the above expression is the total path-error. In the present case, since the light returns along the line of incidence, the path-error will be twice this expression, but since the same spectrum line is obtained with half the value of $\sin\theta$ and therefore with less than half the value of $\tan\theta$, it is seen that the path-error for the same line is less than half of what it is in Rowland's case.

DISADVANTAGES OF THE PRESENT MOUNTING

To set against the foregoing advantages of the present mounting over that of Rowland are the following disadvantages:

1. The instrument does not remain in automatic focus.

Once the series of focusing plates has been taken, however, the trouble of focusing the instrument again for any of these regions is practically nil, since we have only to set three known readings on their respective scales. If the regions of the focusing plates have been carefully chosen it will be seldom that a region not coinciding with one of them is required. If such be the case, either a special focusing plate may be taken for the region in question, or the scale-readings of the different focusing plates may be plotted on a large scale on millimeter paper and the required readings obtained from the curves. Another alternative is to find the corrections which have to be made to the positions of the nearest focusing plate to the region required by theory. This is readily done. If i be the inclination of the camera which is the same as that of the grating, the distance between them is $R\cos i$, while the wave-length at the center of the plate is

$$\lambda=\frac{2\sin i}{Nm}\,.$$

[1] Schuster, *ibid.*, pp. 121, 122.

Hence a change of δi in i will change the wave-length at the center of the plate by

$$\delta\lambda = \frac{2 \cos i}{Nm}\delta i ,$$

while the change in distance between the camera and the grating will need to be

$$R \sin i \, \delta i .$$

The angle i for the grating is readily obtained, knowing the previously determined reading for which the central image is reflected into the center of the field.

From the shift of wave-length which is required from the nearest standard position, δi is calculated and thence $R \sin i \, \delta i$, the required shift of the grating carriage. The value of i used in this formula should strictly be the value of i for a position midway between the region required and that of the nearest focusing plate With regions focused in this manner we have secured the same perfection of definition as in the standard positions.

2. Another disadvantage of the new mounting arises when photographic plates made on poor quality glass are employed, owing to the fact that the light is not incident normally on the plate. i being the angle of incidence, it is clear that a local depression of depth h will displace a line happening to fall in it by $h \tan i$. Such displacements are revealed by an irregular curve of errors when wave-lengths are determined. It is not difficult, however, to obtain plates which do not thus introduce sensible errors, and in one batch of plates which gave rise to them it was clear by looking obliquely over the surface that it was very irregular.

This is not a disadvantage which is peculiar to the present spectrograph since in many instruments in use the plate is very considerably inclined to the incident light, angles of over 60° being used in some cases.

3. A third disadvantage is that the spectrum obtained is not quite normal. This however is not so detrimental as might at first sight be supposed. The greatest deviation between the true wave-length and that calculated from a linear interpolation formula taken over a three-inch (7.6 cm) range in the green of the first order is only 0.2 Å. U. Such a deviation makes it generally practicable to employ a simple

linear interpolation formula between two standards and to draw a curve of errors showing the difference between the calculated and the true wave-lengths on intermediate standard lines From this curve the correction to be added to the unknown wave-lengths can be determined. In all cases of the determination of accurate wave-lengths, even when using a Rowland mounting, such a curve of errors should be constructed, if only to detect and eliminate accidental errors of setting on the standard lines from which the equation is calculated. So far, then, the fact that the spectrum is not normal makes no difference whatever.

When however it is required to obtain a uniform reduction over a much longer range it is best to use a formula of the type

$$\lambda = a + bs + cs^2,$$

which represents the spectrum with the same accuracy as it is represented by a linear formula in Rowland's case. Let λ_1, λ_2, λ_3 be the wave-lengths of three lines from which it is required to determine the constants of the above equation and s_1, s_2, s_3 their scale-readings, of which the second should be about midway between the first and third. The following solution is perhaps the most convenient for use:

$$\lambda = \left\{ a - \frac{c}{4}(s_1 - s_3)^2 \right\} + bs + c(s - s_m)^2$$

where

$$b = \frac{\lambda_1 - \lambda_3}{s_1 - s_3},$$

$$a = \lambda_1 - bs_1$$

$$c = \frac{\lambda_2 - a - bs_2}{(s_2 - s_1)(s_2 - s_3)};$$

and

$$s_m = \frac{s_1 + s_3}{2}.$$

In this form the constants are adapted to logarithmic calculation, while the last term, which is small, is of a very convenient form for evaluation with a slide-rule, which will generally be found sufficiently accurate. For the evaluation of the first two terms of the expression for λ, I am in the habit of using an arithmometer. With the above equation it will be seen that the extra trouble involved in not dealing with a normal spectrum is very small.

Even when this equation is used, a curve of errors should still be drawn as before by means of intermediate standards.

It may be shown that if the plate on which the spectrum is photographed be bent into a circle of radius R instead of into one of diameter R, the spectrum obtained will be as normal as that obtained with Rowland's mounting. Hence those who consider a normal spectrum a necessity cannot on that account alone reject the present mounting, as it may be obtained on a plate which neither requires so much bending as is necessary to fit the focal curve, nor goes out of focus as fast as a flat plate.

This great dependence of the law of dispersion in the spectrum upon the form of the plate may readily be shown as follows:

We have

$$\sin i + \sin (i+\theta) = Nm\lambda$$

where $i+\theta$ is the angle of diffraction.

Hence

$$\frac{d\theta}{d\lambda} = \frac{Nm}{\cos (i+\theta)} = \frac{Nm}{\cos i}\left\{1+\theta \tan i\right\}$$

$$= \frac{Nm}{\cos i}\left\{1+\frac{s}{R}\tan i\right\},$$

approximately, since, as the plate must very nearly fit the focal circle, θ must still be very nearly equal to $\frac{s}{R}$, where s is the distance of any line from the center of the plate; i. e., $s=0$ corresponds with $\theta=0$.

Now

$$\frac{ds}{d\lambda} = \frac{ds}{d\theta}\frac{d\theta}{d\lambda}.$$

The determination of $\frac{ds}{d\theta}$ is a purely geometrical problem. Let APB (Fig. 4) represent the photographic plate. Let $AP=s$, $OP=r$, and $P\hat{O}A=\theta$. Let the equation of APB in polar co-ordinates be

$$r = a + b\theta + c\theta^2 .$$

Then we have

$$\frac{ds}{d\theta} = \left\{r^2 + \left(\frac{dr}{d\theta}\right)^2\right\}^{\frac{1}{2}} = \left\{a^2+b^2+(2ab+4bc)\theta\right\}^{\frac{1}{2}},$$

approximately. Write $a^2+b^2=A$ and $2ab+4bc=B$. Then

$$\frac{ds}{d\theta} = A^{\frac{1}{2}} + \frac{B}{2A^{\frac{1}{2}}}\theta ,$$

approximately. To a first approximation we have from this

$$s = A^{\frac{1}{2}}\theta \, .$$

Substituting this and also the values of A and B we have

$$\frac{ds}{d\theta} = \sqrt{a^2+b^2} \left\{ 1 + \frac{(ab+2bc)}{(a^2+b^2)^{\frac{3}{2}}} s \right\} \, .$$

The radius of curvature of the plate at A is found by means of the usual formula to be

$$\rho = \frac{(a^2+b^2)^{\frac{3}{2}}}{a^2 - 2ac + 2b^2} \, .$$

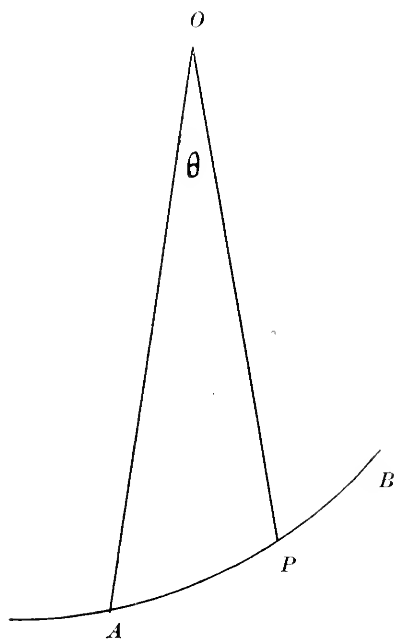


FIG. 4

Using this to eliminate c we obtain

$$\frac{ds}{d\theta} = \sqrt{a^2+b^2} \left\{ 1 - \frac{b}{a}\left(\frac{1}{\rho} - \frac{2}{(a^2+b^2)^{\frac{1}{2}}}\right) s \right\} \, .$$

Since the plate must very nearly coincide with the focal curve we may put $a = R \cos i$ and $\frac{b}{a} = -\tan i$. Then

$$\frac{ds}{d\theta} = R \left\{ 1 + s \tan i \left(\frac{1}{\rho} - \frac{2}{R}\right) \right\} \, .$$

Hence

$$\frac{ds}{d\lambda}=\frac{RNm}{\cos i}\left\{1+s\tan i\left(\frac{1}{\rho}-\frac{1}{R}\right)\right\}$$

or

$$\frac{d\lambda}{ds}=\frac{\cos i}{RNm}\left\{1-s\tan i\left(\frac{1}{\rho}-\frac{1}{R}\right)\right\}$$

to the order of approximation to which we have been working.

From this it will be seen that if $\rho=R$, $\frac{d\lambda}{ds}$ will be independent of the first power of s and therefore λ will be expressed in terms of s by an equation of the form

$$\lambda=a+bs+ds^3\ ,$$

showing that the spectrum is normal at $s=0$. We also note that the coefficient of s in the expression for $\frac{d\lambda}{ds}$ has equal values but of opposite signs when we make $\rho=\infty$ and $\rho=\frac{R}{2}$. That is, if we use plates to fit the focal curve the coefficient of s^2 in the equation

$$\lambda=a+bs+cs^2$$

will have the same value but the opposite sign that it has for a flat plate. This was discovered experimentally when curved plates were first substituted for the flat ones previously used.

In Rowland's mounting also, the law of dispersion is considerably influenced by the form of the plate; for instance, the spectrum will deviate from normal three times as much if a flat plate be employed as it will if a curved plate of the proper radius be used.

We have in this case

$$\sin\theta=Nm(\lambda-\lambda_n)\ ,$$

where θ is the angle of incidence and λ_n is the wave-length diffracted along the normal. For a curved plate $\theta=\frac{S}{R}$ where s is the distance of any line from the center of the plate.

Hence

$$Nm(\lambda-\lambda_n)=\sin\frac{s}{R}=\frac{s}{R}-\frac{1}{6}\frac{s^3}{R^3}+\ \ldots\ .$$

If on the contrary a flat plate be used we have $s/R=\tan\theta$, and hence

$$Nm(\lambda-\lambda_n)=\frac{s/R}{(1+s^2/R^2)^{\frac{1}{2}}}=\frac{s}{R}-\frac{1}{2}\frac{s^3}{R^3}+\ \ldots\ .$$

showing that the coefficient of s^3 is three times as great as in the former case.

If a spectrum be represented by the equation

$$\lambda = a + bs + cs^2 + ds^3$$

it can be shown that the deviation from a linear formula fitting lines at s_1 and s_2 is

$$\Delta\lambda = \{c + d(s + s_1 + s_2)\}(s - s_1)(s - s_2) .$$

From this it may be calculated that if an 18-inch (46 cm) curved plate be used with a 10-foot (305 cm) concave grating mounted in Rowland's manner, a linear formula used over a region near one end of the plate will give a departure of more than 0.01 Å. U. from the true wave-length if the range taken exceed 36 mm in length.

PERFORMANCE OF A CONCAVE GRATING WITH THE PRESENT MOUNTING

Attention has already been drawn to Plate VIII, No. 1, showing the head of the cyanogen band at λ 3883 taken in the fifth order as an illustration of the value of higher orders than can be obtained in Rowland's mounting. No. 2 on the same plate represents a portion of the iron spectrum taken in the other fifth-order spectrum.

The resolving power secured in long exposures has also been mentioned, and it may be noted here that in the case of the four superposed exposures made at intervals of one hour the close double 0.118 Ångström wide, at λ 4240, is seen to be resolved at the tips of the lines, on the negative. This double is always resolved in the first-order spectrum unless overexposed or a specially wide slit be employed. This gives a realized resolving power of 36,000, whereas the theoretical purity with the width of slit employed (0.012 mm) is only 29,000. This fact, that a higher resolving power can be obtained than the theoretical value given by the ordinary formula, has been pointed out by other observers.

In the fifth-order spectrum lines only 0.05 Å. U. apart can be resolved. This, although greater than the theoretical value, which is about 0.03, seems to be limited by the accuracy of the rulings of the grating; 0.05 seems however to be about as small as Rowland was able to resolve in his largest gratings. Possibly also the majority of spectrum lines are not homogeneous to a much higher order than this.

The present mode of mounting and using a concave grating is not merely put forward as a possible alternative to Rowland's manner of using it, but I venture to suggest it as the most convenient and suitable mounting for all purposes and one which adapts the concave grating to the uses of general spectroscopy while sacrificing nothing of the high resolving power and excellent definition which make it suitable for the most refined research. The present mounting makes a compact spectrograph capable of dealing not only with the visible spectrum like a plane grating mounted in the Littrow manner but with the whole of the ultra-violet also.

The suggested mounting would be even more advantageous for a 21-foot (640 cm) grating than it is for a 10-foot (305 cm) one.

The mounting as above described has been constructed for the Spectroscopic Laboratory of the Imperial College of Science and Technology. The author's thanks are due to Professor Fowler for consenting to mount the grating in this manner, and also to Professor Callendar for the facilities he has afforded for having it entirely constructed in the workshop of the college under my supervision. Finally the author's thanks are due to Mr. W. Colebrook, the chief mechanician of the workshop, for the care and attention he has paid to the whole of the work, and also for his valuable suggestions on points of construction which presented difficulties, to the successful overcoming of which the perfection and success of the present instrument is largely due.

Imperial College of Science and Technology
London
December 1909

THE ABSOLUTE WAVE-LENGTHS OF THE H AND K LINES OF CALCIUM IN SOME TERRESTRIAL SOURCES[1]

By CHARLES E. ST. JOHN

The part taken by the H and K lines of calcium in solar and stellar spectroscopic theory is of such importance that all available knowledge of them is valuable, whether it concerns their wave-lengths, their individual characteristics, or their varying appearance under different conditions. The purpose of the present investigation is the determination, with the accuracy obtainable through the use of gratings of high dispersion, of the wave-lengths of the H and K lines referred to secondary standards based upon the red cadmium line adopted as the primary standard by the International Union for Co-operation in Solar Research at Meudon in 1907. The work is preliminary to a comparative study of the corresponding solar lines. For such an application of the results it is desirable that the wave-lengths in terrestrial sources should be known with the highest obtainable accuracy. The wave-length of the cadmium standard has been fixed as 6438.496 Ångströms, which defines the Ångström as equal to 10^{-10} m with an accuracy of one part in ten million. In the following paper wave-lengths based on this system are indicated by the subscript A.

	H	K
Rowland (arc)*	3968.617	3933.809
Kayser and Runge (arc)†	3968.63	3933.83
Jewell (arc)‡	3968.603	3933.794
Adams (arc)§	3968.629	3933.818
Exner and Haschek (spark)†	3968.62	3933.63
Eder and Valenta (spark)†	3968.638	3933.803
Cooper (spark)¶	λ_A 3968.488	λ_A 3933.686

* *Astronomy and Astrophysics*, **12**, 332, 1893.

† Eder und Valenta, *Beiträge zur Photochemie und Spectralanalyse* (Vienna, 1904), p. 336.

‡ *Astrophysical Journal*, **3**, 112, 1896.

§ *Contributions from the Mount Wilson Solar Observatory*, No. 6; *Astrophysical Journal*, **23**, 45 1906.

¶ *Astrophysical Journal*, **29**, 332, 1909. In a personal letter Dr. Cooper corrects an error in the published value of the wave-length of K. His corrected value is given above.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 44.

The differences in the values assigned to the wave-lengths of the H and K lines of calcium by different observers probably depend in great part on the standards involved. Some representative determinations are shown in the table on p. 143.

In the case of the determinations by Rowland, Jewell, and Adams, H−K=34.808, 34.809, 34.811, respectively. This close agreement in the relative values for H and K indicates a high degree of accuracy in the separate determinations and makes it probable that the absolute differences are to be referred to the different standards used. In view of these discrepancies, it was necessary to redetermine the wave-lengths in terms of accepted standards which can also be applied to the solar lines. The recently published absolute wave-lengths for iron lines by Fabry and Buisson,[1] prepared in accordance with the resolution adopted at the Meudon meeting of the International Solar Union,[2] furnished the standards used. The five iron lines $\lambda\lambda$ 4021, 3977, 3935, 3906, and 3865 include the desired region and are made the basis of the following determinations in arc, spark, and electric furnace.

APPARATUS

Three entirely different instruments, all spectrographs of the Littrow form, were used and served as checks upon each other in determining the wave-lengths of H and K.

1. The 13-foot spectrograph in the Pasadena laboratory of the Mount Wilson Solar Observatory.[3] This consists of a 5-inch (127 mm) objective of 13 feet (4 m) focal length and a Rowland plane grating having 14,438 lines to the inch (568 lines to the cm) placed in a deep dry well. It was used in the third order where the scale is approximately 1 mm=1.38 Å.

2. The 18-foot spectrograph used in connection with the Snow horizontal telescope and mounted above the 5-foot spectroheliograph. In this spectrograph was used an 8-inch (202 mm) Michelson grating having 15,000 lines to the inch (590 lines to the mm), and a 6-inch (152 mm) lens of 18 feet (5.5 m) focus. The grating gave excellent

[1] *Astrophysical Journal*, **22**, 169, 1908.

[2] *Transactions of the International Union for Co-operation in Solar Research*, **2**.

[3] *Contributions from the Mount Wilson Solar Observatory*, No. 27; *Astrophysical Journal*, **28**, 244, 1908.

definition in the third order, though having a small amount of astigmatism due to a ruling error. The third order was used in the following investigation, the scale of which is very nearly 1 mm = 1.02 Å.

3. The 30-foot spectrograph of the tower telescope.[1] In this a Rowland plane grating, class A, with 14,438 lines to the inch was used, placed at the bottom of a deep dry pit of practically constant temperature. The performance of this grating is always excellent. The third order was used, the scale being about 1 mm = 0.60 Å.

CALCIUM IN THE ARC

The source of light for this part of the investigation was an electric arc between two iron poles or between iron and carbon poles when the absorption lines of H and K were to be measured, and between copper poles when it was desired to use the emission lines of H and K. Metallic calcium or a dilute solution of calcium chloride was introduced into the arc according as absorption or emission lines were desired. With large amounts of calcium vapor in the arc the appearance of these lines is familiar showing fine sharp reversals in the middle of the bright emission lines. The majority of the measurements were made upon the reversals, but some were made on the fine sharp symmetrical bright lines produced with very small quantities of calcium in the arc.

In the case of the reversals, the calcium being in the iron arc, the comparison spectrum was photographed simultaneously with that of calcium, and there can be no question of any disturbance due to a movement of any part of the apparatus. Two of the iron lines have very nearly the same wave-length as H and K, respectively, and partially overlap and tend to broaden them on the violet side when the calcium lines are very narrow. As it was desired to compare the wave-lengths for large and very small amounts of calcium in the arc, a copper arc with extremely small quantities of calcium was used for obtaining the narrow emission lines, the iron comparison spectrum being introduced by means of an occulting bar and spanning the narrow calcium spectrum. The agreement of the wave-lengths for the two classes of lines indicates no instrumental shift between the calcium and iron spectrum.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 23; *Astrophysical Journal*, **27**, 204, 1908.

The plates were measured on a Gaertner machine, with red right and red left, and in all cases, at least four settings were made on each line, the cross-hair being brought alternately from the right and left to the middle of the line. In the reduction of the measurements the factors were obtained for each of the four intervals given by the five standards and plotted as ordinates with the mean wave-lengths of the corresponding intervals as abscissae. These gave uniformly smooth curves with which the points agreed well even when a large scale was used. The standards proved perfectly satisfactory in this regard. In the calculation of the wave-lengths, H was determined from its distance from λ 3977 and λ 3935, and K from its distance from λ 3977, λ 3935, and λ 3906. Two iron lines, $\lambda\lambda$ 3948 and 3930, were also measured on each plate as a control, and in the case of λ 3930 its absolute wave-length was used in the later investigation. The factors can be read from the curve to the fourth decimal place directly and to the fifth approximately; this degree of accuracy is necessary for a run of 40 Å, as in determining K from λ 3977. The factor-curve for plate No. 521, which is an average example, is shown herewith.

The results for each line, as calculated from this curve, are given below.

From	H	λ 3948	K	λ 3930
λ 3977	3968.475	3948.784	3933.666	3930.302
λ 3935	.476	.783	.672	.297
λ 3906			.667	.298
	3968.476	3948.784	3933.668	3930.299

The best standard wave-lengths hitherto determined are those of Kayser's iron lines.[1] To compare the internal agreement of the two systems, the points for the factor-curve of plate No. 521 were obtained by using Kayser's wave-lengths for the corresponding Fabry and Buisson standards and were plotted as crosses on the same sheet. The curve for these points would be quite different from that shown, and owing to its irregularities the reduction factors could not be determined from it with the same accuracy. The close agreement between the values for H and K determined from the Fabry and Buisson

[1] *Astrophysical Journal*, **12**, 329, 1901.

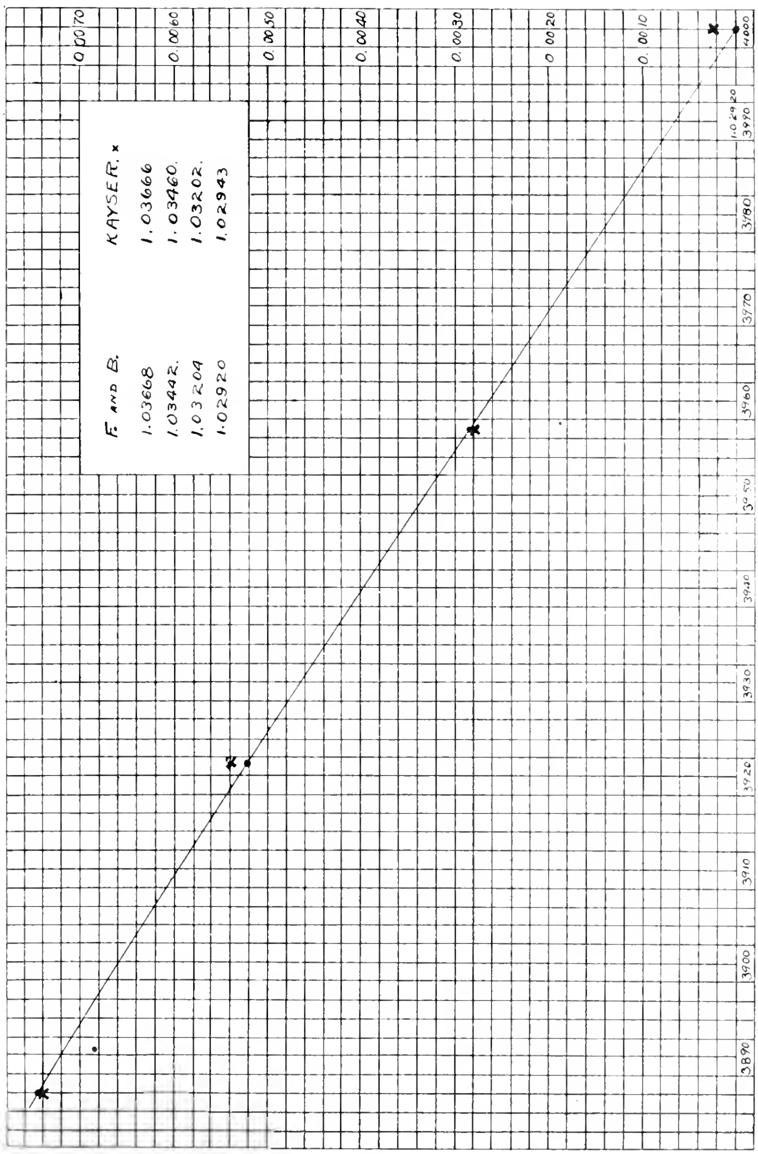


FIG. 1.—Factor-Curve for Plate No. 521

standards in the case of a single plate, depends upon the accuracy with which the factors could be interpolated from the curve.

Table I contains the results from the twenty-five spectra selected for measurement.

TABLE I

Plate	H	λ 3948	K	λ 3930	Spectrograph
348	3968.475 r	3948.784	3933.667 r	3930.300 r	18-foot
349	.476 r	.783	.668 r	.300 r	"
420	.476 r	.785	.667 r	.297 r	"
422	.477 r	.785	.666 r	.297 r	"
423	.478 r	.783	.667 r	.297 r	"
432 (1)	.473 r	.784	.668 r	.300 r	30-foot
(2)	.472 r	.781	.665 r	.302 r	"
(3)	.474 r	.782	.664 r	.300 r	"
476 (1)	.478 r	.787	.667 r	.299 r	18-foot
(2)	.479 r	.784	.665 r	.301 r	"
479	.475 r	.783	.667 r	.302 r	"
480 (1)	.478 r	.784	.665 r	.302 r	"
(2)	.478 r	.784	.665 r	.302 r	"
519 (1)	.479 r	.784	.667 r	.301 r	"
(2)	.475 r	.786	.669 r	.299 r	"
520 (1)	.478 r	.782	.671 r	.303 r	"
(2)	.478 r	.783	.670 r	.301 r	"
521 (1)	.476 r	.784	.668 r	.299 r	"
(2)	.475 r	.784	.668 r	.300 r	"
522 (1)	.473 r	.782	.667 r	.300 r	"
(2)	.473 r	.783	.666 r	.303 r	"
523 (1)	.476 r	.784	.666 r	.304 r	"
(2)	.476 r	.784	.667 r	.301 r	"
B 44 (1)	.473 b	.780	.667 b	.302 r	30-foot
(2)	.475 b	.780	.667 b	.302 r	"
Means of all	3968.476	3948.783	3933.667	3930.301	
Mean residuals	1.7	1.3	1.1	1.5	
Means, 18-foot	3968.476	3948.784	3933.667	3930.300	
Means, 30-foot	3968.473	3948.781	3933.666	3930.301	
Means, reversals	3968.476	3948.783	3933.667	3930 301	
Means, emission	3968.474	3948.780	3933.667	————	

In the above table the reversed and bright lines are indicated by the letters "r" and "b," respectively. An inspection of the table shows the close agreement between the results obtained with the two spectrographs, and the practically identical wave-lengths for both the reversed and bright lines of calcium. The K line was found to be more satisfactory for measurement than H; as an absorption line it is sharper and cleaner, when narrow, than the H line. The residuals represent fairly the difference between the two lines in this particular.

To compare these results with those found previously it is neces-

sary to express them in the Rowland-Kayser system. The reduction factor was obtained by using Kayser's wave-lengths.[1]

λ_A	λ_K	$\lambda_K \div \lambda_A$
4021.872	4022.029	1.000390
3977.745	3977.892	370
3935.818	3935.966	376
3865.526	3865.670	373
Mean		1.000377

Using this factor the following equivalents were found: The H line λ_A 3968.476 = λ_K 3968.625 and the K line λ_A 3933.667 = λ_K 3933.815. H − K = 34.810, which agrees closely with the mean difference 34.809 obtained from the Rowland-Jewell-Adams determinations, while the wave-lengths of H and K differ by 0.004 and 0.003 Å, respectively, from those found by Adams.

The standards used are known for only one of the observers, namely Adams. With respect to these Adams says:

The first four plates were reduced with Rowland's values for the aluminium lines and λ 3973, and Kayser's value for the iron line λ 3928. The last eight plates were reduced with the use of Kayser's iron standards wholly. . . . . That the choice of standards may vitally affect the results is evident from the difference in wave-length shown by 0.013 tenth-meters assigned by Kayser and Rowland to the iron line λ 3928.

For the Kayser iron standards the internal disagreement is sufficient to account for the difference between Adams and the writer, as shown below.

λ_A	λ_K	$\lambda_K \div \lambda_A$
3927.921	3928.073	1.000387
3935.818	3935.966	376
3966.068	3966.219	381
3969.261	3969.411	378
Mean		1.000380

From Mr. Adams it was learned that he used the wave-lengths of the following iron lines from Kayser's table in reducing his last eight plates, namely $\lambda\lambda$ 3969, 3966, 3935, and 3928 The line at λ 3935 is a Fabry and Buisson standard, and as the writer had

[1] *Loc. cit.*

determined the other lines in terms of the Fabry and Buisson standards used for H and K, it was possible to obtain the factor for reducing Adams' wave-lengths to the same scale.

The values for H and K found by Adams from the last eight plates were λ3968.628 and λ3933.817, respectively, which become λ3968.477 and λ 3933.668 on the absolute scale. These differ from the results given in this paper for H and K by 0.001 Å only. Mr. Adams used a different combination of portions of the same apparatus to form the Littrow spectrograph with which his measurements were made.

CALCIUM IN THE SPARK

The same standards were used in the determination of the wave-lengths of the H and K lines in the spark as in the arc. The spark was produced between terminals of metallic calcium for the absorption lines, and between copper terminals moistened with a dilute solution of calcium chloride for the bright lines, which were purposely made of weak intensity. The source was a high-voltage transformer charging an oil-immersed condenser. In series with the spark there was a variable self-induction. The comparison spectra were from an iron arc. A lens kept in a fixed position formed the image of the spark and the arc on the slit, a mirror being introduced into the optical train to bring the light from the arc into the axis of the system. The spectrograph was the 13-foot Littrow instrument in the Pasadena laboratory, referred to as spectrograph No. 1. The plates were measured on the same machine and in the same manner as for the calcium arc plates. Table II exhibits the results.

TABLE II

Plate	H	λ 3948	K
4	3968.476 r	3948.783	3933.668 r
5 (1)	.474 r	.781	.668 r
(2)	.476 r	....	.672 r
6 (1)	.475 b	.783	.668 b
(2)	.476 b	.784	.666 b
7 (2)	.474 r	.781	.665 r
(3)	.479 r	....	.672 r
Means	3968.476	3948.782	3933.668

The agreement between these results and those for the calcium arc

is extremely good, particularly in view of the character of the spark lines when compared with the arc lines, and the lower dispersion used. It indicates the accuracy of a grating for comparative measurements over a limited region when such excellent standards are available, and shows the practically identical wave-lengths of H and K in the arc and spark. The control line, λ 3948 of iron, serves in this case only as a check on the measurements of the comparison spectrum and the reduction of the plates. That there was no instrumental shift is shown by the agreement of the plates with each other.

CALCIUM IN THE ELECTRIC FURNACE

Dr. King placed at the writer's disposal a plate of the region of the spectrum under investigation taken with calcium in the electric furnace at an approximate temperature of 2600° C. The graphite tube was a new, clean one in which some iron dust had been placed that furnished a comparison spectrum showing four of the standards previously used, viz., $\lambda\lambda$ 4021, 3977, 3906, and 3865. The weakest line of the five, λ 3935, was absent. The line λ 3930, whose wave-length was determined in the first part of the investigation in terms of the Fabry and Buisson standards, was used in place of λ 3935, so that the results should be strictly comparable with those for the arc and the spark. The H and K lines were from impurities in the graphite. On this plate the lines of calcium were bright. The results of the measurements are given in Table III.

TABLE III

Exposure	H	λ 3948	K
1	3968.479	3948.789	3933.667
2	.470	.772	.661
3	.472	.787	.667
4	.473	.775	.663
Means	3968.474	3948.781	3933.665

The plate was not taken with a view of wave-length measurements and was not as satisfactory as plates taken expressly for the purpose would have been. In every exposure one or more of the standards was very difficult to set on and the accuracy is much less than that of the corresponding arc measurements. Nevertheless, the final result

shows a satisfactory agreement with the arc measurements, considering the quality of the plate.

In Table IV are assembled the results from the arc, spark, and furnace measurements. The figures in parentheses indicate the number of lines on which the measurements were based.

TABLE IV

Source	H		K	
	Absorption	Emission	Absorption	Emission
Arc..........	3968.476 (23)	3968.474 (2)	3933.667 (23)	3933.667 (2)
Spark........	.476 (5)	.476 (2)	.669 (5)	.667 (2)
Furnace......	....	.474 (4)	.665 (4)	.665 (4)
Means........	3968.476	3968.475	3933.667	3933.666

In determining the following means equal weights were given the results from arc, spark, and furnace, respectively. The final weighted means are:

$$H=3968.476 \qquad K=3933.667$$

The close agreement of the results for the three sources, the two classes of lines and the three spectrographs, shows that the wavelengths of the H and K lines of calcium are independent of the source of the radiation and the density of the vapor, and that gratings of high dispersion may be depended upon to give results over a limited region with an error not greater than 0.001 Å when the best lines referred to sufficiently homogeneous standards.

CHARACTERISTICS OF THE H AND K LINES

The appearance and behavior of the H and K lines of calcium are very similar under all conditions; both, as King has shown,[1] are high-temperature lines, and in the furnace increase in strength in proportion to the temperature. They shift equally under pressure, but only to one-half the amount that λ 4227 shifts for the same increment of pressure, as Humphreys has shown.[2] But in the magnetic field H consists of four and K of six components.[3] Both lines are charac-

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 32, p. 7; *Astrophysical Journal*, **28**, 389, 1908.

[2] *Astrophysical Journal*, **3**, 114, 1896.

[3] Kayser, *Handbuch der Spektroscopie*, **2**, 671.

teristic of the rarer vapor of calcium, as shown by Sir William and Lady Huggins, who found that the other calcium lines disappeared, leaving H and K alone, when by repeated washings they reduced the quantity of calcium vapor in the spark between platinum poles moistened with a dilute solution of calcium chloride. They state:

> Once more the electrodes were washed, with the expectation of having removed completely the last of any trace of calcium. To our surprise when the photograph was developed the H and K lines came out alone.[1]

The extraordinary persistence of these lines was shown in the present investigation when using the arc between copper poles moistened with a drop of very dilute solution of calcium chloride. After repeated washings the lines still appeared on the plates and the observations of Liveing and Dewar, as given in Professor Liveing's letter to Sir William Huggins,[2] could not be confirmed. In this he says:

> I have been looking up some observations of Dewar's and mine on the H and K lines made in 1879. We found that when we used for the arc carbon poles which had been heated for two days in chlorine to remove metals the calcium lines were not at first visible in the arc, but after a time H was seen alone and not strong; after a further time K was seen and then other calcium lines came out. No doubt the calcium had been pretty well removed from the carbon rod to some depth, but not from the interior, so that as the carbon burnt away in the arc the calcium in the interior became manifest.

They found a similar behavior for the H and K lines on diluting the calcium vapor in the arc by a stream of hydrogen, the H line remaining longer as the proportion of hydrogen increased, and appearing before the K line as the proportion of hydrogen was decreased. In the present investigation it has not been possible to obtain either of the lines alone on the photographic plate. K is always wider and stronger than H, even when the lines are extremely narrow and faint. From the measurement of twenty-five emission and absorption lines, the mean ratio of the width of K to H is 1.28. A wide range of widths was covered, which, expressed in Ångström units, are as follows:

[1] *Atlas of Representative Stellar Spectra*, p. 98.

[2] *Loc. cit.*

	Emission		Absorption	
	K	H	K	H
Maximum width	0.455	0.405	0.143	0.106
Minimum width	0.072	0.065	0.056	0.040
Mean width	0.177	0.147	0.087	0.064

The mean ratio of the widths for absorption lines is 1.36, and for emission lines 1.20. The difficulties of width measurement are great and are different for the two classes of lines, and this difference in ratio was at first attributed to errors of measurement, but a similar result was found for the ratio of the intensities of the emission lines. The intensity ratio of bright K to H was obtained by comparing them directly with an artificial scale, made, on the suggestion of Mr. Adams, by using a narrow slit and photographing the direct reflection of sky light from the grating, with exposure times varying from one to sixty seconds. For eight lines of mean width 0.160 Å, the ratio of the intensity of K to H was 1.23; for six lines of mean width 0.083 Å, the ratio was 1.70. In the case of emission lines, K is always stronger and broader than H, and with extremely small quantities of vapor in the arc the preponderance of K over H increases, both in width and intensity. The narrow absorption line produced by the outer cool and thin layer of the arc corresponds to the emission line with rare vapor, and shows a similar preponderance of K over H in width and intensity.

The reversal of K is always stronger than H, and often shows a clean reversal when H shows scarcely a trace of reversal; and in the case of a double reversal, as described later, the emission line of K showing in the reversal is much stronger than the corresponding line of H. In view of the greater width and intensity of K for minute quantities of calcium vapor in the arc, it is difficult to understand how K could disappear before H, as was observed by Liveing and Dewar, unless the observations were made visually The wording in Professor Liveing's letter indicates visual observations, as he speaks of their being "visible" and "seen" In that case it might easily happen that the line of shorter wave-length would disappear first, as the sensibility of the eye falls off very rapidly in the region of H and K.

DOUBLE REVERSALS

Humphreys obtained double reversals of the H and K lines of calcium by using two arcs in series, the one nearer the slit containing a small amount of calcium while that 5 cm farther away contained a large quantity.[1] Barnes attempted to obtain double reversals of these lines by the method of Konen and Hagenbach,[2] which consists in taking a plate of very short exposure of the arc when quickly remade after having been extinguished and just on the point of burning, but he was unsuccessful.[3] In the course of the present study four cases of double reversal of the H and K lines were observed in photographing the spark. The spark was between terminals of metallic calcium and was projected on the slit by a lens so that the image was slightly larger than the spark. The terminals were 5 mm in diameter and pointed, and the gap 5 mm long. The spark was explosive and very brilliant, except occasionally for a few seconds when it was in a hissing state and much less vapor was being evolved It is very doubtful whether the conditions for a true double reversal were present. It seems more probable that the absorption lines were produced by the outer and cooler layers of vapor during the explosive condition, and the fine sharp emission lines were superposed during the hissing stage—though this last was of short duration compared to the total exposure time of about sixty seconds—or possibly during the occasional displacements of the image when for an instant the edge would be on the slit. Out of hundreds of plates taken with the arc, only one showed a double reversal, and that was probaby caused by an extinction and remaking of the arc, as in the Konen and Hagenbach method.

ABSOLUTE WAVE-LENGTHS OF SOME IRON LINES

During the investigation the wave-lengths of nine lines of iron in the region near H and K were carefully determined from the five Fabry and Buisson standards and may serve as additional reference lines in this region, as they are well adapted for measurement. In the following list the secondary standards of Fabry and Buisson are

[1] *Astrophysical Journal*, **18**, 204, 1903.

[2] *Ibid.*, **19**, 111, 1904.

[3] *Ibid.*, **27**, 156, 1907.

marked with an asterisk. The figures in parentheses indicate the number of plates on which the corresponding lines were measured, and the letter "r" the lines which are easily reversed.

4021.872*	3940.882 (6)
3977.745*	3935.818*
3969.261 (10) r	3930.301 (25) r
3966.068 (10)	3927.921 (10) r
3956.678 (4)	3922.913 (9) r
3956.462 (10)	3906.481*
3948.783 (25)	3865.526*

Mr. Kristian Lows was a visitor at the observatory for some weeks during the past summer. The writer wishes to express his appreciation of the assistance rendered by Mr. Lows in making some of the observations and in the reduction of plates for this investigation.

SUMMARY OF RESULTS

1. The wave-lengths of the H and K lines of calcium are 3968.476 and 3933.667, as determined from the secondary standards of Fabry and Buisson, with an uncertainty of not more than 0.001 Å.

2. The wave-lengths are identical for absorption and fine emission lines, and are the same for the arc, spark, and furnace; that is, they are independent of the density and temperature of the vapor, and the source of the radiation.

3. The mean ratio of the width of K to H is 1.28, and the mean ratio of the intensity of K to H 1.47, for the lines measured. Both H and K persist in the arc as the density is decreased, with an increasing preponderance of K over H in width and intensity, pointing to the disappearance of H before K.

4. With sufficiently homogeneous standards, an accuracy of 0.001 Å can be obtained with a grating of high dispersion over a limited region of the spectrum, in the case of the best lines.

Mount Wilson Solar Observatory
December 1909

SOME PHOTOGRAPHIC PHENOMENA BEARING UPON DISPERSION OF LIGHT IN SPACE

By HERBERT E. IVES

Star photographs obtained by Tikhoff have been interpreted as evidence for the dispersion or scattering of light in space.[1] Photographs of the same constellation were made through differently colored glasses. In a "red" photograph the faint stars were more numerous than in a "blue." Therefore, since the faint stars are on the whole more distant, and since blue light is scattered more by passage through a turbid medium than is red, this could mean that an appreciable scattering of light takes place in space.

Considerable discussion in astronomical and physical journals has been aroused by this work. Of most interest from the standpoint of the present article is the discussion by J. A. Parkhurst[2] of the photographic side of the question. The exponent p of the Schwarzschild equation $It^p = iT^p$, upon the different values of which Tikhoff founded part of his conclusion, is shown by Parkhurst to have a wide range of values for different densities upon the same plate, values as large as Tikhoff's "red" values, and as small as his "blue." Photographs by Parkhurst of the *Pleiades* (the constellation chiefly cited by Tikhoff) through visual color-filters and without filter, magnitudes being measured on an absolute scale, showed no evidence for space dispersion. The conclusion he drew was that "the cause lies somewhere in the instruments and plates used, with a probability that it is mainly in the plate and filter." The exact cause was, however, not determined. It is the object of the present paper to show how results may be obtained similar to those of Tikhoff, due to peculiarities of the photographic plate.

In Tikhoff's work the assumption is made that the scale of gradation of the photographic plate is the same for all colors. In other words, the relative photographic action by two differently colored light-sources will be the same, no matter what the time of exposure or

[1] *Comptes Rendus*, **148**, 267, 1909.

[2] *Astrophysical Journal*, **30**, 33, 1909.

the absolute intensity. According to Eder and to Abney[1] this is not so. Abney thus expresses the conclusions from his experimental work: "The least gradation was given at the wave-lengths to which the salt was most sensitive." Leimbach,[2] in the most recent investigation in which this matter is treated, finds no difference for different colors. The evidence is, therefore, apparently conflicting. There is, however, the possibility of these different results being explainable as due to differences in the manipulation of plates. If so, the manipulation in the astronomical work in point may have been such as to cause differences of gradation, as noted by Eder and by Abney.

Two photographic phenomena with which the writer has been familiar presented themselves as possible causes of a different scale of gradation for different colors. First is the fact that with certain developers and plates an image obtained through a red glass develops more slowly than one through a blue glass, although the final density is the same. This is sometimes referred to as a photographic "Purkinje effect." Second is the fact that, in a plate sensitized by bathing, the sensitive layer is very thin. This is shown by microsections obtained by the writer in studying the Lippmann film.[3]

An explanation of the first effect which offers itself is that the effective thickness of film is different for the two colors. The bromide of silver emulsion is comparatively opaque to blue light, transparent to red. We might, therefore, expect the photographic action of blue light to become progressively less through the film, while that of red light is more nearly constant. By long development the red image would continue to gain density by the penetration of the developer into the film after the whole depth of the blue image had been reached. (The red sensitizer is assumed to be incorporated in the emulsion.) There would result upon development to the same densities for the most exposed parts a red image of greater depth than the blue image. A thick layer of emulsion having a longer scale of gradation than a thin one, we should expect a longer scale for the red than for the blue.

If this hypothesis is borne out by experiment we would have an effect just the opposite of that obtained by Tikhoff. His photographs

[1] *Instruction in Photography*, p. 452.

[2] *Zeitschrift für wissenschaftliche Photographie*, **7**, 205, 1909.

[3] *Astrophysical Journal*, **27**, 348, 1908.

would therefore indicate a larger dispersion than he deduced. If, however, we investigate the bathed plate with the same idea of film thickness in mind we find a different condition. The layer sensitized by bathing being thin, it might well happen that the emulsion layer sensitive to red was thinner than that sensitive to blue. This would give a shorter scale to red than to blue.

With this theory as a guide the experiments described below were carried out. Since for the effects to be marked the developer should act through the depth of the film, slow, dilute developers of the non-fogging type were employed, hydrochinone at first, and later glycin (as giving less fog). Development lasted from twenty to thirty minutes. In certain cases, to test the theory, other methods of development were tried, as noted in the appropriate places.

To investigate the scale of gradation, simultaneous exposures of increasing length were made, through red and blue glasses, upon the same plate. A long, narrow plate-holder ($2\frac{1}{2}\times 8$ inch plate), sliding in a wooden frame, permitted successive portions of the sensitive plate to be exposed by means of a "Volute" shutter to the light of a 16-candle-power incandescent lamp eight feet distant. The lamp was maintained at constant voltage throughout a series of exposures. The blue glass was a combination of cobalt blue and signal green glass; the red a methyl-violet and tartrazine stained gelatine film on glass. Neutral-tint glasses placed over either red or blue made it possible to obtain exposures of any relative magnitude desired. Exposures in a series ranged from $\frac{1}{2}$ to 60 seconds. Densities were measured in a spectrophotometer in which the comparison light was varied by a Brodhun sector. The opacities plotted are the total opacities of glass, gelatine, and reduced silver.

The results of the investigation are given diagrammatically in Figs. 1 and 2. With a Cramer "Instantaneous Isochromatic" plate, developed as described, the red gradation was steeper than the blue. With a "Seed 26" bathed for two minutes in a $\frac{1}{100,000}$ solution of pinacyanol the blue gradation was steeper than the red. In both the characteristic curves given the relative exposures were so adjusted that the curves crossed. By proper adjustment the red and blue densities could be made equal' for the longest exposures, in which case, with the bathed plate, the less exposed blue images

were very much fainter than the corresponding red; with the isochromatic plate the reverse condition held. These effects are also obtained if instead of varying exposure the intensities are varied, as was tested by exposing plates at different distances from the lamp, the time of exposure remaining constant.

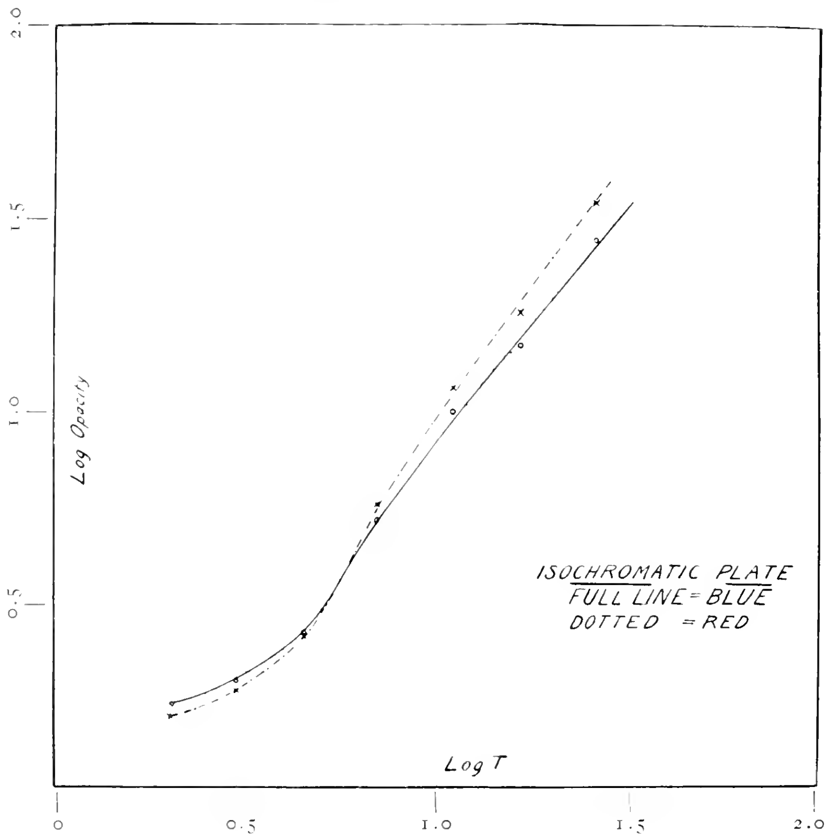


FIG. 1

The conclusion to be drawn from these experiments is this: The relative photographic action of different colors is different for different exposures and for different absolute intensities. With an isochromatic plate we may have a "Purkinje effect," with a bathed plate we may have the reverse of the "Purkinje effect." In the existence of these effects we have a possible explanation of the photographs

obtained by Tikhoff. At any rate, their existence shows the necessity for ascertaining the properties of the plates which he used, under the conditions which held, before accepting conclusions from his results. His plates were bathed, and his results were of just the kind obtained in this investigation with bathed plates.

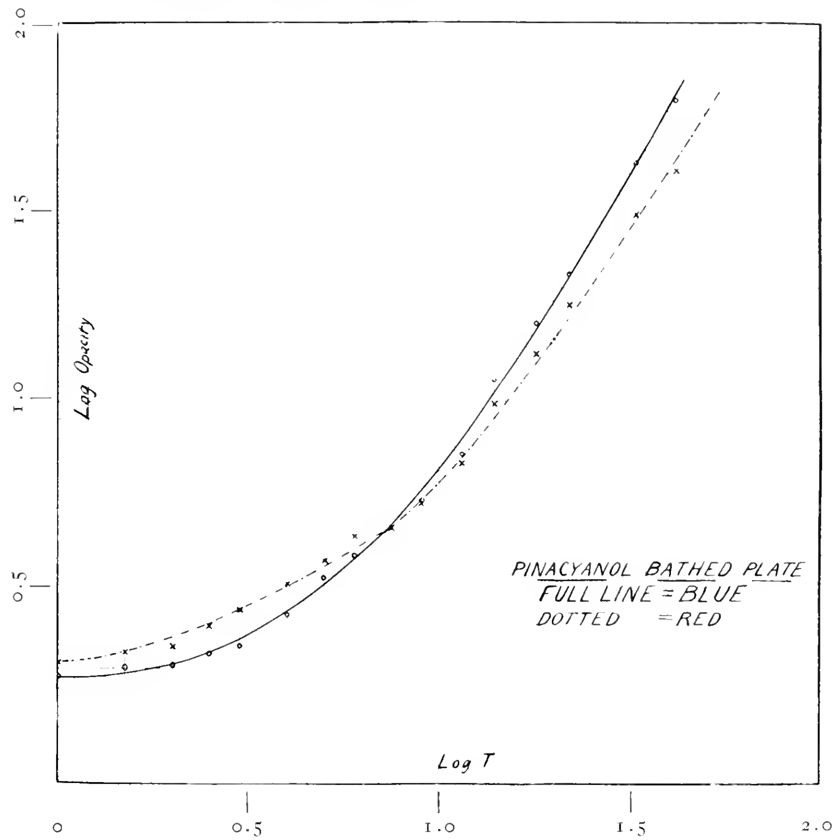


Fig. 2

With a view to testing the validity of the theory upon which the experiments above were based, several others were performed. An isochromatic plate was developed with pyrogallic acid, which, as shown by the writer's sections of Lippmann films, acts chiefly on the surface of the film up to the point of fogging. It was found necessary greatly to increase the red exposure to secure the same density in red

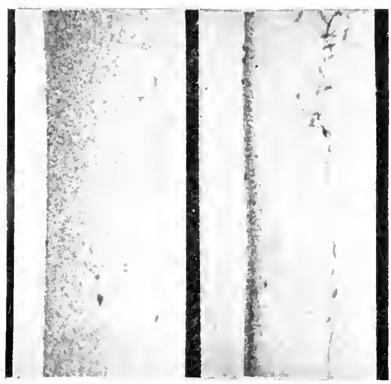
as in blue. When this was done the difference in gradation of the two was no longer evident. In each, therefore, the effective film depth was the same. Another plate was developed with strong hydrochinone, but the development limited to two minutes, with a similar result. These experiments, while appearing to confirm the theory worked upon, also offer an explanation of the conflicting results of Abney and Leimbach. The latter used quite short development with ferrous oxalate in order to avoid all trace of fog. Longer development, or perhaps experiments with other developers would have given other results.

A result obtained with another plate, a slow "Cramer Spectrum," does not appear to fit in so well with the film-thickness theory. These plates are exceedingly sensitive to red, with the sensitizer in the emulsion. With them no difference in scale of gradation was appreciable with any development. With them, however, it is not necessary to depend upon the whole thickness of film to secure sufficient density in the red to match the blue. For the same developed density it is probable that the effective film thickness was nearly the same. Of course it is possible that other factors than film thickness enter in certain cases. There may be a "specific gradation" of the sensitive emulsion for each wave-length, more evident with some sensitizers than with others. If this is the case, Abney's statement of the manner in which gradation changes with color may be nearer expressing the fact than the explanation in terms of thickness. In any case the thickness is of significance. If the thin layer of sensitiveness in the bathed plate is to give as much photographic action as the thicker layer of the unbathed plate, the sensitiveness per unit volume must be greater. We have then actually the condition stated by Abney, that the least gradation is given for the color to which the salt is most sensitive. It is also possible that in the "spectrum" plate, either the sensitizers or the method of manufacture may tan the surface, as pyrogallic acid developer does, and so retard the penetration of the developer. A different amount of photographic inertia for different colors may also play some part.

Whatever the complete explanation of the effects obtained, it is made evident by microscopic sections of the films that the different relative thickness of bathed and unbathed emulsion is a reality, and

therefore probably responsible for a large part of the effects. In Fig. 3 are given two microsections of film from the same plate, *a* for blue light, *b* for red. The small depth of the bathed sensitization is evident. In an isochromatic emulsion the film shows a section very similar in red and blue.

The question naturally arises: What is the magnitude of these two effects? Are they sufficient completely to account for the results obtained by Tikhoff? The extent of the difference in photographic action for different colors obtained by him is unfortunately given only by the values derived for the Schwarzschild exponent p. This, as Parkhurst has shown, is indefinite; widely different values may be obtained where no color difference enters. The writer, on measuring several plates exposed in a manner to give data for calculating p, found the same conditions noted by Parkhurst; a different value could be obtained from each density represented. For the bathed plate p was larger for red than for blue, but the values were so unsatisfactory as values of a supposed constant that no weight could be attached to them. It is indeed open to question whether this exponent, which is a constant for the region of normal exposure, is necessarily so for the region of comparative under-exposure, in which it is evident from the characteristic curves lie the photographic effects with which we are concerned. Another basis of comparison was, therefore, adopted. A bathed plate was exposed through red and blue glasses for the same length of time at varying distances from a source of light, thus securing a gradation scale due to different intensities. The density values were plotted against intensities, giving crossing curves similar to those of Fig. 2. From the curves it was possible to determine the relative intensities for the two colored lights which would give any chosen density. The curves were then replotted with the intensity values for one color multiplied



a *b*

Fig. 3

by a constant factor, so that the curves crossed at a point of considerable density. This is exactly equivalent to making the exposures alike for high intensity by the interposition of the proper neutral tint glasses. With the intensity corresponding to this density (about 3 per cent. transmission) as unity, the intensities corresponding to successive star magnitudes (each 0.4 of the one above) were marked off. The curves then showed that the decrease in density corresponding to a drop of five magnitudes in the red was given by a drop of just over four magnitudes in the blue. Therefore, if two such plates, exposed through red and blue glasses for such times as would give the same density for first-magnitude stars, were examined for the apparent magnitudes of fainter stars, a difference of a whole magnitude would be found by the time the fifth magnitude was reached. This is a difference which should be plainly evident on inspecting the negatives, as Tikhoff says was the case with his photographs.

The conclusion from this work is that the assumption underlying Tikhoff's experiment—that the scale of gradation of the photographic plate is the same for all colors—is not true. The relative photographic action of different colors depends upon the time of exposure and the absolute intensity. The experiment performed by Tikhoff has meaning only if the scale of gradation of the plates employed is known for each color, under the conditions used, and allowance made for it. This is equivalent to the use of such an absolute system of measuring photographic magnitudes as that of Parkhurst and Jordan,[1] provided, of course, that the comparison densities are obtained for each color used. The "Cramer Spectrum" plate, which is highly red sensitive, though not bathed, is apparently largely free from these spurious effects, and would be much better adapted for a research of this kind than an isochromatic of low red sensitiveness, or a bathed plate. In every case the plates employed should be subjected to careful laboratory test before using.

It is evident that a difference in the relative photographic densities of faint and bright stars by differently colored light may be entirely a photographic phenomenon, and hence no evidence for scattering of light in space. Should, however, the effect be found to be real, when tested under conditions as indicated above, there is another possible

[1] *Astrophysical Journal*, **26**, 299, 1907.

explanation. According to Tikhoff's reasoning the faint stars are on the whole more distant. Now we know that many faint stars are as near as some of the brighter ones; certainly in many stellar groups faint and bright stars are grouped together at about the same distance from us. Could we not then state with equal justification that the faint stars are on the whole the smaller ones? Being smaller they would in any group of common origin be farther along in their life-history and so, it might be argued, cooler, and more yellow. The same photographic test which has been applied to the question of light scattering might, therefore, equally well have been called upon to test the hypothesis that the faint stars are as a class smaller than the bright ones—had such a hypothesis been necessary to astronomical problems. If this reasoning is correct, positive photographic evidence of the kind we have been considering would not alone be sufficient to prove scattering of light in space. Reliable conclusions could be drawn only by knowledge of the size and distance, as well as the color, of a large number of stars, the line of investigation which Kapteyn[1] is now pursuing.

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[1] *Astrophysical Journal*, **30**, 284, 1909.

ON THE APPLICATION OF THE LAWS OF REFRACTION IN INTERPRETING SOLAR PHENOMENA

By J. A. ANDERSON

During the past few years Professor W. H. Julius has published a number of very interesting papers dealing with the subject of refraction and anomalous dispersion in the atmospheres of the sun and other heavenly bodies (*Astrophysical Journal*, from Vol. **12** onward). The conclusions at which he arrives are in many cases radically different from the views which have hitherto been held by astrophysicists, and it is not surprising therefore that the investigators in this subject have been rather slow in adopting them. They are naturally unwilling to discard their own views in favor of new ones until these new ones shall have been established beyond any possibility of doubt. Now, the earlier experiments of Julius, although they illustrated the principles of anomalous dispersion admirably, did not imitate the conditions existing in the atmosphere of the sun well, because the source of white light invariably subtended only a very small angle as seen from the position occupied by the refractive medium in question, while in the supposedly parallel case in the solar atmosphere the angle subtended is very nearly 180°. In a later paper by Julius, however, entitled "Regular Consequences of Irregular Refraction in the Sun,"[1] a source of much greater angular diameter is used, although the description of the apparatus does not enable one to say just how closely conditions existing in the solar atmosphere are copied.

In the present paper we shall make an attempt to examine by elementary methods the consequences of irregular refraction in such an atmosphere as that of the sun, and then try to define the conditions which an experiment must fulfil if it is to have any application to solar phenomena in general.

Let us assume in the first case that what is called the photosphere may be represented by a perfectly uniform self-luminous surface. Any point in the solar atmosphere (neglecting the corona) lies so

[1] *Proc. of Roy. Acad. Sci. of Amsterdam*, **18**, 266, September 25, 1909.

close to the photosphere that the latter subtends an angle very nearly equal to 180° and we may therefore fairly represent conditions by replacing the photosphere by an infinite, plane, self-luminous surface. The irregularities in the solar atmosphere may be assumed to be of the nature of "Schlieren" which are usually more or less nearly circular cylinders of gas whose refractive index differs from that of the surrounding medium, the change in refractivity being gradual from the circumference toward the axis.

Let A (Fig. 1) represent a section of such a cylinder, and $BCDE$ represent the path of a ray of light passing through it. As far as the

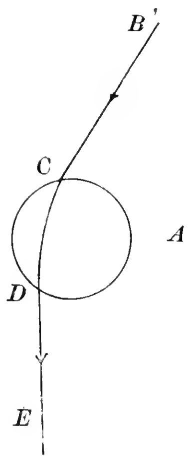

FIG. 1

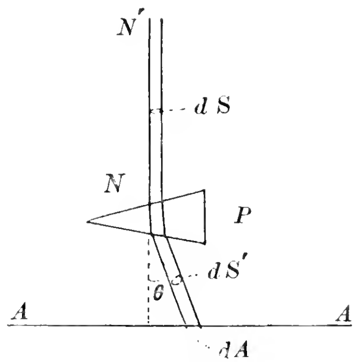

FIG. 2

ray $BCDE$ is concerned the action is the same as that of a prism of suitable angle, and for the sake of clearness we will first suppose this cylinder replaced by a prism. After finding what is the action of a prism, we will make the changes necessary to fit the case under consideration. Let AA (Fig. 2) be the uniform plane self-luminous surface, and let P be a prism of any transparent substance whatever, so oriented that light of wave-length λ emerging from it in the direction NN', which is perpendicular to AA, shall have passed through it with minimum deviation. Consider a small cylinder of the emerging beam of cross-section dS, trace this back through the prism to the surface AA, where it incloses an area dA. Call the cross-section of

the cylinder between P and AA dS'. Since the beam has traversed the prism with minimum deviation we have $dS = dS'$. If the condition for minimum deviation had not been fulfilled we should have dS greater than dS' or dS less than dS' according as the angle of incidence is greater or less than the angle of emergence. For the case of minimum deviation we have, letting θ be the angle of deviation, $dS' = dA \cos\theta$. Let I_λ be the intensity of light of wave-length λ emitted in a direction normal to AA, then the intensity in a direction making the angle θ with the normal will be $I_\lambda \cos\theta$, and hence the amount of energy passing through any section of our cylinder between AA and the prism per second will be $dA \times I_\lambda \cos\theta = dS' \times I_\lambda$. If we now neglect losses by reflection from the surfaces of the prism we have for the energy passing any section of the cylinder NN' per second $I_\lambda dS$, since $dS = dS'$; that is, the intensity is I_λ or the same as it would be in the absence of the prism.

If the light did not traverse the prism with minimum deviation we should have the intensity as seen through it greater or less than what it would be in its absence according to whether the angle of incidence is less or greater than the angle of emergence. For small deviations, however, the change in intensity would be slight, a deviation of 10° producing a change in the intensity which can never exceed $1\frac{1}{2}$ per cent.

For an actual prism the above statements will require some slight modification owing to the reflections at the two surfaces, but if we substitute for the prism a mass of gas such as that represented in Fig. 1, then, since the change in refractive index is gradual, we have no reflection and the above statements are correct. Moreover, for the case represented in Fig. 1, no ray of light can pass through except at minimum deviation, and this is very approximately true even if the cross-section instead of being circular is distorted into the form of an ellipse. Therefore it is evident that *a uniform luminous surface would appear uniformly luminous to an eye at a distance, even if covered by an atmosphere full of "Schlieren," provided the deviation produced by the "Schlieren" does not exceed 90°.*

Since this holds for light of any wave-length, it will hold for such substances as the metallic vapors showing anomalous dispersion, except inside the limits of the absorption band. In this region absorp-

tion would of course take place, extinguishing the light more or less completely. But *unless the deviation for light anomalously refracted exceeds 90°, it follows that the absorption band would have exactly the same width and character that it would have if produced by a perfectly homogeneous atmosphere of the same absorptive power.*

This applies directly to the center of the solar disk. It also applies to any point of the sun's visible surface if we substitute for 90° the angle at the center of the sun between the radii drawn to the point in question and to the nearest point on the limb, respectively. *Now, since such phenomena as sun-spots, faculae, flocculi, etc., do not change materially in appearance as they approach the limb it becomes at once evident that the angles of deviation with which we have to deal never approach 90° and possibly never even 5° or 10°.*

We can get a better idea of the order of magnitude of the deviations to be expected by inquiring what irregular density-gradients we may reasonably expect to find in the solar atmosphere. In the paper by Julius referred to above it is shown that if we have a density-gradient equal to the vertical gradient at the earth's surface, the radius of curvature of a ray at right angles to the gradient will be about $\frac{1}{50}$th of the radius of the photosphere. There are two causes which tend to destroy irregular density-gradients in an atmosphere. One is the pressure of the gas, and its effect will be the same on the sun as on the earth, since it depends upon the inertia of the gas. The other one is the weight of the gas, which causes it to move toward its own proper level in the atmosphere. This is 27.3 times as great on the sun as on the earth, and, besides, the distance the gas will have to move to find its own proper level is only $\frac{1}{27.3}$ as great as on earth. It follows therefore that irregular density-gradients in the sun's atmosphere will have, in general, only $\frac{1}{(27.3)^2}$ or about $\frac{1}{750}$ of their value in the earth's atmosphere. These gradients in the earth's atmosphere perhaps never exceed one inch of the barometer in ten miles (1 cm in 6 km), or about $\frac{1}{50}$th of the vertical gradient at the surface. The corresponding radius of curvature for a light-ray at right angles to the gradient would equal the radius of the sun's photosphere. For the gradients to be expected in the sun's atmosphere the radius of curvature would be 750 times this, or about 325 million miles (525 million kilometers).

Let us allow a ray of light a path of 10,000 miles (16,000 km) in a direction at right angles to this gradient, *and the corresponding deviation amounts to just about 6 seconds of arc!* In cases of light-rays suffering anomalous dispersion the deviation may perhaps reach 100 times this value, or a matter of a few minutes of arc. And this on the extravagant assumption of an undisturbed path 10,000 miles long!

In conclusion we may say, then, that irregular refraction and anomalous dispersion undoubtedly do modify the true appearance of the solar surface, but the order of magnitude of the effect is such that with our present instrumental equipment it is very doubtful if we shall be able to detect it.

Physical Laboratory
Johns Hopkins University
January 18, 1910

GLASS AND METALLIC REPLICAS OF GRATINGS

By J. A. ANDERSON

Replicas of gratings were first made by Thorpe in England; later by Wallace and by Ives in this country. The method used by both Wallace and Ives is to pour upon the grating a solution of gun cotton in amyl acetate, or some similar substance, and after this is dry to allow it to peel off under water, and then to mount it upon a piece of plane glass. One surface of the film of collodion, the one which was in immediate contact with the surface of the grating, is found to be a fairly accurate copy of the ruled surface of the grating itself, while the other one is more or less perfectly flat. The first will be spoken of simply as the ruled surface, or as the face.

Thorpe mounted his replicas with the ruled surface up, while Wallace speaks of mounting the film either ruled surface up or down, preference being given to the latter. Ives mounts all of his with the ruled surface down, in contact with the glass, I believe.

When a replica is mounted face up it may be transformed into a metallic reflection grating simply by coating it with platinum by means of cathode disintegration in a vacuum, as has been lately described by E. Gehrcke and C. Leithäuser.[1] As a rule, however, this surface is perhaps never quite plane, owing to the unavoidable differences in the thickness of the film in different places, and hence I imagine that gratings made in this way will never perform very well when subjected to a really severe test.

The method of making the ordinary transmission replicas as used by Wallace and Ives has already been described by Wallace[2] and hence need not be described here in any more detail.

During the past two years the author has been experimenting with the making of replicas in the hope of finding a method of duplicating the gratings in metal; and in the course of the work a method was also found by which gratings can be copied in glass or quartz, which of course makes them very permanent. Besides this, much

[1] *Verhandlungen der Deutschen Physikalischen Gesellschaft*, 1909.

[2] *Astrophysical Journal*, **22**, 123, 1905; **23**, 96, 1906.

was learned about the characteristics of gratings and their replicas which must have escaped other workers in the same field, since the author had at his disposal about 100 Rowland gratings to work with, while others have had only a very limited number. This will, however, be the subject of a future paper. At present we shall touch on only one point which is important in connection with the subject of this paper, and that is, "How perfect is it possible to make a replica?" or, what amounts to the same thing, "How nearly will the resolving power of a replica equal that of the grating from which it was made?"

At first some small gratings of $1\frac{1}{4}$ inches and $2\frac{1}{2}$ inches width were used and it was found that the replicas if carefully handled were all good when examined by an ordinary laboratory spectroscope having about a 1-inch objective of perhaps 10 inches focal length. On trying them with our plane grating spectroscope whose focal length is about 10 feet it was soon seen that they fell far short of equaling the original grating in definition and resolving power. This became more and more evident as larger gratings were used. The replicas from a 6-inch flat grating were practically worthless, although if an inch square or so of their surface was used it performed beautifully. The explanation is evident. The films in drying shrink somewhat and tend to shrink a little unevenly, so that the lines are no longer absolutely straight, parallel, and equidistant, but deviate from these conditions more or less. This phenomenon can be made very evident to the eye by placing the replica in contact with the original grating so that the lines of the two are parallel to each other. In this case a series of dark and bright fringes are seen where the lines of the replica alternately get in step and out of step with the lines of the grating. Wallace mentions that in his replicas these fringes are always more or less curved but that if the curvature is small the replica is found to perform well.

The author attempted to make these fringes all parallel and equidistant by mechanically stretching the replica wherever it was required, this being of course done before all the water between it and the glass surface had evaporated. At first he succeeded only in making things worse, but with a little practice it was soon found that a replica could be corrected to within a small fraction of a fringe,

which of course corresponds to the same small fraction of the grating-space in the replica itself. A replica corrected in this manner gives the same resolving power as the grating from which it was made within a very few per cent., provided that the glass on which it is mounted is optically perfect.

In order to transform one of these replicas into a metallic grating it is not sufficient simply to turn its face up and then deposit a film of metal over the ruled surface, since the film is never of absolutely even thickness and hence the ruled surface in this case will not be plane, as it must be, accurately, if the resulting grating is to be of any value; nor is it sufficient to silver the glass surface upon which the replica is placed face down, for very obvious reasons. What is wanted is some substance which will fill up the grooves *AA* (Fig. 1) between the replica and the glass surface after the replica has been corrected and dried, and which will adhere to the glass sufficiently

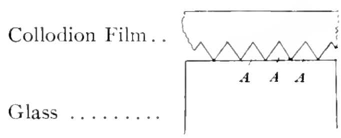

Fig. 1

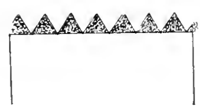

Fig. 2

to allow the replica to be stripped off. The result will then be similar to Fig. 2. This is evidently a fair copy of the grating from which the replica was made.

Such a substance was accidentally found. If certain gums are dissolved in the collodion solution they will, on gently heating the glass plate upon which the corrected replica is placed, slowly ooze out, filling up the grooves as indicated, and on cooling will harden and allow the replica to be stripped off. The resulting grating (Fig. 2) may now be treated in one of the two following ways:

1. It may be covered by a thin film of platinum, nickel, or other suitable metal in a vacuum, which coating may be improved by subsequent electroplating, thus producing a durable metallic grating having a perfect optical surface.

2. It may be treated with hydrofluoric acid gas, thus transforming it into a glass or quartz transmission grating with an equally good optical surface, as the gum used is not affected by the acid, while the glass or quartz between the ridges is rapidly attacked.

In conclusion it may be remarked that the method is equally well applicable to concave gratings, the apparatus required being a convex surface of the same radius of curvature as the grating to be duplicated to be used as a test plate for the correction of the replica, and then a concave glass mirror on which the replicas are mounted and treated as just described.

At the present time the author has a number of plane platinum, nickel, and gold gratings made by the above process, which perform admirably, as well as a number of glass ones made by the hydrofluoric acid process.

I wish to express my thanks to Mr. Sparrow, who has given me much valuable aid in this work.

Physical Laboratory
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January 18, 1910

Minor Contributions and Notes

NOTE ON THE CALCIUM BANDS AT λ 6382 AND λ 6389

An important feature of sun-spot spectra is the presence of the calcium bands with heads at λ 6382 and λ 6389. Their presence was discovered by Professor Fowler. In a previous paper[1] by the writer it was shown that these groups appear in the metallic arc, burning in air under greatly reduced pressure, and their wave-lengths were measured. The purpose of the present note is to report some further observations concerning these bands.

The concave grating, inclosed arc, and the other apparatus used have all been described in former publications.[2]

In air.—No signs of these bands can be found on the plates of the spectra of the arcs in air at atmospheric pressure. The electrodes were pure metallic calcium and carbon poles filled with $Ca(OH)_2$, $(CaSO_4)_2H_2O$, $(C_3H_5O_3)_2Ca$, and $(C_7H_7SO_3)_2Ca$. They are also absent from the arcs in air at a pressure of half an atmosphere, but when the pressure is reduced to about 3 cm of mercury and less they come out strongly and the plates show that the intensities of the heads are as strong as any of the lines in this neighborhood, such as $\lambda\lambda$ 6439, 6462. This observation was made only with metallic poles. Every precaution was taken to exclude water-vapor from the inclosed arc at the low pressures by covering the bottom with a layer of phosphorus pentoxide.

In hydrogen.—Olmsted[3] found these bands present in the arc burning in hydrogen. This observation was repeated; the bands appear clearly on the plates but never with a relative intensity greater than those from the arc *in vacuo*. When steam was continually driven into the vessel the bands did not appear. Olmsted, however, found them when the steam entered through a hole in an electrode.

In nitrogen.—Nitrogen carefully prepared and thoroughly dried was admitted into the vessel until the pressure was about that of an

[1] *Astrophysical Journal*, **30**, 14, 1909.

[2] *Ibid.*, **27**, 152, 1909, and **30**, 15, 1909.

[3] *Ibid.*, **27**, 68, 1908.

atmosphere. The bands appear on the plates but never as strong as those from the arc *in vacuo*.

In sulphur dioxide the arc does not contain these radiations.

It has been suggested that these bands are due to a calcium and hydrogen compound. Although it can always be claimed that even *in vacuo* sufficient hydrogen may be liberated from the hot poles to form the "hydride" necessary for the production of these bands, nevertheless in the light of the above experiments it seems very doubtful, since they do not appear in the spectra of compounds containing hydrogen, and even in the arc in hydrogen their intensities are no greater than in the arc in air under reduced pressure. In a recent article by King[1] on the radiations from an electric furnace the same view is expressed.

Brooks[2] remarks in a very interesting paper upon the magnesium spectrum that the so-called "hydride" spectrum is still an open question. He suggests that the flutings may be due to the metal itself and not radiations from a compound, and cites the views of Hemsalech, Hartley, and Ramage in support of this idea.

So far as my observations go, they seem to indicate that these calcium bands may also be considered as true metallic radiations. The presence of hydrogen and nitrogen surrounding the arc does not influence them to any great amount; air and sulphur dioxide destroy them completely.

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BRYN MAWR COLLEGE
January 1910

FOUR STARS HAVING VARIABLE RADIAL VELOCITIES

Of the measures recently made on Bruce spectrograms the following are of immediate interest.

The place of *α Cygni* in the spectral classification has been a subject of considerable discussion, but it is properly included in Vogel's type Ia2, and is characterized by the numerous and sharp enhanced metallic lines, as was long ago pointed out by Lockyer.

[1] *Astrophysical Journal*, **29**, 381, 1909.

[2] *Proc. R. S.*, **80**, 218, 1907.

The lines are seldom blended and the spectrum is thus very satisfactory for measurement.

a Cygni ($\alpha = 20^h\ 38^m$; $\delta = +44°\ 55'$; Mag. = 1.3)

Plate	Date		G. M. T.	Taken by	No. Lines	Velocity	Quality
						km	
B 121	1901	Mar. 31	$20^h\ 52^m$	E	16	−9.1	g.
B 131		May 10	21 45	F, E	15	−6.9	g.
A 167		Aug. 14	15 30	F	12	−4.3	g.
A 168		Aug. 14	15 54	F	13	−4.4	g.
B 326	1902	April 16	20 30	A	13	−2.6	g.
B 463		Nov. 27	15 47	F, A	13	−3.0	g.
B 603	1905	July 1	17 6	F	10	−5.2	g.
A 525		Oct. 23	14 11	F	17	−0.1	v. g.
IB 1074	1907	June 1	21 39	Fox	12	−1.9	v. g.
1075		June 1	21 53	Fox	18	−2.5	v. g.
2177	1909	Nov. 8	15 18	L	16	−9.4	v. g.
2182		Nov. 19	11 28	B, L	13	−9.3	g.

A=Adams, B=Barrett, E=Ellerman, F=Frost, L=Lee. g=good, w=weak, v=very. Mr. Sullivan assisted as usual in guiding on all the plates in this note.

The only published velocity determinations of *a Cygni* are those by Vogel and Scheiner[1] from plates taken at Potsdam in 1888–1889. Using four plates which give values ranging from −4.5 to −10.4 km, Vogel derived a mean of −6.0 km, Scheiner a mean of −10.0 km. Imperfections in the apparatus and method of measurement in that early day of spectrography would, no doubt, account for this range even if the velocity of the star had been constant.

a Cygni has been casually observed with the Bruce spectrograph since 1901. All spectrograms but the last four were taken with three prisms. Camera A is a Zeiss anastigmat of 449 mm focus; B, a Hastings triple of 608 mm focus. The last four plates have the dispersion of one prism.

Plate No. B 326 was measured by Mr. Walter S. Adams, B 463 by Professor Frost. The other measures are mine. On our one-prism plates the calcium lines H and K are sharp, and they were used in the measurement, being in satisfactory agreement with the other lines. The excellent character of the lines had suggested to Mr. Frost the use of this star as a control for low-dispersion plates. On measuring the one-prism plates taken with this in view, the variable velocity became apparent, and this was confirmed by the early

[1] *Publikationen der Astroph. Obs. zu Potsdam*, **7**, Theil 1, 1892.

three-prism plates that had not been measured. The range of 9 km happens to be the same on the one-prism and three-prism plates.

The orbit of *α Cygni* should be determined from spectrograms taken with the highest possible dispersion, but this does not preclude the desirability of a careful study of the velocity derived from the H and K lines.

The velocities given above do not suggest the period.

58 Tauri ($\alpha = 4^h\ 15^m$; $\delta = +14°\ 51'$; Mag. $= 5.3$)

Plate	Date	G. M. T.	Taken by	No. Lines	Velocity	Quality
					km	
IB 1867......	1908 Nov. 16	$20^h\ 33^m$	B	9	+41	g.
2179......	1909 Nov. 8	18 51	B	4	+32	w.
2195......	Nov. 25	17 50	F, L	11	+17	v. g.
2262......	1910 Jan. 18	14 28	L	9	+15	g.

This star, and the one following, are members of the *Taurus* stream discovered by Professor Boss;[1] this is No. 1007, the next is No. 1092, of his catalogue. From the radial velocities of three stars, γ, δ, and ϵ, as determined by Küstner, Boss has predicted velocities for the other stars of this group. His prediction of +39.9 km for *58 Tauri* seemed to be substantiated by our first plate. The last two plates, however, have proved the binary character of the star. The spectra of both *58* and 7°681 *Tauri* are classified as A in the *H. R.* notation. Each has a great many lines that apparently might be measured, but, as a matter of fact, so many of them are difficult blends that only the comparatively few lines used seemed reliable for velocity determinations. The region of the spectrum used extends from λ 4045 to $H\beta$.

It is a singular fact that of the eight spectroscopic binaries in this group that have so far been investigated here only two have thus far shown variations in velocity extending appreciably above the predicted values. These are θ^2 and *69* (upsilon) *Tauri*, found by Professor Frost.[2]

The remarkable abundance of spectroscopic binaries among the stars of this stream, pointed out (*loc. cit.*) by Mr. Frost last year, is

[1] *Astronomical Journal*, **26**, 31, 1908.

[2] *Astrophysical Journal*, **29**, 237, 1909.

confirmed by the subsequent observations. Two-thirds of those stars of which three or more spectrograms have thus far been obtained here are found to vary in radial velocity.

B. D. 7°681 *Tauri* ($\alpha = 4^h\ 34^m$; $\delta = +7°\ 40'$; Mag. = 5.6)

Plate	Date	G. M. T.	Taken by	No. Lines	Velocity	Quality
					km	
IB 1830......	1908 Nov. 6	$18^h\ 45^m$	B, L	10	+34	g.
1974......	1909 Feb. 1	14 30	F, L	14	+42	g.
2226......	Dec. 29	14 2	L	10	+29	g.
				7	+28	
2267......	1910 Jan. 21	14 37	B	11	+17	g

The first two values are the means of accordant duplicate measures. The second measure on plate No. 2226 is by Professor Frost. Boss predicted a radial velocity of +41.8 km for this star. Measures of the first plate indicated a departure from constant velocity, which succeeding plates have confirmed.

θ Pegasi ($\alpha = 22^h\ 5^m$; $\delta = +5°\ 42'$; Mag. = 4.1)

Plate	Date	G. M. T.	Taken by	No. Lines	Velocity	Quality
					km	
IB 803......	1906 July 13	$20^h\ 8^m$	F	3	−11	g.
810......	July 20	21 0	F	4	+ 4	v. g.
1771......	1908 Oct. 5	14 5	F, L	4	+ 8	g.
1816......	Nov. 2	13 47	L	6	−31	v. g.
1826......	Nov. 6	14 58	B	6	+10	v. g.
1841......	Nov. 9	13 8	L	6	+19	v. g.
2117......	1909 Aug. 27	17 3	B	4	−32	g.

This star has a spectrum of type Ia2 in the notation of Vogel. There are only a very few metallic lines that are strong enough to measure. Measures on the hydrogen lines were given most weight.

The binary character of this star was long ago suspected by Professor Frost upon examining the early plates. Plate No. 1816 shows a very faint component at +62 km.

OLIVER J. LEE

YERKES OBSERVATORY
February 16, 1910

REVIEWS

Annals of the Astrophysical Observatory of the Smithsonian Institution. Vol. II. By C. G. ABBOT AND F. E. FOWLE.

In the bolometric study of the infra-red solar spectrum conducted by the Astrophysical Observatory of the Smithsonian Institution it was found that this region of the spectrum was the seat of great terrestrial atmospheric absorption, that the intensity of absorption of the lines was variable, and that the relative intensity of energy in different parts of the spectrum changes appreciably. A thorough and detailed account of the investigations is given in the first splendid volume of the *Annals*. With the publication of these results, the work of mapping the spectrum was discontinued and attention was directed specifically toward the determination of the total solar radiation, the distribution of energy in the solar spectrum, and its variations. The present volume records in detailed form the methods and results of these and allied investigations.

Three distinct lines of observations are discussed, and they all point harmoniously and more or less clearly to the possibility of variation of the so-called solar constant. Such a variation would be of the greatest interest to astronomers, climatologists, and geologists.

Part I deals directly with the determination of the solar constant of radiation. High and low solar observations were made simultaneously with spectrobolometer and pyrheliometer at Washington and at Mt. Wilson, California. The pyrheliometer measures the total energy of the solar radiation as it reaches the instrument. The spectrobolometric observations enable the observer to estimate the losses the beam has suffered in the earth's atmosphere. The accuracy with which this estimate can be made finds noteworthy illustration in the comparison of the results from the two stations. Although the pyrheliometer at Washington showed only three-quarters of the quantity of energy recorded at Mt. Wilson, the estimation of atmospheric transmission was of such accuracy that its application brought the results into sensible accord. Early chapters give a full description of methods and apparatus, sample observations and reductions, and a discussion of the sources of error. From the last the authors conclude that separate determinations at Mt. Wilson will seldom differ by as much as 1.5 per cent. or at Washington by 3 per cent. The lower curves of Plate XV are of particular interest in showing that the

methods are of such sensitiveness that they reveal the slow variation of the solar radiation depending on the sun's varying distance. It seems unavoidable to ascribe the outstanding variations which amount to 10 to 15 per cent. to a variation in the solar constant.

In Part II there is a discussion of the dependence of terrestrial temperatures on solar radiation. From the Stefan-Boltzmann law we find that a fractional change of absolute temperature of a perfect radiator is one-fourth the fractional change of radiation which accompanies it. For the earth, if it were a perfect radiator, a change of 1 per cent. in the solar constant would produce a change in temperature of $\frac{2}{3}°$. But the curve for response in temperature-change of a perfect radiator following fluctuating insolation is more or less modified and straightened by the curves of actual terrestrial stations, much as the erratic track of the fore wheel of a bicycle is reproduced in modified form by the rear wheel. As we increase the distance between the wheels the rear wheel modifies more and more the deviations of the front wheel; so also as we go from inland meteorological stations to coast and island stations and the insulation against temperature-change exerted by great water masses is more and more potent, the response to varying insolation is more and more sluggish. It is estimated that a fluctuation of 5 per cent. in solar radiation with a period of a year would produce a change of 1° for inland and 0°.3 for island stations. The authors selected 47 well-distributed inland stations and examined their temperature records. Their conclusion is that there are well-marked deviations from normal mean temperatures of these stations which embrace them all. No comparison was made between the curves of temperature deviation and the solar-constant curves. Such a comparison shows, however, that the curves follow fairly well during 1903, but for no other time is there striking agreement.

Part III gives results of study of the radiation from different parts of the solar disk. Spectrobolometric observations giving the transmission coefficients for different wave-lengths at varying distances from the center of the disk show that the transparency of the solar envelope varies. Here is independent evidence of the variation of the solar constant and an indication of its cause as well. A comparison of departures from mean coefficients of transmission with solar constant results gives, however, only questionable agreement. This lack of agreement suggests the necessity of seeking another cause for the variation of radiation.

Professor C. L. Poor[1] has published three papers on "The Figure of

[1] *Astrophysical Journal*, **22**, 103, 305, 1905; Contrib. Columbia University Obs., No. 26.

the Sun. His observations indicate a variation in the diameter of about $0''.1$. Moulton[1] has shown that this would produce a prohibitive change of solar temperature in the order of 1400° C. A small oscillation might, however, exist, and he found that an oscillation of $0''.01$ would produce the variation of 10 per cent. in the solar constant first reported by Langley[2] and confirmed in this volume. Such an oscillation may well be the true cause, and if so, the investigators in any given determination catch the radiation in one point of its rapid change. For Moulton finds that the period of oscillation is short, and Emden in his *Gaskugeln* (p. 453) states: "Betrachtet man aber die Sonne in jedem Moment als adiabatische Gaskugel, so bemisst sich die Dauer ihrer Gravitationsschwingungen nur nach Stunden." If the solar radiation follows this rapid oscillation it is hopeless to expect agreement between the results of the three investigations included in the *Annals*. Between the solar atmospheric transmission coefficients and terrestrial temperatures there may be some connection. Data given here can be compared only in certain months of 1905. A longer interval is needed.

The by-products of these investigations are scarcely less important than the main results, but they can only be mentioned. The development of a standard pyrheliometer; a determination of the reflecting power of clouds, 65 per cent.; the albedo of the earth, 37 per cent.; new determinations of the solar temperature; a very interesting solar theory, constitute other material treated.

The volume contains the results of observations of the most difficult character made with extreme care. The volume is a model typographically, the plates are excellent, the arrangement and indices perfect.

PHILIP FOX

The Inequalities in the Motion of the Moon Due to the Direct Action of the Planets. (An Essay Which Obtained the Adams Prize at the University of Cambridge for the year 1907.) By ERNEST W. BROWN. Cambridge: The Cambridge Press, 1908. Pp. xii+92.

Professor Brown has spent many years on the Lunar Theory, the greater part of which is concerned with the orbit of the moon as disturbed by the sun. He has carried out the extremely laborious piece of work of calculating the perturbations produced by the sun, following the general lines marked out by Hill in his celebrated *Researches*, and his results have a degree of accuracy, when considered as a whole, not before attained.

[1] *Astrophysical Journal*, **29**, 278, 1909.

[2] *Ibid.*, **19**, 305, 1904.

It is sufficient to state that, assuming that the series converge, the computations give the position of the moon with at least as great accuracy as it can be observed. Hence, in order to test the sufficiency of the theory and incidentally to make the most rigorous demands on the law of gravitation which are capable of being made, it became necessary to add the perturbations due to other causes, such as the direct action of the planets. Brown's prize memoir is devoted to this problem.

In computing the effects of the direct action of the planets on the motion of the moon there are certain practical difficulties, the chief of which are finding the derivatives of the moon's co-ordinates with respect to its mean motion, whose numerical value is used in Brown's theory, and the fact that the sensible terms appear only at rare intervals, and are likely to be overlooked. Brown overcame the first difficulty by an ingenious method invented in 1903, and the second by the construction of a "sieve" by means of which those terms which it is necessary to compute could be separated out from the others. Words of praise for the excellence of the work are superflous when it is noted that the memoir obtained the Adams prize.

F. R. M.

Magneto- und Electrooptik, von WOLDEMAR VOIGHT. Leipzig: B. G. Teubner, 1908. 8vo, pp. xiv+396, with 75 figs. M. 14, bound.

It will be commonly conceded that there is no one living who is better qualified to present the existing status of the electron theory of the Zeeman, Faraday, and Kerr effects than is the pre-eminent Göttingen physicist whose name appears upon the title-page of this book, and who has himself made such important contributions to the theory of these phenomena.

The book was published about a year ago, and at once took its place as the most authoritative and the most complete treatment of this subject which has yet appeared. And although the field is one in which new experimental data are continually appearing and demanding a place in existing theory, there can be little doubt that for years to come Professor Voigt's book will hold its place as a most important reference work both for the experimental and the theoretical student of magneto-optics. For it contains not only all the experimental data which had appeared in this field up to the time of publication, but also an admirably clear and logical development of the whole electron theory of light, particularly in its relations to the phenomena of dispersion and absorption. One of the most noteworthy features of the book is its beautiful interweaving of theory

and experiment, and the careful working-out of numerical examples to support theoretical conclusions.

The author has avoided the notation of the vector analysis, first, because, as he says, his book is designed for the experimental as well as for the theoretical physicist, and, second, because he finds that in the problems with which he has to deal this notation offers no appreciable advantage.

Roughly speaking, the first half of the book has to do with the presentation of experimental data and the simpler aspects of the electron theory as applied to dispersion, absorption, the Zeeman and the Faraday effects; while the last half presents the theory, largely due to the author, of the more complex types of Zeeman effect, the Kerr effect, and the magneto-optics of non-isotropic media. In the last fifty pages are found the theory of the vibrations of bound electrons under the influence of an electric field.

R. A. MILLIKAN

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A General Index to Sidereal Messenger (Vols. 1–10), *Astronomy and Astrophysics* (Vols. 11–13), *Popular Astronomy* (Vols. 1–16). By W. W. PAYNE. Northfield, Minn., 1909. Price $1.50; bound $2.50.

This index to the journals successively edited by Professor Payne and his associates will be welcomed as an important addition to astronomical bibliography. It is arranged first by authors, and then by subjects. Natural abbreviations of one letter readily distinguish the journal in which a paper appeared. The type and presswork are excellent. A few typographical errors have been noticed, but the references to volumes and pages have doubtless been carefully checked for their accuracy.

Annuaire astronomique de l'observatoire royal de Belgique. Bruxelles: Hayez, 1909. Pp. 534, with diagrams and plates.

This excellent little book appears in the same style as in recent years. Besides its accurate data of an astronomical kind, it contains an admirable sketch by Professor Stroobant on the progress of astronomy in 1908.

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ON THE ORIGIN OF BINARY STARS

BY HENRY NORRIS RUSSELL

1. One of the fundamental problems of cosmogony may be stated as follows: Will a rotating mass of fluid, in equilibrium under its own gravitation, and free from sensible external disturbance, but subject to loss of heat and consequent contraction, eventually break up into separate parts; and if so, how will the separation take place? The only case which has been subjected to detailed mathematical investigation is the classical one of a homogeneous incompressible fluid. The general results (associated with the names of Maclaurin, Jacobi, Poincaré, and Darwin) are widely known. The originally spheroidal mass becomes more and more flattened, tends to lose stability, and recovers it by changing into an ellipsoid of three unequal axes, which similarly goes over into an elongated "pear-shaped" figure with one end larger than the other. According to Darwin, this last is also stable;[1] but at this stage the analysis becomes too complicated to pursue further. Darwin has, however, shown[2] that two masses of similar fluid cannot be brought close enough to coalesce without previously becoming unstable, and hence that if fission of a single mass occurs, it must be accompanied by a "period of turbulence." It is not, however, definitely known what would be the ratio of the resultant masses, or even into how many pieces the original mass would divide.

[1] *Phil. Trans.*, A, **200**, 251–314, 1903.

[2] *Ibid.*, **206**, 161–248, 1906.

But the actual stellar or nebulous masses with which astronomy must deal are gaseous—highly compressible, and much condensed toward their centers. The general mathematical investigation of the figures of equilibrium and their stability would probably in this case be very difficult. It is known that central condensation diminishes the ellipticity of the spheroid when the angular velocity is small, and very probable that it increases its stability.

On the other hand, Jeans has shown[1] that the compressibility of the gas tends toward instability, which may set in for relatively small angular velocity, and is of such a nature that the centers of the inner surfaces of equal density move away from the center of gravity in one direction, and those of the outer layers move in the opposite direction; while later these surfaces themselves become pear-shaped. The later stages of this process have not been followed mathematically. The detailed results (such as the exact rate of rotation at which a given change of form or stability occurs) would depend to an unknown extent upon the assumed law of distribution of temperature and density within the gaseous mass.

There remains therefore a good deal of uncertainty as to what would actually happen to a rotating and contracting mass of gas.

Sir George Darwin, in a recent essay,[2] summarizes the theory of fission as follows:

> Originally the star must have been single, it must have been widely diffused, and must have been endowed with a slow rotation. In this condition the strata of equal density must have been of the planetary form. As it cooled and contracted the symmetry round the axis of rotation must have become unstable, through the effects of gravitation, assisted perhaps by the increasing speed of rotation. The strata of equal density must then become somewhat pear-shaped, and afterwards like an hour-glass, with the constriction more pronounced in the internal than in the external strata. The constrictions of the successive strata then begin to rupture from the inside progressively outwards, and when at length all are ruptured we have the twin stars portrayed by Roberts and by others.

On the other hand, Chamberlin,[3] in a paper recently published by the Carnegie Institution, has expressed the view that "such rotat-

[1] *Phil. Trans.*, A, **199**, 1–53, 1902.

[2] "The Genesis of Double Stars," *Darwin and Modern Science*, p. 563 (Cambridge University Press, 1909).

[3] "The Bearing of Molecular Activity on Spontaneous Fission in Gaseous Spheroids," *Carnegie Institution, Publication 107*, p. 167.

ing gaseous spheroids must shed portions of their matter molecule by molecule, if they do so at all," and Moulton,[1] in the same volume, concludes that it is probable "that if a fluid mass ever gets into the state where fission occurs, there is at least great danger of its breaking into many pieces," and hence that "if binary stars and multiple stars of several members have developed from nebulae, the nebulae must originally have had well-defined nuclei" (corresponding to the members of the final system).

2. Between such divergent opinions, in the absence of any well-developed theory, the appeal evidently lies to the facts of observation.

Those stars which are merely double give us very little relevant evidence. The close pairs, almost in contact, revealed to us among the variable stars, may be accounted for on either theory. The apparently universal fact that the components of a binary system are comparable in mass is what might be expected as a consequence of the fission theory, but would probably have to be a postulate of the other.

When it comes to the triple and multiple systems, the situation is different. There seems to be no a-priori reason why systems originating from independent nuclei should show any definite relations of mass or relative distance. We might expect a purely random arrangement, without grouping into well-defined pairs.[2] On the contrary, such a grouping is a necessary consequence of the fission theory, depending upon elementary dynamical principles, quite apart from the more complex questions of stability, etc. It is the purpose of the present discussion to develop these consequences, and to see how far the results agree with observation.

3. In any system subject to internal forces alone the total moment of momentum about any axis through the center of gravity must remain constant, whatever the configuration assumed. There is a certain axis for which this is a maximum. Let us take this as the z-axis. Then the moments of momentum about the x- and y-axes

[1] "Notes on the Possibility of Fission of a Contracting Rotating Fluid Mass," *ibid.*, p. 157.

[2] Conditions of stability of the resulting orbits might, however, lead to such a grouping, as is shown by Moulton's work on *70 Ophiuchi* (*Astronomical Journal*, **20**, 33, 1899).

will vanish. Let us also choose the units of mass, length, and time so that the constant of gravitation is unity.

Suppose now that the system consists of two rotating bodies, of masses m_1 and m_2, in orbital motion about one another. The moment of momentum M of the system will be the sum of that due to the rotations of m_1 and m_2 about their axes, and that due to the orbital motion of their centers of gravity about that of the system. Let k_1 be the radius of gyration of the mass m_1; i_1 and Ω_1 the inclination and node of its equator on the invariable plane $z=0$, and ω_1 its angular velocity of rotation. Then its rotational moment of momentum about an axis through its center of gravity parallel to the z-axis will be $M_1 = m_1 k_1^2 \omega_1 \cos i_1$; and similarly for m_2. We cannot express this in terms of the mean density or radius of the mass unless we know its internal constitution as well as its form; but if r_1 is the equatorial radius of m_1 (or its maximum radius, if the equator is not circular), we may set $k_1^2 = c_1 r_1^2$ and write

$$M_1 = c_1 m_1 r_1^2 \omega_1 \cos i_1. \tag{1}$$

For a homogeneous sphere the constant c is 0.40. Central condensation diminishes it; for example, $c=0.26$ for a sphere whose density decreases according to Laplace's law and vanishes at the surface, and 0.20 for one whose density follows the law of adiabatic equilibrium for a monatomic gas.[1]

Polar flattening has little or no influence upon the ratio c as here defined. Ellipticity of the equator again diminishes it. If the curves of equal density in the equatorial plane are changed from circles into ellipses whose minor axis is b times the major axis, c will be reduced to $\frac{1+b^2}{2}$ times its original value. This ellipticity will be greatest when the masses are homogeneous, and will increase as they approach one another.

In the extreme case of two equal homogeneous masses in contact, Darwin's calculations[2] give the maximum and minimum radii of the equator as 1.17 and 0.645, whence $b=0.55$, and $c=0.26$—an approximation only, as the actual form is only roughly ellipsoidal, but a close one. This configuration, however, is unstable. Two

[1] T. J. J. See, *Astronomische Nachrichten*, **169**, 321, 1905.

[2] *Phil. Trans.*, A, **206**, 246, 1906.

fluid masses in contact can be in stable equilibrium, if at all, only when condensed toward their centers. This condensation will diminish c, and it seems probable that for actual masses in contact c can hardly exceed 0.20, and may be much less.

The relative orbital motion may as a first approximation be regarded as in a Keplerian ellipse. Let its semi-axis major be a, its eccentricity e, and its node and inclination be Ω and i. The orbit of m_1 about the center of gravity of the system will have the semi-axis $\frac{m_2 a}{m_1+m_2}$. Its orbital moment of momentum about the z-axis will be $\frac{m_1 m_2^2}{(m_1+m_2)^2} n a^2 \sqrt{1-e^2}\cos i$, where n is the "mean motion" in the orbit. Adding the momentum m_2, and remembering that $n^2=\frac{m_1+m_2}{a^3}$, we find for the whole orbital momentum

$$N=\frac{m_1 m_2}{\sqrt{m_1+m_2}}\sqrt{a(1-e^2)}\cos i\,.$$

The expression under the radical is the semi-parameter p of the orbit.

When the masses are not spherical, these relations will no longer hold exactly. It is easy to see that both polar flattening and elongation of the masses toward one another tend to increase their mutual attraction. This will increase the mean motion, and hence the orbital momentum.

We may write

$$n^2=\frac{m_1+m_2}{a^3}(1+\zeta) \tag{2}$$

where ζ is always positive, and increases as the bodies become more elongated, and as they approach one another, varying approximately as $\frac{r^5}{a^5}$.[1] For the case already mentioned, of two equal homogeneous bodies in contact, Darwin's formula gives $\zeta=0.22$, which must be a close approximation to the truth. For two equal ellipsoids of homogeneous fluid, separated by but one-third of their longer diameters, ζ is about 0.07. If they are separated by one diameter, it is less than 0.01.

Central condensation tends strongly to diminish ζ, as the surfaces

[1] Darwin, *Phil. Trans.*, A, **206**, 195, 1906.

which inclose successive equal portions of the whole mass become smaller and more nearly spherical.

We may assume as a guess that $\zeta = 0.1$ for two gaseous masses in contact. It will be seen later that the uncertainty of its value will not seriously influence our results.

The orbital momentum about the z-axis is therefore

$$N = \frac{m_1 m_2}{\sqrt{m_1+m_2}} \sqrt{p(1+\zeta)} \cos i. \tag{3}$$

If K is the whole moment of momentum of the system, we have then

$$c_1 m_1 r_1{}^2 \omega_1 \cos i_1 + c_2 m_2 r_2{}^2 \omega_2 \cos i_2 + \frac{m_1 m_2}{\sqrt{m_1+m_2}} \sqrt{p(1+\zeta)} \cos i = K. \tag{4}$$

The corresponding equations for the x- and y-axes may be obtained by substituting $\sin \Omega \sin i$ and $\cos \Omega \sin i$ for $\cos i$, and zero in place of K.

4. Let us now consider an initial configuration consisting of two equal and similar masses, revolving in contact with uniform angular velocity. Let m represent their combined mass. Then

$$m_1 = m_2 = \frac{m}{2}\ ; \quad \omega_1 = \omega_2 = n = a^{-\frac{3}{2}} m^{\frac{1}{2}} (1+\zeta)^{\frac{1}{2}}\ ; \quad \text{and } r_1 = r_2 = \frac{a}{2}\,.$$

Also c is the same for both stars, and $e = i = i_1 = i_2 = 0$. The rotational momentum of each mass is

$$M = \tfrac{1}{8} c m^{\frac{3}{2}} a^{\frac{1}{2}} (1+\zeta)^{\frac{1}{2}},$$

and the orbital momentum

$$N = \tfrac{1}{4} m^{\frac{3}{2}} a^{\frac{1}{2}} (1+\zeta)^{\frac{1}{2}},$$

whence

$$M = \tfrac{1}{2} c N\,, \quad K = N + 2M = (1+c) N.$$

We are now in a position to follow the future course of the system. As the separated masses contract, their rotation will become more rapid, and tidal interaction will tend to transfer their rotational momentum to that of the orbital motion.

a) Let us first suppose that this process proceeds to its limit. Then all the momentum will be orbital. If p is the semi-parameter of the final orbit, we shall have $\frac{1}{4} m^{\frac{3}{2}} p^{\frac{1}{2}} = K$ (ζ vanishing in this configuration), whence

$$p = a(1+c)^2 (1+\zeta). \tag{5}$$

Setting $c = 0.2$, $\zeta = 0.1$, we have $p = 1.58a$ or $a = 0.63p$.

This assumed final condition is practically that of many binary systems. We may therefore say:

If a binary star, whose components are single, and of equal mass, has originated by fission, the distance of the centers of the two masses at the time of fission must have been fully $\frac{5}{7}$ of the present semi-parameter of the orbit (the exact ratio varying with the internal constitution of the separating masses).

If the final period is measured in years, the density at the time of separation must have been exceedingly small.[1] This was first pointed out by Moulton,[2] who discussed the case where the bodies are spheres and the orbit a circle. The equation (5) may be reduced to his equation (which involves the period), when $c=\zeta=0$, but is somewhat simpler in form.

b) Let us next assume that tidal friction is negligible. Then the moment of momentum of the detached masses remains constantly equal to M, and in time, as they contract, they may split up again. Suppose that one of them divides again into equal parts. If a^1, c^1, etc., are the constants defining the state of the new system at the time of fission, we shall have $m^1=\frac{1}{2}m$, and the whole momentum of the new system will be

$$K^1=(1+c^1)\cdot 2^{-\frac{7}{2}}m^{\frac{3}{2}}a^{1\frac{1}{2}}(1+\zeta^1)^{\frac{1}{2}}\,.$$

Equating this to its initial value M, we find

$$\frac{a^1}{a}=\frac{2c^2}{(1+c^1)^2}\,\frac{1+\zeta}{1+\zeta^1}\,. \tag{6}$$

If the law of density within the separating masses is the same as at the first fission (which seems probable if the gas is in both cases rare enough to obey the ordinary laws of gases closely), the values of c and ζ will be the same, and we shall have simply

$$a^1=\frac{2c^2}{(1+c)^2}a\,.$$

Setting $c=0.2$, we have $a^1=0.056a$.

This is a result of great importance, which may be expressed verbally as follows:

If a mass divides by fission into equal parts, and one of these

[1] Of the order of magnitude of one-millionth of that of air under ordinary conditions.

[2] *Carnegie Institution, Publication 107*, p. 133.

divides again in the same fashion, owing to its rotation alone, the initial distance of the secondary pair cannot be greater than about $\frac{1}{18}$ of that of the primary pair; and hence the mean density of the mass at the time of the second separation must be at least 2500 times as great as at the time of the first.

If tidal evolution proceeds to the limit in this new system, the final semi-parameter of the secondary orbit will be given by the equation

$$p^1 = a^1(1+c^1)^2(1+\zeta^1) = 2c^2(1+\zeta)a \,. \tag{7}$$

If there is no tidal friction, each component will again split up into a much closer pair, until the increase of density sets a physical limit to the process.

c) If, as is most probable, tidal influence has diminished the rotational momentum of the separated masses, so that at the time of the second fission it has x times its initial value, we have only to set cx instead of c in (6) or (7) and $c(1-x)$ instead of c in (5), to find the dimensions of the corresponding bodies or orbits in terms of the initial distance a. Since in this case the final orbits need not be in the same plane, we must also set $p \cos^2 i$ in place of p in (5) and (7); and the equations of momentum about the x- and y-axes will give relations between the positions of the orbit planes.

5. The results so far obtained hold good only in the case of fission into equal parts. If the masses resulting from fission are unequal, we may set

$$m_1 = my \,, \quad m_2 = m(1-y) \,,$$

and, at the moment of separation,

$$r_1 = az \,, \quad r_2 = a(1-z) \,, \tag{8}$$

(where y and z lie between zero and unity).

Proceeding as above, keeping the subscripts for the two masses separate, and setting for brevity $m^{\frac{3}{2}} a^{\frac{1}{2}} (1+\zeta)^{\frac{1}{2}} = \mu$, we find:

$$M_1 = yz^2 c_1 \mu \,, \quad M_2 = (1-y)(1-z)^2 c_2 \mu$$
$$\text{and } N = y(1-y)\mu \,,$$

and hence, if tidal evolution proceeds to the limit, for the semi-parameter of the final orbit

$$p = a(1+\zeta)\left(1 + \frac{z^2 c_1}{1-y} + \frac{(1-z)^2 c_2}{y}\right)^2 . \tag{9}$$

If tidal action does not enlarge this orbit at all, its final semi-parameter will be

$$p_0 = a(1+\zeta) . \tag{10}$$

The actual value must in all cases lie between these limits.

If the detached mass m, without loss of momentum, divides again into two portions, the ratio of whose masses is $u_1 : 1-u_1$, and tidal evolution proceeds to the limit in the resulting system, the semi-parameter p_1 of the final orbit will be given by:

$$p_1 = \frac{z^4 c_1^2}{y u_1^2 (1-u_1)^2} \cdot a(1+\zeta) . \tag{11}$$

If the rotational momentum of this mass is reduced by tidal action to x_1 times its initial amount before the second fission, we must as before replace c_1 by $c_1 x_1$ in (11), and by $c_1(1-x_1)$ in (9), and p by $p \cos^2 i$ in both. We thus obtain the equations of the most general case.

6. The radii of the two separating masses can be very approximately determined when the ratio of their masses is known.

The final severance of the neck connecting the two masses will take place at that point on the line of centers where the resultant of the attractions of the two masses is exactly balanced by the centrifugal force. The distances of this point from the centers of gravity of the masses, and of the system, are respectively az, $a(1-z)$, and $a(y+z-1)$.

Let ζ_1 and ζ_2 denote the fractions by which the attractions of the two masses at this point are increased by their departure from sphericity. (As this influence increases rapidly with diminishing distance, these will be greater than the quantity ζ, previously defined, which represents the increase of the attraction of each body on the other as a whole.)

The condition of equilibrium may then be written:

$$(1+\zeta_1)\frac{m_1}{a^2 z^2} = (1+\zeta_2)\frac{m_2}{a^2(1-z)^2} + n^2 a(y+z-1) ,$$

which reduces to:

$$\frac{(1+\zeta_1)y}{z^2} = \frac{(1+\zeta_2)(1-y)}{(1-z)^2} + (1+\zeta)(y+z-1) . \tag{12}$$

Differences between ζ_1 and ζ_2 can only arise from differences in the form and internal structure of the two masses. These will not be

great when they are comparable in magnitude. Moreover, if ζ_1 increases (other things being equal), z must increase. But increase of ζ_1 corresponds to increased ellipticity of the mass m_1, and hence to a decrease of c_1. Its moment of inertia, which is proportional to $c_1 z_1^2$, will therefore suffer little change; and this alone appears in the equations (9) and (11).

We may therefore, for our purpose, safely assume that $\zeta_1 = \zeta_2$. If now we set $\frac{1+\zeta}{1+\zeta_1} = k$, (12) becomes:

$$y\left(\frac{1}{z^2} + \frac{1}{(1-z)^2} - k\right) = \frac{1}{(1-z)^2} + k(z-1) \; . \qquad (13)$$

For such mass-ratios as are known to occur among double stars, the solution of this equation is almost independent of k, as is illustrated by the following examples:

y	If $k=0$	If $k=1$	Adopted
$\frac{1}{5}$	$z=0.333$	$z=0.360$	$z=0.35$
$\frac{1}{4}$	0.366	0.390	0.38
$\frac{1}{3}$	0.414	0.429	0.42
$\frac{1}{2}$	0.500	0.500	0.50

The actual value of k will probably be much nearer unity than zero. It is clear that the values of z adopted in the last column can be very little in error.

7. With these data, we proceed to calculate the ratio of the greatest possible final distance (semi-parameter) of the close secondary pair to the least possible distance of the wide primary pair. By (10) and (11) this is

$$\frac{p_1}{p_0} = \frac{z^4 c_1^2}{y u_1^2 (1-u_1)^2} \; \frac{\cos^2 i}{\cos^2 i_1} \; . \qquad (14)$$

This depends both on the ratio y of the mass in question to the original mass and on the corresponding ratio u_1 for the second fission. If the masses formed by the latter are equal, we find (neglecting the inclinations):

$y=$	$\frac{4}{5}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	
$\frac{p_1}{p_0}=$	$3.57c_1^2$	$3.15c_1^2$	$2.72c_1^2$	$2.00c_1^2$	$1.48c_1^2$	$1.33c_1^2$	$1.20c_1^2$	(15)

If the masses resulting from the second fission are unequal, these values of $\frac{p_1}{p_0}$ must be multiplied by the following factors:

Ratio of masses:	1 : 1	2 : 1	3 : 1	4 : 1	(16)
Factor:	1.00	1.26	1.78	2.44	

To find the initial distance a_1 of the close pair (at the time of fission) we see by (9) and (10) that we must divide the ratios just obtained by

$$\left(1+\frac{z^2c_1}{1-y}+\frac{(1-z)^2c_2}{y}\right)^2$$

(in which the values of y and z are those corresponding to the second fission).

Setting $c_1 = c_2 = 0.2$ and combining the results with (16), we find that $\frac{a_1}{a}$ may be obtained by multiplying the quantities (15) by the following factors:

Ratio of masses:	1 : 1	2 : 1	3 : 1	4 : 1	(17)
Factor:	0.69	0.80	0.98	1.16	

8. The following consequences may easily be deduced from the above data:

Given a gaseous mass, which divides by fission, without external disturbance, into two parts:

1) The distance of centers at the time of separation is greater, and the density less, the more unequal these parts are.

2) The ratio in which the initial distance can be increased by tidal action increases as the masses become more unequal.

3) The smaller mass has the greater density just after separation.

4) The ratio of contraction necessary to bring about a second fission (other things being equal, and tidal friction absent) is less for the greater mass.

5) The ratio of the dimensions of the separating masses at the time of the second fission to that at the first involves the factor c^2, and is always small.

6) The same is true of the final orbits resulting from the successive fissions.

7) The increase of density between the fissions is very great.

The greatest disparity in mass of which we have satisfactory evidence among the few binary systems which have been investigated with any approach to accuracy is about 3:1. In most cases the components are much more nearly equal.

For mass ratios within this limit, the maximum ratio of the parameter of the secondary orbit to that of the primary is $5.6c^2$ which, as c is at most 0.2, cannot be greater than 0.22. The corresponding ratio of the initial distances is at most $3.19c^2$, or 0.124. As in this case the separating mass contains $\frac{3}{4}$ of the original material, its density at the time of the second fission must be at least 380 times as great as at the time of the first; which is the minimum ratio of increase.

The smaller mass produced by the original fission will give rise to a still closer pair, with greater initial density.

Tidal action before the second fission can only increase the distance of the wide pair, and decrease that of the close pairs. It is easy to see that its influence will be relatively greater upon the smaller mass. This will increase the difference in distance of the corresponding pairs, and, if powerful enough, may keep the smaller mass from dividing, and so produce a triple instead of a quadruple system.

If neither mass divides again, we find from (9) (setting $c=0.2$, $\zeta=0.1$) that the final semi-parameter of the orbit is at most equal to twice the initial distance (so long as one mass is not more than three times the other).

9. For such a distribution of masses as is found among binary stars, the results of the fission theory are quite definite. Multiple systems arising in this way must be pairs, one or both of whose components are themselves double, with a distance less than about one-fifth that of the wide pair—usually much less. Some of the components of these close pairs may be still closer pairs, after the same fashion.

It would therefore seem that we ought to be able to tell by a mere glance whether a multiple star can have originated by fission or not. But the situation is not really quite so simple.

We must first of all have some evidence (such as common proper motion) that the stars really belong together. Among the many systems of this sort there is not one in which we can yet determine the orbital elements of the wide pair; and in most cases this is still impossible even for the close pairs.

We therefore usually know, not the real distances between the stars, but only their projections on the celestial sphere, which may be much foreshortened. Moreover, on account of the eccentricity of

the orbits, the actual distances at a given time may differ considerably from the semi-parameters of the orbits.

The ratio of the observed distances of the close and wide pairs may therefore be very different from that of the parameters of their orbits. We cannot determine one from the other, in any particular case, unless we know the orbital elements; but the theory of probability enables us to estimate how many cases there will be, out of a large number, in which the apparent ratio will exceed the true one by more than any given factor.

Since we know of no reason why the smaller orbit should be more eccentric, or more highly inclined to the line of sight, than the larger, it is a priori equally likely that the larger or the smaller distance will be most affected by these influences; and hence it is equally probable that the apparent ratio will exceed or fall short of the true one; that it will be more than double or less than half the latter, etc.

In general, let r_1, r_2, denote the real, and s_1, s_2, the apparent distances, and let $f(x)$ be the probability that, owing to a given cause, $s_1 < r_1 x$. By hypothesis, the same function represents the corresponding probability for s_2. There will be certain values a and b which mark the extreme limits of the given influence (as for example 0 and 1 in the case of foreshortening). Then $f(x)=0$ if $x<a$, and $f(x)=1$ if $x>b$. The probability that $\frac{s_1}{r_1}$ lies between x_0 and x_0+dx is $f'(x_0)dx$. In order that $\frac{s_2}{s_1}$ may be less than k times $\frac{r_2}{r_1}$ we must in this case have $\frac{s_2}{r_2}<kx_0$. The probability of this is $f(kx_0)$. Combining the two probabilities and integrating over the whole range of possible values of x, we find for the whole probability $F(k)$ that $\frac{s_2}{s_1}$ is less than $k\,\frac{r_2}{r_1}$.

$$F(k)=\int_a^b f(kx)\,f'(x)dx.$$

It is easy to show that $F(1)=\frac{1}{2}$, and that $F\left(\frac{1}{k}\right)=1-F(k)$, these being merely formal statements of the propositions already proved by considerations of symmetry.

If now we have two independent influences, which separately give rise to probabilities $F(k)$ and $\Phi(k)$, of the kind just discussed, it

follows in the same way that the probability $\Psi(n)$ that under their joint action $\frac{s_2}{s_1} < n\frac{r_2}{r_1}$ is given by the equation

$$\Psi(n) = \int_\alpha^\beta F(nk)\ \Phi'(k)dk\ ,$$

the limits being those appropriate to $\Phi(k)$. As before, by symmetry

$$\Psi\left(\frac{1}{n}\right) = 1 - \Psi(n)\ .$$

In the case of foreshortening, if we assume that all directions of r_1 and r_2 are equally and independently probable, we find easily $f(x) = 1 - \sqrt{1-x^2}$ and hence, if $k < 1$,

$$F(k) = \frac{k^2}{3} + \frac{k^4}{3\cdot 5} + \frac{k^6}{5\cdot 7} + \ldots .$$

In the case of an elliptical orbit, since we observe all the systems under discussion in the same relatively short interval of *time*, the probability that $\frac{r}{p}$ is less than any given value is simply the fraction of the whole period of revolution during which it is below this limit. The necessary integration is best performed graphically, and so is the combination of the result with the influence of foreshortening.

In the numerical work the orbital eccentricity has been taken as 0.5, about the average for double-star orbits.

The results are as follows:

s represents the observed distance, r the real distance, and p the semi-parameter of the orbit.

$F(n)$ is the probability that $\frac{s_2}{s_1}$ is *greater* than n times $\frac{r_2}{r_1}$

$\Phi(n)$ is the probability that $\frac{r_2}{r_1}$ is *greater* than n times $\frac{p_2}{p_1}$

$\Psi(n)$ is the probability that $\frac{s_2}{s_1}$ is *greater* than n times $\frac{p_2}{p_1}$

n	4	3	2	$\frac{3}{2}$	1	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$
$F(n)$	0.02	0.04	0.09	0.17	0.50	0.83	0.91	0.96	0.98
$\Phi(n)$	0.00	0.00	0.10	0.20	0.50	0.80	0.90	1.00	1.00
$\Psi(n)$	0.035	0.07	0.16	0.28	0.50	0.72	0.84	0.93	0.965

10. It is now possible to compare the observed facts regarding multiple stars with the predictions of our theory.

To be sure of dealing only with physical systems, discussion has been confined to cases where there is good evidence that all three or more stars have a common proper motion. The data are those of Burnham's *General Catalogue of Double Stars*, supplemented at times by Lewis' catalogue of the Struve stars (*Memoirs R. A. S.*, **56**).

A		*B*		*C*		*D*	
s_1 less than 100 years' p. m.		s_1 between 100 and 300 years' p. m.		s_1 between 300 and 1000 years' p. m.		s_1 greater than 1000 years' p. m.	
No.	s_2/s_1	No.	s_2/s_1	No.	s_2/s_1	No.	s_2/s_1
2109*	0.07	152	0.03	1036	0.03	1448	0.13
2279	0.12	648	0.03	1457	0.08	1755	0.14
4414	0.11	672	0.06	2605	0.02	1985	0.13
4477*	0.16	1070*	0.03	2883	0.02	2027	0.005
4771*	0.07	1262	0.26	. . .	0.61	2406	0.04
. . . .	0.17	1471	0.01	3099	0.01	2883	0.42
4866	0.09	1559	0.10	3402	0.33	4456	0.40
7040	0.15	2857	0.02	3962	0.04	4481	0.06
7487*	0.10	3402	0.32	4122	0.08	5833	0.06
7878	0.22	3559	0.18	5331	0.02	6482	0.02
7929	0.06	3757	0.19	5841	0.05	8783	0.02
8162*	0.04	6155	0.03	7259*	0.01	8785	0.01
8642	0.09	6296	0.04	7493	0.07	9602	0.004
9114	0.06	6571	0.05	7533	0.02	9617	0.03
10643	0.09	7332*	0.01	. . . .	0.05	10112	0.004
11928	0.27	9011	0.04	7905	0.01	11160	0.49
13055	0.14	9090	0.10	8669	0.01	11839	0.57
13264	0.06	9643	0.04	11129	0.02	. . . . .	0.53
		9660	0.03	11160	0.14	12573	0.01
		9782	0.10	12378	0.05		
		10057	0.15	. . . . .	0.05		
		12257	0.01	12571	0.04		
		12946	0.13				
		13025	0.07				

The whole number of double stars for which these catalogues record common proper motion is about 800. Of these, 74 are triple or multiple, a not inconsiderable percentage of the whole.

As the full data are so easily accessible, it is only necessary to give here the numbers of the stars in Burnham's catalogue, and the ratio $\frac{s_2}{s_1}$ of the apparent distances of the close and wide pairs. These have been divided into four groups, according to the distance of the wide pair (in the case considered), measured in years' proper motion of the system—thus roughly classifying them according to their real dimensions.

As in some cases both components of a wide pair are double, and in others the wide pair itself forms one component of a still wider pair, certain systems appear two or three times in the list, raising the whole number of entries to 83.

The distance of the wide pair is measured from the mid-point of the closer pair when this correction affects the results appreciably. In a few cases, when the orbit of the close pair is known, its semi-major axis replaces the observed distance s_2. (This is chosen instead of the semi-parameter because it is nearer the mean value of r_2.) These cases are indicated by an asterisk *.

The distribution of the individual values of s_2/s_1 in each group is as follows (values at the limit between two lines being divided equally between them):

s_2/s_1	*A*	*B*	*C*	*D*	*ABC*	Theory
Over 0.40 . . .			1	5	1	1
0.40 to 0.30		1	1		2	2
0.30 to 0.20 . . .	2	1			3	3
0.20 to 0.15	$2\frac{1}{2}$	$2\frac{1}{2}$			5	$4\frac{1}{2}$
0.15 to 0.10	4	3	1	3	8	$9\frac{1}{2}$
0.10 to 0.05	$8\frac{1}{2}$	4	5	2	$17\frac{1}{2}$	16
0.05 to 0.025 . . .	1	$8\frac{1}{2}$	5	2	$14\frac{1}{2}$	8
Under 0.025 . . .		4	9	7	13	1

The average distance of the wide pair increases rapidly from group to group. This explains the deficiency in the last lines of group *A*, for close pairs of such small separation could not usually be seen double.

Apart from this there is evidence of a progressive change in the distribution from group to group. This may be partly due to the fact that foreshortening of the wide pair at the same time diminishes its apparent distance and increases the ratio $\frac{s_2}{s_1}$; which tends to heap up the larger ratios in the earlier columns of the table. The marked discrepancy for group *D* cannot, however, be explained in this way.

Setting it aside for the moment, and combining the other groups, we have in the column headed *ABC* the observed distribution for all systems whose extent is less than 1000 years' proper motion.

The last column gives the distribution which might be expected,

according to paragraph 9, among 45 systems for all of which $\frac{p_2}{p_1} = 0.09$. The agreement is very good, except in the last two lines. That is, we may account for the observed facts by assuming that in 45 of the systems $\frac{p_2}{p_1} = 0.09$, while in the other 19 it is very much less, averaging about 0.02.

Of course it is not to be supposed that $\frac{p_2}{p_1}$ has this exact value in all these cases. The point is that in order to account for the facts we are not obliged to assume that it is ever greater than 0.09, while we are forced to conclude that it is often very much less. This is exactly what the fission theory demands, and so far it accounts completely for the facts.

For the stars of group *D* the proportion of large values of $\frac{s_2}{s_1}$ is far too great to be explained in this way. In fact, they fall sharply into two groups, with this ratio less than 0.15 or greater than 0.40.[1] The former can easily be accounted for on the fission theory, but the latter cannot. It would seem that at about this limit of distance we begin to come upon systems of different origin—perhaps evolved from separate nuclei in the original nebula, as suggested by Moulton.

These wide pairs, however, with a separation of many thousand astronomical units, showing as yet no sign of relative motion, whose periods, if they are really in orbital motion, must be counted by hundreds of thousands of years, are very far from what are usually called binary stars. It is probable that if we could extend our survey to systems of still greater linear extent we would find them grading into the irregular star-clusters, like the *Pleiades*, whose members have a common proper motion.

The average separation of the 50 pairs (close and wide) which show sensible relative motion is but 20 years' P. M., and the greatest among them 71 years' P. M. Of the 16 pairs which are moving at the rate of 1° per year or more, the greatest separation is 27 years', and the average only 6 years' P. M. These systems therefore lie far within

[1] The familiar "Trapezium," θ *Orionis*, which was not included in the above discussion on account of the smallness and uncertainty of its P. M., would fall in this latter group, and increase the force of the argument.

the limits of distance up to which the fission theory accounts for the phenomena.

11. When both the wide and close pairs are in motion, a further test of the theory is possible, for it demands that both pairs shall revolve in the same direction—though not necessarily in the same plane. In seven of the eight cases in our list the apparent motions are in the same direction. In one, ξ *Scorpii*, they are different; but a single case of this sort is not surprising, for occasionally the planes of the two orbits will pass on opposite sides of the sun, so that, as projected on the sky, they are apparently described in opposite directions.

The numerous cases in which one or both components of a visual binary are spectroscopic binaries (usually of short period) are also in agreement with the fission theory—representing cases where tidal friction has considerably retarded the rotation of the separated mass before its second division.

The few examples of stars spectroscopically triple (*Polaris*, *Algol*) show a short period superposed on a very much longer one, and we meet again the close and wide pairs of the theory.

In the entire range of triple systems which are accessible to observation, we therefore find everything in harmony with the fission theory, up to a distance exceeding more than tenfold that of the widest pairs which so far show signs of relative motion.

It may still be questioned whether the examples which have been discussed are sufficiently numerous and typical to be representative of binary stars as a whole. In answer to this it may be said that there are 34 binary stars whose orbits, according to Burnham, may be regarded as fairly well determined. Eight of these are visual triples, which appear in the preceding table, and the bright components of two more are spectroscopic binaries, so that 30 per cent. of the whole give direct evidence favorable to the fission hypothesis. It is probable, in view of the limitations of our observing powers, that the actual percentage of triple systems is considerably greater. They are certainly enough anyhow to give a good sample of the whole. The singleness of the components of the remaining binaries in no way discredits the theory, but, according to it, is a result of relatively great tidal action.

12. As the fission theory accounts so well for the observed facts, it is worth while to consider the objections to it in some detail.

a) As regards the stability of such dividing masses as are here postulated, practically nothing is known theoretically, owing to the great difficulty of investigation. The facts already detailed establish a presumption that when the subject is mathematically explored, some series of figures of equilibrium of a *compressible* gas, ending in fission into two comparable masses, will be found to be stable. The continued existence of many variable stars, such as β *Lyrae*, whose light-changes can be best accounted for on the hypothesis that they are composed of two ellipsoidal masses of very small density revolving practically in contact, raises this presumption to a very high degree of probability; and it may well be accepted, on these physical grounds, unless direct mathematical evidence is produced to the contrary.

b) Professor Chamberlin's theory of the escape of gas, molecule by molecule, from the equatorial region of a rotating spheroid presupposes that the velocity of the gaseous molecules in question, relatively to the neighboring gas as a whole, is greater than the difference between the rotational velocity of the surface and the orbital velocity of a particle revolving in a circle under gravitation just outside. His conclusion is that "the critical stage of exact balance between the centrifugal and centripetal factors of the spheroid is never reached. If so, bodily separation is excluded by the conditions of the case."[1]

Such loss of gas may of course occur; but its amount, which will depend upon the temperature and surface-gravity of the mass, can hardly be predicted. It would have to be very considerable to prevent the arrival of the mass at the critical stages preceding fission, which consist in deep-seated changes in the distribution of its matter, which come into play long before the centrifugal force at any point on the surface equals the attraction, and while the difference between the rotational velocity of the surface and the orbital velocity of a particle just outside is a considerable fraction of the latter.

The lightest gases, hydrogen and helium, would in any case be lost first. But in stars such as β *Lyrae*, which are apparently almost in the act of fission, these are just the gases which are spectroscopically most prominent. In general, the extreme rarity of stellar spectra

[1] "The Tidal Problem," *Carnegie Institution, Publication 107*, p. 167.

which lack the lines of hydrogen seems conclusive evidence that this process is of little importance in stellar evolution. A star whose hydrogen had escaped into a sort of ring of loose molecules surrounding its equator would show the hydrogen lines if viewed from a point near its equatorial plane, but not from the direction of its pole; and numerous cases of this sort ought to occur if the phenomenon was at all common among the stars.

c) Professor Moulton's argument against the fission theory may best be given in his own words.[1]

> At the time of fission all parts are rotating at the same angular rate, and one of the two parts must have a mean density less than, or at most equal to, the mean density of the original mass. Consequently one of the two fragments, because of its lower density and equal rotation, must have at least as great a tendency to fission as that which led to the division of the original mass, unless either its form is one of greater stability, or the tidal forces of the other member of the pair tend to keep it from breaking up. If, as seems probable, the approximate spheroid is the most stable figure of equilibrium, and if the mass under consideration has suffered fission by evolution along this line of figures, as is assumed, then the former alternative is eliminated. It does not seem that the tidal factor can tend towards stability. . . . . We observe next that the binary stars are now actual stars of considerable density. Consequently if they have originated from the fission of nebulas they have undergone enormous contraction. The contraction implies increased rotation which would increase the already dangerous tendency for at least one part to suffer further fission. Tidal friction would offset this tendency by decreasing the rotations, but considering all the factors involved, it is seen that if a fluid mass ever gets into the state where fission occurs, there is at least great danger of its breaking into many pieces.

Before discussing this, it is well to consider for a moment the consequences of loss of stability in such a case. Whenever "exchange of stability" occurs, the result is a *change of form* (such as that from a spheroid of revolution to an ellipsoid, in the case of a homogeneous fluid). The new form at the start differs but infinitesimally from the old, but deviates from it more and more as contraction proceeds. At first it increases in stability, but later it may become unstable, and go over into still another form, and so on. If contraction ceased at any time, the mass would remain permanently of the form which it then possessed.

All through this sequence the succession of figures of equilibrium

[1] "The Tidal Problem," *Carnegie Institution, Publication 107*, pp. 156–157.

is *physically* continuous, though the mathematical expression for them changes abruptly when exchange of stability occurs. It is of course possible that the series might terminate at some point beyond which no stable figures of equilibrium existed; but, as we have seen, the stars themselves furnish evidence against this.

So long as this does not happen, the question of physical importance is not: Is the form approaching instability? i. e., Is a change in the mathematical expression for it imminent? but: How far has it gone in the orderly series of evolution forms which begins with the spheroid and ends with two separate masses?

The increase in the density of the gas while passing through this series of forms may be, and in all probability is, very great, and the time consumed in the process indefinitely long.

The determining factors in the physical problem, where the mass is given, are not density and angular velocity, but density and moment of momentum. Bodies of the same mass, but with different moments of momentum, will reach similar forms at densities which vary inversely as the sixth power of the latter.[1]

Now there are often two figures of equilibrium, of different forms, which have the same mass, density, and angular velocity, but differ considerably in moment of momentum. On this account they cannot represent different stages in the history of the same system; but it is evident that the one with the smaller angular momentum represents a system in an earlier stage of evolution than the other.

For example, in the case of a homogeneous fluid, the ellipsoid of three unequal axes, when just ready to change into the pear-shaped figure, is so much flattened that its longest axis is 2.90 times the shortest one, about which it rotates. But there exists also a spheroid of revolution, highly stable, with the same density and angular velocity, whose equatorial radius is 1.42 times the polar. If the two are of equal mass, the moment of momentum of the first is 1.67 times that of the second. It follows that the spheroid will not assume the form of the ellipsoid until it has contracted to 22 times its original density.

[1] Let M be the moment of momentum, m the mass, and r any linear dimension. Then, for similar forms, $M \propto mr^2\omega$, $\rho \propto \frac{m}{r^3}$, and $\omega^2 \propto \rho$. Hence $M \propto m^{\frac{5}{3}}\rho^{-\frac{1}{6}}$, and $\rho \propto \frac{m^{10}}{M^6}$. (Cf. Moulton, *op. cit.*, p. 148.)

The case of the masses produced by fission is exactly similar. Just before separation the shape of the whole mass is much like an hour-glass. Just afterward the shape of the detached pieces is much like an egg. The density and angular velocity are practically unchanged. Consider *one* of these eggs, and imagine an hour-glass of the same mass and density (at corresponding points). The angular velocities of the two will be the same, but it is obvious, almost intuitively, that the moment of momentum of the hour-glass must be much the greater of the two. The egg then, considered by itself (without thought of its origin or of its twin egg), represents a much earlier stage of evolution than the hour-glass, and will have to contract a great deal before it reaches a similar form and divides into two. The detailed calculations of momentum in paragraph 7 show that its density must increase several hundred fold before this happens.

It therefore appears that the fear that a mass liable to fission would soon break up into many pieces is without foundation.

The final density might in some cases be so great that the gas laws no longer held in their simple form, and the character of the figures of equilibrium was changed. But in the case of all double stars, except spectroscopic binaries of short period, the mean density of the mass at the time of separation must have been much less than that of air under ordinary conditions, so that the effects of departures from the gas laws would probably be insensible.

It may be appropriate to add that the argument here discussed has no bearing upon the main thesis of Professor Moulton's paper in which it appears, and that the equations of paragraph 5, if applied to the earth and moon, may be used to illustrate the difficulties which attend the hypothesis of their origin by fission, in very much the way in which he has discussed the problem.

13. The alternative theory—that multiple stars have developed from nebulae which originally had well-defined nuclei, corresponding to the members of the system[1]—must in any case be invoked to account for those wide and irregular groups (such as the Trapezium in *Orion*) for which the fission theory gives no explanation. But the arrangement in close and wide pairs, so characteristic of the large majority of multiple stars, is on this theory a positive difficulty. Not only is

[1] Moulton, *op. cit.*, p. 157.

there no apparent reason for it, but if we try to retrace in imagination the history of such a system, through stages of greater and greater diffusion as we penetrate farther into the past (keeping in mind that the moment of momentum of the whole system must remain constant), it is hard to form any idea of the history of the nuclei which finally form a close and rapidly revolving pair, attended by a distant companion.

In any case, by this theory (in Professor Moulton's words) "we do not explain anything—we only push by an assumption the problem of explaining the binary systems a little farther back into the unknown."

The fission theory, on the other hand, not only accounts for the existing peculiarities of arrangement, but gives a simple and fairly detailed account of the manner of their origin.

While, therefore, it is apparently necessary to assume that star-clusters have developed from originally separate nuclei, it is more reasonable to suppose that binary stars have originated by fission: and this theory may well be adopted as a working hypothesis until some evidence, either observational or theoretical, is produced to oppose it.

Princeton University Observatory
January 26, 1910

OBSERVATIONS OF THE AURORA, MADE AT THE YERKES OBSERVATORY, 1902–1909

By E. E. BARNARD

In the *Astrophysical Journal*, **16**, 135–144, October 1902, I have given my observations of the aurora at the Yerkes Observatory in the years 1897–1902. I have continued these observations, but not with the closeness of the previous records. I have not failed, however, to record every aurora that I have seen.

During this time there have been few striking auroras, with the exception of that of October 30, 1903, which was comparable to some of the brighter ones seen in the early years of the Yerkes Observatory. From newspaper accounts, this aurora was widely extended over this country from New York to Oregon. This was one of the large magnetic disturbances that once in a long while cripple the telegraph systems of the country. A few of the newspaper accounts of this disturbance are worth preserving and I have copied some of them here.

From the Chicago *Daily News*, Saturday, October 31, 1903, 5 P.M. edition:

Peculiar electrical effects were experienced in Chicago today. Frisky waves of electric force zigzagged through the atmosphere and at times caused temporary suspension in the transmission of messages over telegraph wires. . . . .

TELEGRAPH WIRES MADE USELESS

Beginning shortly after midnight the waves scattered all over the country. The Western Union Telegraph Company's western wires, extending as far as Denver, at intervals were rendered useless. Messages in the process of transmission were stopped. Postal Telegraph wires were affected but not disabled. The French Cable Company reported disturbance in its service. . . . .

By noon the effect in Chicago had subsided and all telegraph wires were nearly normal. Ten years ago was the last time any similar disturbance was experienced here, according to L. K. Whitcomb, chief operator of the Western Union Telegraph Company. . . . .

The French Cable Company, according to an Associated Press dispatch from New York, gave notice early that owing to extraordinary electrical disturbances it was informed by the European administration that business will be subject to heavy delay.

AURORA BOREALIS IN NEW YORK

New York had an aurora borealis display early today, says another Associated Press dispatch. It interfered with telegraphic service. Both telegraphic companies reported wire trouble on account of the electrical display, and all cablegrams were accepted subject to heavy delay. This was the first [?] display of the aurora in New York for ten years and it lasted several hours.

NORTHERN LIGHTS IN MINNESOTA

From Duluth, Minn., comes a report through the Associated Press that the northern lights illuminated the heavens for half an hour about midnight. The display was pronounced by several to be the most beautiful ever seen in Duluth. It took the form of huge, waving plumes, blown by the wind, the tips extending directly overhead.

Another paper, elsewhere, reports the following:

Salt Lake (Utah), October 31.—A remarkable display of the aurora borealis was visible here early this morning, continuing for several hours. So intense was the light that many were of the belief that a large fire was raging north of the city. Telegraph service throughout the Northwest was badly crippled for some time.

Seattle (Wash.), October 31.—The display of aurora borealis probably reached its climax as viewed in the Puget Sound country. The display lasted over two hours. The city was illuminated as if by moonlight. The rays met in a focus in the zenith. The coloring was mostly brilliant emerald and blood red. The effect was startling.

Portland (Ore.), October 31.—The aurora borealis was observed here during the early part of last evening for the first time in about ten years. A dense fog settled down over the city soon after dark and brought the display to an early termination. The peculiar electrical conditions greatly hampered the telegraph companies in the transmission of messages.

A striking feature of this aurora, as seen at the Yerkes Observatory, was the rapidly ascending waves of light that succeeded each other with a startling rapidity. They had their origin apparently in the upper part of the bright arch, as if thin rims of the arch were expelled upward. This spasm occurred at intervals, and would last for a minute or so, and then cease for a short time, as if to renew its energies for another display. These waves shot up to beyond the zenith with very great velocity, succeeding each other at intervals of about a third or a fourth of a second.

Some of the phenomena of the previous auroras have not been repeated; perhaps from the fact that in general these later auroras

have been of a milder type. And yet this period has included a sun-spot maximum. In these last observations I have not again seen the bluish-green masses which moved along the summit of the arch toward the east and which were such striking features of the aurora of February 11, 1899.

One fact that is evident in the observations made here is that the auroral arch varies greatly in size. For if we assume that it is the segment of a circle whose center is essentially stationary with respect to the earth it must fluctuate as much as a diameter of itself in size. The arch sometimes extends half-way to the celestial pole, and at other times its entire extent is beneath the horizon, as indicated by the streamers coming up from below the horizon.

Perhaps with the exception of the narrow ray—a degree in width and eighty degrees or more in length—which was seen rising from the west horizon on August 21, 1903, no new phenomena have been seen.

The streamers which spring from the arch as a base, and which always have a decided lateral motion and last for a minute or so only, almost always move to the west. On several occasions, however, I have seen them divide the arch, with respect to their motion, so that the ones to the west moved west and those to the east moved east. This is very rare. The motion is about 2° in one minute (and not 2 minutes to the degree as I stated in *Astrophysical Journal*, **16**, 143). I have wished to determine this motion more accurately, but we have had so few ray-producing auroras in late years, and the rays are so transient, that I have not been able to do so. It would be interesting to know if this motion is constant in a streamer and for all streamers.

The pulsating bright masses that usually appear in the northeast or northwest, but which are sometimes seen under the pole, are among the most interesting phenomena. They are sometimes present when there are no other evidences of an aurora.

In the auroras seen here there is comparatively little in the way of color. Referring again to the great aurora of April 16, 1882, which I saw at Nashville, Tenn., there were magnificent brilliant crimson curtains in the west, and brilliant colors in other parts of the sky. But as seen from the Yerkes Observatory in the past ten

or twelve years, there have not appeared any such striking colors. [This was written before the great aurora of October 18, 1909.]

In some of the auroras seen here, there was a double arch—one several degrees higher than the other.

Extracts from my notes now follow, in continuation of those in the paper already referred to. Time of the ninetieth meridian ($6^h\ 0^m$ slow of Greenwich mean time) is used throughout.

1902

November 23. $6^h\ 30^m$: Faint auroral light in northern horizon. $7^h\ 20^m$: Strong auroral arch, very low. Top of brightness half-way to *β Ursae Majoris*. Dark under arch. No decided streamers, but bases of streamers moving to left. $8^h\ 0^m$: The aurora had almost died out. There was still a feeble arch. $13^h\ 0^m$: There was still a feeble glow.

November 24. $11^h\ 40^m$: At this time there were auroral streamers shooting above the northern horizon. The arch seemed to be below the horizon. For several hours there had been a feeble glow but no streamers or arch. $11^h\ 30^m$: The aurora seemed to have all died out.

1903

August 21. $9^h\ 0^m$: A rather strong auroral glow in the north. $10^h\ 30^m$: The aurora was active. There was a poorly defined arch very low; estimated 5° high, with frequent streamers moving to the left. Specially active under the handle of the Great Dipper. It was so bright that it strongly illuminated a few clouds overhead. $10^h\ 50^m$: There was a strongly defined streamer near *Arcturus* in the west. $11^h\ 35^m$: The aurora had died down. For the past hour it had been quite active, and for the past half-hour there had been rapid fluctuations ascending vertically. This activity gradually passed to the right. The sharply defined streamer in the west remained stationary for a quarter of an hour. A second one a little to the south of *Arcturus* sprang up and was inclined 20° from the vertical toward the south. It was faint at first, then became brighter—very thin, and rose to the altitude of *Vega* and a little to the left of that star. It was most remarkable, not being over 1° in width. It stretched up for 80° or so, and gradually became curved—convex to the south or left and then broadened out and finally faded from view. $11^h\ 45^m$: The aurora had quieted down. There was considerable light in the northeast and a great amount of fluctuating light, especially in the northeast. The streamers to the left of the pole moved to the left. The arch was not definite. The display extended over a long reach of the horizon. The greatest activity seemed to be near 11^h. The light was much broken by luminous regions. Mr. Sullivan says it started at dark. $12^h\ 30^m$: There was a strong auroral light all over the north but it was not active. It was active again with streamers at about 14^h. This was the first aurora I had seen for a large part of a year.

September 18. $14^h 45^m$: There was a low aurora with an ill-defined arch—dark part 4° high, and a brighter rim 2° wide above this. It got a little brighter but by 16^h was very much faded out. No activity.

September 19. $7^h 0^m$: There was an ill-defined arch 6° high with another arch of light above, but not bright. By $8^h 0^m$ it had brightened considerably. $9^h 0^m$: The dark arch was 6° high with its center 10°–15° to east of the north point. It was fairly well defined. The light was very much stronger but did not extend far above the dark arch. At $9^h 30^m$ there were a few feeble streamers, after which the arch broke up and disappeared. By $10^h 0^m$ it had all gone except a feeble suggestion of a diffused glow in the north—no arch.

September 23. At a little before 16^h there was a very feeble auroral glow low in the north.

September 28. A slight aurora about midnight.

October 30. At $15^h 0^m$ a magnificent aurora was in progress. Mr. Sullivan says that there had been an arch, not conspicuous, from early in the night and that it had culminated in a great display at $15^h 0^m$. I first saw it at $15^h 30^m$. The whole north was full of great streamers and rapid pulsations were fluttering all over the sky. The greater part of the heavens—especially north—was full of hazy light, and masses of streaky brilliancy were conspicuous at different points. These streaks ran toward the north. The streamers seemed to converge toward the zenith where frequently masses of curved and irregular light appeared, fluctuating brightly and then drifting to the south. The fluctuations extended down to and below *Orion*. The streamers were broad—a few brilliant ones were narrow. They did not make much inclination to the horizon except to the northeast and northwest, and at their upper parts they seemed to diffuse toward the zenith, as if struck by a wind. For a certain height the streamers were straight, then they seemed to lose their force and become subject to drifting and irregularity. The ascending pulsations came from the north—not in arch form exactly. They would go to the zenith and beyond with extreme rapidity. There were several waves a second, but the irregular pulsations would last only $\frac{1}{10}$ second. There was frequently a good deal of red and a tendency of the streamers to form curtains. But there was no definite arch—it was diffused and extended, 10° or 15° high. The sky, except overhead and north, was more or less covered with smeary clouds, especially from *Orion* south. Though these were illuminated when the light was strongest they did not show the effect of the pulsations which seemed to pass by them in waves, showing that the pulsations were beyond them. There were a few streaky clouds in the north below the pole. I could not be positive that there was any of the auroral light this side of them. Several bright streamers were occulted by these clouds and everything seemed to be beyond the clouds. The activity extended nearly from east to west, through the north. At any one time the activity would be confined to either the northwest or northeast.

The streamers to the west of the pole moved west about 2° a minute; those to the east moved to the east. I watched this carefully to see where the line of demarkation occurred. Usually here the summit of the arch is from 15° to 20°

east of north, and one would suppose that there would be a stationary point for these streamers at the summit of the arch. Though there was no definite arch to go by I noticed that some of the streamers west of where the summit of the arch ought to be moved east as shown by the notes.

$15^h\ 30^m$: The whole north luminous with streaky streamers; patches of streaky light overhead and north. Some of these appeared very much like the streakiness of the *Merope* nebula in photographs of the *Pleiades*. Rapid pulsations everywhere. $15^h\ 50^m$: Very active. Red in north. There was no distinct arch. $16^h\ 5^m$: Red in northeast. $16^h\ 10^m$: Dying down. Most active in northwest. $16^h\ 25^m$: Little activity at this time—quieting down. $16^h\ 35^m$: Streaky streamers in the north. Not much activity; no fluctuations. $16^h\ 40^m$: A fine streamer appeared 5° west of β *Ursae Minoris*, and moved toward β and to the east. $16^h\ 43^m$: A streamer shot up through *Polaris* and moved to the east. Aurora active, with pulsations. Fine streaky streamers all over the northeast. $16^h\ 50^m$: Active, with big broad streamers all east of north—none to the west. No pulsations, but active pulsations a few minutes later. $16^h\ 55^m$: Pretty nearly died out. $17^h\ 10^m$: All out at this time except some luminous appearance in the north.

The pulsation periods, or periods of activity, were spasmodic—that is, every once in a while the ascending pulsations would be at work and then quit. This was the finest display I have seen since September 9, 1898. At the brightest the light did not cast a shadow.

October 31. $8^h\ 30^m$: There was an arch visible in the north reaching one-third of the way to *Polaris*, not very definite on account of moonlight. It was dusky beneath and the summit was 15°± east of north. Some streamers shot up at different points on the arch, but a hazy sky and nearly full moon prevented any display to amount to anything. By $9^h\ 0^m$ the activity had ceased, but the arch remained feebly. This soon disappeared and did not return during the night. $11^h\ 15^m$: No trace of aurora. $14^h\ 30^m$: Thick sky with few stars. No aurora. $17^h\ 0^m$: No trace of aurora. Sky misty but more or less clear.

November 1. No aurora.

November 17. $9^h\ 0^m$: A slight aurora low in the north. There were a few streamers, but there did not seem to be any arch and the streamers appeared to come from the horizon. The aurora seemed to die away soon. An hour later there was none. Frequent looks after midnight did not show any further aurora in the north. My view was confined to the immediate north. $17^h\ 30^m$: There was no aurora in the north but there was a large curved half-lune of pulsating light resting on the north by east horizon and curving up through *Cassiopeia* to 5° or 10° under the pole. This slowly fluctuated in brightness and was tinged with warm color. Temperature +10° F.

November 18. $6^h\ 0^m$: A slight aurora in the north. This rapidly brightened and formed an arch, not very dark beneath. A half-hour later there were feeble efforts at streamers. There were two clouds of light at about 25° altitude, one in the northwest, the other in the northeast. They would brighten up, then fade

away. The one in the northwest was the most persistent. These faded away soon but a fragment of an arch formed half-way from the true arch to the pole, under *Polaris*, then it died away. At $7^h\ 0^m$ there was an auroral glow but not active in the north. Temperature $+14°$. $7^h\ 30^m$: There was a rather dull aurora with low arch, but no streamers. $12^h\ 0^m$: There was only a glow in the north. Nothing in the east or west. $17^h\ 0^m$: There seemed to be no aurora.

November 19. Mr. Sullivan reported that there was a strong arch at 8^h to 9^h, brightest at 9^h, after which it disappeared. There were no streamers.

December 29. $15^h\ 30^m$: A strong aurora in the north. It was badly broken by clouds, so one could not make much out of the arch. Feeble effort at streamers.

1904

April 18. $8^h\ 40^m$: A strong auroral arch in the north. The aurora was strongest about midnight. It seemed to disappear about $15^h\ 30^m$ just before dawn. It did not become active, except that a few feeble streamers were attempted after midnight. The arch was a low one—not among the lowest, however. The summit was some 20° to 30° to the right of the pole.

April 19 (R). Aurora from 7 P.M.

June 15. $9^h\ 0^m$: A strong auroral glow in the north. By 10^h and 11^h it was very strong with very low arch. There was some activity—efforts at streamers. At $14^h\ 0^m$ there were rapidly appearing and disappearing long luminous spots to the west of the pole at 10° or 15° altitude.

June 16. In the first part of the night there was no aurora, but there was possibly a slight auroral glow about 13^h.

October 5. $14^h\ 0^m$: A faint auroral light low on the northern and northeast horizon. It had been visible for half an hour or more. No arch. Not anything seen of it in early part of night.

October 6. $8^h\ 30^m$. A feeble auroral glow at northern horizon. $9^h\ 0^m$: The arch was forming. $9^h\ 27^m$: Arch active. Short and rapidly changing streamers. $10^h\ 10^m$: The arch was very low. It was not strong and there was almost no activity. $10^h\ 25^m$: The arch had risen decidedly, was better defined, and was brightening rapidly. There was a second arch above it, but not so well defined. $10^h\ 30^m$: The upper arch was very bright also but not active. $10^h\ 35^m$: The second arch had almost disappeared; the main one was fainter.

When clouded at $13^h\ 30^m$ the arch was very bright but not active. The aurora seemed most active from about $9^h\ 30^m$ to $10^h\ 0^m$. There was quite an effort at streamers, some of them were distinct and long, and those under the pole moved to the left.

October 11. $11^h\ 0^m$: A slight auroral glow a little to the east of the northern horizon. $13^h\ 0^m$: It had almost entirely died away.

November 3. $14^h\ 30^m$: An auroral glow, not seen at any time earlier tonight. This then formed into an irregular brightening along the northern horizon, but no distinct arch. When I quit after moonrise, at $15^h\ 30^m$, it was quite brightish, but without activity.

November 5. $11^h\ 0^m$: A decided auroral glow at the northern horizon, extending 15° or 20° high. Unmistakable. No arch.

November 15. After moonset, at midnight, there was a strong aurora. No definite arch but dark underneath and very low. $13^h\ 0^m$: The arch was very strongly marked and dark beneath. It was very low. The bright part was not broad and was quite bright. No action. $13^h\ 25^m$: The arch was much broken up into bright masses. $13^h\ 50^m$: There was a thin streaky second arch above but no streamers. $14^h\ 35^m$: Some effort at streamers. $14^h\ 50^m$: It seemed to have died out, and dusky, hazy patches were over the brightest part which was all broken and smeary. $16^h\ 42^m$: It still feebly existed. $17^h\ 10^m$: It had disappeared.

1905

February 1 (R). Aurora from $9^h\ 30^m$ to $10^h\ 30^m$.

February 2 (R). Bright aurora all night.

March 1 (R). Auroral arch from $12^h\ 30^m$ to 15^h. No streamers were seen.

March 3 (R). Faint aurora from $10^h\ 30^m$ to 12^h.

September 25 (R). Aurora nearly all night after 10 P.M.

November 26. $8^h\ 35^m$: There seemed to be a feeble aurora near the northern horizon.

The observer was absent at Mount Wilson, in California, from January 1 until the middle of October. The observations marked (R) are from the meteorological records of the Yerkes Observatory.

A few other records so marked (R) were either missed by the observer or made in his absence.

1906

February 18. Heavy aurora at midnight. Did not notice any streamers. Very bright glow to nearly an arch. Did not look out earlier, and did not look again until $16^h\ 0^m$; then it was heavily clouded.

Mr. Stillhamer says there were two peculiar straight vertical rays, one east of north, and the other west of north. They were not visible at the same time.

February 25. $8^h\ 0^m$: Strong but low auroral glow along northern horizon. $13^h\ 0^m$: It was very much stronger and much broken as if it had been active. Some activity with attempt at streamers. $14^h\ 0^m$: About the same. $15^h\ 0^m$: The aurora had died down to a distinct, but not strong arch about 4° or 5° high.

March 24. $9^h\ 0^m$: A strong aurora with one or two streamers. By midnight it had died down to a faint glow and by 15^h it had disappeared.

May 18. $8^h\ 45^m$: A strong aurora started. No definite arch but strong light with ill-defined, uncertain dark arch beneath. $12^h\ 0^m$: It was very strong and a strong arch had formed. An hour before it was almost dead. There were no streamers. Mr. Sullivan saw several streamers in first part of the night. The aurora remained strong until daylight, but no streamers.

May 19 (R). Aurora 9^h to 13^h.

June 1. 13^h: A strong aurora, without any well-defined arch. There were some streamers.

August 12. $9^h\ 40^m$: There was an active aurora; broad sheets of light shooting up. A few minutes later it had died out except for an auroral glow which remained unchanged as late as $11^h\ 0^m$. An hour before $9^h\ 40^m$ there were no signs of an aurora.

September 22. $12^h\ 0^m$: A faint aurora in the north. Mr. Sullivan says that in the first part of the night it formed an arch but no streamers.

September 23 (R). Aurora.

September 24. $10^h\ 30^m$: There were fluctuating spots of light above the northern horizon. Later a strong light above a dark region. This died out and lighted up again farther to the east, where it was very strong in the northeast but not under the pole. It was a very strong light all along above a bank of what appeared to be dark clouds, but a star was seen through it. $13^h\ 30^m$: A feeble scattered glow. At $14^h\ 0^m$ the aurora was showing spasms of renewal—bright spots appearing and disappearing rapidly under the pole and to the east. The aurora was very fitful.

October 13. $7^h\ 5^m$: A very feeble auroral glow east of the north point. $16^h\ 50^m$: It died out soon, no increase of its light—simply a feeble glow.

1907

February 7. 8^h or 9^h: There was a strong aurora—very dark below. I did not see any streamers. Could not watch it because I was photographing. At midnight it seemed to have died out.

February 9. Sky cleared suddenly just before 7^h. The whole northern heavens from the east point to the west and to the zenith was covered with a very bright aurora—just a uniform light. In a few minutes this became very active. A very large arch half-way to the pole formed, then broke up into cloud masses which extended all over the north. Bright masses and great streamers all over the north. Then a very low arch formed, quite definite, dark below. Then this faded out and a somewhat larger arch formed, dusky beneath. Then later this disappeared and there was only a uniform glow over the northern horizon. At midnight it was very dim and quiet. Cloudy from north with haze all over sky. This aurora was the most active in several years but it did not compare with the magnificent one of October 30, 1903.

February 10. 9^h: A spot of auroral light 6° to 8° in diameter in the northwest, some 15° high. This would become very bright and then in a few seconds disappear entirely. $15^h\ 0^m$: A bright mass of fluctuating light 6° to 8° in diameter was visible in the low west. *Jupiter* was exactly half-way to the horizon from it and directly under it. The light was at 6° or 8° altitude. It became intensely bright once in a while, of a bluish-green color. Under the pole near the horizon there was frequently a long spot of light (not bright) that came and went.

February 11. An aurora was observed here through the clouds at 10^h, 13^h, and 17^h.

February 12. $14^h 30^m$: A pretty large spot of light in the west by north, very near the horizon, that flared up very bright every few seconds, then faded out again, and a faint auroral glow near the northern horizon which when brightest showed a low arch, dark beneath. $16^h 15^m$: A feeble, very low, long spot due north that pulsated every few seconds. I did not again see the spot near the northwest horizon.

February 13. Professor Frost reported that at different times in the night he saw spots of auroral light shining through the clouds—not through breaks.

February 14. At $7^h 15^m$ in moonlight (moon very thin) there were several streamers in the north and northwest. These seemed to come from the horizon, there being no arch. I saw no more streamers, but later there was a decided auroral glow in the north. This was unmistakable as late as $13^h 40^m$.

February 16. From $10^h 55^m$ to $12^h 20^m$ an auroral glow was visible in the north.

February 17. A feeble auroral glow in the north during the night.

February 19. $8^h 30^m$: There seemed to be a faint aurora before it got too cloudy. Clouded at $10^h 37^m$ and remained so.

February 20. $13^h 30^m$: Before moonset there was a low glow near the northern horizon. At $15^h 0^m$ there was a brilliant cloud close to the eastern horizon. Sighting along the Bruce Observatory (running east and west) it was seen that this cloud was perhaps $1°$ north of the east point—almost exactly at the east point and only $2°$ or so high. It was $5°$ or so in diameter. This died out a few minutes later and did not reappear, but there were several thin bright strips just above the northern horizon that came and went. At $16^h 20^m$ there was a long, brilliant cloud in the northwest, low and slanting up at its north end. Its south end was near the horizon under *Castor* and *Pollux*, while its north end was above *Capella*. The aurora was then dead under the pole. This cloud died out completely by $16^h 35^m$. At $17^h 0^m$ there was no trace of aurora anywhere. $17^h 30^m$: Still no traces of the aurora.

February 21. Between 15^h and 16^h there was a display of long, bright masses under the pole and near the horizon. These would brighten up and then fade out. They appeared like horizontal bright clouds. I did not see anything of bright clouds in the east or west but I only glanced out once in a while, so they may have been there in the intervals. At $16^h 45^m$ it seemed dead under the pole.

March 5. It was reported that there was an aurora last night—that is, evidences of aurora.

March 10. A strong aurora at $7^h 30^m$. At $8^h 10^m$ it was sending up a number of streamers. It was more or less active till $9^h 30^m$, when it seemed to have died out. There was no distinct arch, unless it was hidden by some low clouds. At $10^h 30^m$ there was perhaps a feeble glow, but the aurora was dead. The arch was again strong at about 12^h. At 15^h–16^h the aurora was again evident as a glow.

March 19. $13^h 55^m$: A very strong aurora beneath some clouds in the north. It was all along the northern horizon, the lower part of the arch evidently

being below the horizon. No streamers. Later: The aurora seemed to have the summit of the dark part of the arch just at the horizon—that is, it could be seen not exceeding 1° high and for only a short arc.

April 5. $7^h 30^m$: A dull auroral arch in the north, which soon died out without becoming active. At $14^h 15^m$ there was no trace of aurora, but at $14^h 30^m$ there was a great, long, brilliant auroral cloud, say 60° long and averaging 10° broad, slanting a little to the west. It was one-third of the way up to the pole. This was very bright and somewhat irregular, smaller at the ends. The stars covered were seen through it. This rose slowly, and at about $14^h 50^m$ was half-way to the pole. A few minutes later it had disappeared, all but a faint smudge. The north end of the great cloud ended among the bright stars of *Cassiopeia*. There was no question as to its auroral nature. It was very bright, pearly white, even in moonlight. There were no other evidences of an aurora anywhere.

April 13. $13^h 20^m$: A strong auroral glow in the north. $14^h 0^m$: There were a few streamers to the west. No arch (arch below horizon?). Reported to be brighter just before dawn with distinct arch and a few streamers.

May 12. $10^h 5^m$: A strong auroral glow with feeble arch in the north. $13^h 20^m$: The aurora was very much brighter. The arch was at the horizon. It was shooting up streamers to the west. $13^h 30^m$: It had almost died out. There was only a widely diffused glow.

August 20. Bright arch and sheets of streamers. $14^h 55^m$: The sheets of streamers were in the northeast, at the northeast extremity of the arch. Arch very low and broad. The aurora died out soon and was faint before dawn.

September 11. 11^h: A strong auroral light in the north. $11^h 30^m$: An occasional streamer, but no arch. By 13^h it was very strong with very low arch and no streamers, but there were long sheets of pulsating light just above the arch. These would flare up for a few seconds and then entirely disappear. Toward dawn it seemed, if anything, to be brighter. The pulsations continued. Could only get out to see it once in a while, as I was photographing. It was reported to have been very active about 9^h with streamers, etc.

October 2. At $8^h 20^m$ saw a number of streamers shooting up from the northern horizon. Looked out again at $8^h 50^m$ and they were gone. Did not see any further evidences of aurora as late as when it clouded over at 14^h.

October 10. There seemed to be a very faint auroral glow at $11^h 0^m$. Mr. Sullivan also thought there was one.

October 13. $11^h 0^m$: A bright, active aurora, with arch almost on the horizon. It was shooting up streamers all along the arch. $12^h 30^m$: It was dying down. $13^h 0^m$: There was only a feeble glow in northern horizon. $16^h 30^m$: There was still a feeble auroral glow.

November 23 (R). $6^h 15^m$: Bright below. 10° wide.

December 10. $11^h 0^m$: A clear space in the north showed a feeble aurora was visible. This got higher until a very low uncertain arch was formed. This clear space remained for a long while. The aurora was brighter at midnight but

not active. By $13^h\ 0^m$ it had faded almost out. At $17^h\ 30^m$ it had disappeared entirely.

1908

January 8 (R). Aurora all night.

January 28 (R). Auroral arch.

March 20. $7^h\ 50^m$: A strong auroral glow in north and northeast. It was still visible at moonrise.

March 29. $11^h\ 15^m$: There were two pulsating masses at an altitude of 17°, one west, and the other east, of north. The west one was the bigger. It was in the bright stars of *Cassiopeia* and was 20° long by 5° broad. Its east end was elevated 10°. The west end was at the double cluster of *Perseus*. It became very bright and then faded out to almost invisibility. The fluctuations were irregular—sometimes the intervals were 5 or 6 seconds and at others 15 or 20 seconds. The east one was narrow and long and roughly parallel with the horizon. It was bright at times but later almost faded out. No other evidence of aurora. $11^h\ 30^m$: The pulsating clouds seemed to have disappeared. There were several faint streamers rising from the northern horizon as if the arch was below the horizon. $12^h\ 15^m$: All evidence of the aurora was gone. $13^h\ 30^m$: There was a pulsating cloud to the left of *Capella* about 5° above the northwest horizon. It would become very bright and then almost fade out. Mr. Sullivan saw it 15^m before and it was then very much nearer *Capella* and higher. It must have been moving west by south. $14^h\ 15^m$: There was no trace of aurora or pulsating clouds. Looking out several times after this before daylight there were no more evidences of aurora.

April 5. $15^h\ 40^m$: A low arch—only 3° or 4° high—with dark beneath, nearly due north. It was not bright.

May 1. $9^h\ 30^m$: A strong aurora, but no arch or streamers. Widespread auroral light in the north.

May 5. $12^h\ 0^m$: Through clouds a strong auroral light was seen in the north.

May 22. $9^h\ 15^m$: Quite a strong, but low, arch reaching half-way to *Cassiopeia*. $13^h\ 15^m$: The aurora was still bright but the sky was very thick. It died out about $14^h\ 0^m$.

May 23 (R). Aurora from $10^h\ 30^m$ to 13^h.

May 25. The display on this date was one of the most extraordinary that I have seen. The coincidence with a great electrical storm at this time was very singular, though it is probable that the two were not connected. At dark there were great banks of cumulus clouds in the west and northwest. These moved north and were alive with lightning—a most impressive sight.

A few minutes before $9^h\ 0^m$ a great number of narrow, but short, brightish strips like shreds of clouds appeared overhead. They formed a band across the sky from the north of west to the south of east through the zenith. The individual strips were moving west with great rapidity like low clouds driven by a heavy wind and appeared to be only a few hundred feet distant. From their luminosity it was at once evident that they were of an auroral nature. At this time there were

several great streamers in the north, but the northern horizon was covered with clouds. It was therefore not certain that any arch existed. In some ten or fifteen minutes the strips overhead, which were inclined sharply to the general direction of the band and which were $\frac{1}{2}°$ wide and $10° \pm$ long and had been moving westerly, had now blended into a beautiful, soft, but bright, band of cometary-looking light. This band was 10° wide and had its south edge in the zenith. It extended from the east by south horizon (in clouds) to the west by north where it disappeared in the great storm clouds. The sky was beautifully clear elsewhere. The band, though bright, was perfectly transparent to the light of the stars. At its ends it was apparently connected with two narrower and sharper and brighter cometary-looking strips that were perhaps connected with the horizon. The appearance was as if a great scroll had been flung across the sky and the end sticks to which it was attached had been dropped at an angle to the horizon. These end strips were at an angle of some 20° to the vertical, inclining to the south. At $9^h 20^m$, between the great arch and the pole, another band of short strips some 3° apart appeared. Near the meridian these strips pointed to the pole, while away from the meridian they pointed to the northwest. They apparently had no decided motion. These soon disappeared. The great band remained bright and uniform. It, however, was drifting very slowly to the south, and at $9^h 30^m$ it passed through the zenith. Then a parallel strip formed beside it and a few degrees north, which broke into short narrow strips pointing (overhead) toward the pole, but, beyond the zenith, to the northwest. These strips were longer and narrower away from overhead. They were not so well developed in the east. There was little or no motion in these. They fluctuated rather slowly in light and presently disappeared. Shortly after 10^h the great band broke up into short strips—at a sharp angle to its length—which had a rapid westward motion. Finally the band faded out overhead, the two ends becoming thus disconnected. The upper end of the west section drifted south so hat the two would not connect if continued. This kept very bright and became narrow, while the east portion split laterally into several parts. These shattered nto short strips 10° or 15° long which had a quick, short motion northerly. There were apparently other strips nearer to us that moved rapidly over the first ones. Their visible existence seemed to extend over about 10° of motion. About 10^h this easterly portion drifted overhead in a fragmentary form—frequently in this appeared the moving forms. At $10^h 15^m$ or $10^h 20^m$, at the time the masses of light in the zenith were fading out, the sky was blotted out by the storm clouds.

During all this time all over the west and northwest there was a magnificent and rapid electrical display in the clouds—so bright that it frequently blotted out everything else. Just before the storm closed in, a great, black, broad path of clouds came rapidly from the storm clouds in the west. This was perfectly straight at the edges and stretched from the northern to the southern horizons and was uniformly 15° or 20° broad with clear sky on each side. It swept overhead very rapidly—black, opaque, and sharply defined. As it passed I could see a few irregularities in it. These were moving very rapidly along its length to the

north, while the whole extent of the band itself moved rapidly a little to the north of east. When finally low in the east this object looked like an irregular strip of cloud. It was first seen near the west horizon under the great storm clouds and was then very black and sharply defined. There was no lightning in this band. I doubt if it had anything to do with the aurora, but its singular appearance and motion at this time were strange enough for record.

At $9^h 15^m$ the great zone of light stretched directly through the zenith. At the east it would strike the horizon 10° south of east. In the west it passed 10° north of *Venus*. Roughly it was perpendicular to the magnetic meridian. It was nearly 10° broad—bright and transparent. Its light strongly reminded one of that of a comet.

There were some newspaper accounts of this singular phenomenon, which seemed to have been seen at various other places. In general these accounts were of no value in locating the position in the sky. One newspaper had a long editorial on the subject and associated it with the zodiacal light, giving a lengthy history of the latter object. Of course there was no connection between the two phenomena.

According to Dr. Walter L. Rankin, of Carroll College, Waukesha, Wis., who observed the band, its summit was at an altitude of 70° and south of the zenith. Waukesha is 29 miles north and 13 miles east of the Yerkes Observatory. Taking into account the inclination of the band to the east-and-west line, his observation would make it about 140 miles high.

A description in the *Northfield* (Minn.) *News* says: "The peculiar aurora was in the shape of a long search light, like a path running east and west, from 7° to 10° wide, and at an elevation of 70° above the southern horizon." This would place it some 4 or 5 times as high as Mr. Rankin's observations. It seems quite probable that it was, in its later stages, several hundred miles above the earth's surface. The times were not given in any of the observations, and as it had a motion, southerly, of over 5° while under observation here, it is probable that the results given for its height are not very trustworthy.

I have thought that the remarkable nature of this display would warrant a full description, as some of the features were to me unique.

In *Science* for July 10, 1908 (**28**, 51), an account is given of what was apparently a similar display of the aurora on March 27, 1908, by Wilmot E. Ellis, of Fort Terry, N. Y. His confusion of this phenomenon with the zodiacal light is unwarranted. The zodiacal light never exhibits any such phenomena as the above. Indeed it does not show any marked changes which cannot be readily accounted for by the position of the observer or the atmospheric conditions at the time. The phenomenon of March 27 was purely auroral.

The night of March 27 was cloudy here, with a vivid electric storm and heavy rain in the first part.

May 26. $11^h 20^m$: There was a large, luminous, elongated cloud in the low northwest, whose light fluctuated rapidly. The entire sky was covered with patches and smears of luminous haze which I am sure were auroral. The night

was strangely luminous. The sky was apparently covered with a luminous patchy haze. In places it was streaky and sometimes long streams of it were seen. Some of these were nearly as bright as the brightest part of the Milky Way. With the lights out in the dome the windows looked as bright as if the sky were moonlit. With the exception of the pulsating cloud in the low northwest there were no other certain evidences of an aurora. But the unusual brightness of the sky and the patches and strips of luminous haze suggested that a strange auroral effect was on hand.

June 3. $11^h 30^m$: There was some attempt at streamers but no arch. $11^h 40^m$: Large streamers but no arch. There was a great deal of stray illumination. At midnight it began to darken near the northern horizon as if an arch were forming but it did not develop. At $13^h 40^m$ the aurora was dead.

June 4. No aurora during the night.

June 19. $10^h 10^m$: A faint arch with some feeble attempts at streamers.

July 29. Feeble aurora about $11^h 0^m$ for a short time. No arch or streamers.

July 31. At $14^h 0^m$, for a short time, there was the west part of a low arch which was quite strong, but which did not last long.

August 1. $12^h 30^m$: A very low arch and a few streamers. It soon died out and at $14^h 30^m$ there was nothing of it.

August 6. $12^h 0^m$: Rather strong aurora low on northern horizon. Arch either on horizon or below it. Did not last long—an hour or an hour and a half.

August 8 (R). Aurora.

August 18. $8^h 30^m$: A very bright aurora through breaks in clouds. There did not seem to be any arch. $8^h 40^m$: It was sending up streamers vertically under *Cassiopeia*. There was possibly a very low arch. The aurora was seen through breaks in clouds until moonrise.

August 19. $8^h 10^m$: A faint auroral glow low in the north. As late as $9^h 30^m$ it did not seem to be active.

August 20. $9^h 30^m$: An auroral glow in the north and a few streamers. By $10^h 30^m$ it had essentially died out.

August 21. A very bright aurora stretching far along the northeast and northwest horizons but not extending very high. It was very bright and active at $9^h 30^m$, sending up rays and broad sheets of light which were very bright at their bases—of a bluish-white color. The rays to the west of the magnetic north moved toward the west. $9^h 55^m$: Up to this time it was very active but now less so. $10^h 40^m$: Again active. $10^h 55^m$: It was dull but there were a few faint streamers. This was the brightest display I had seen for a long time.

August 22. There were no signs of aurora during the night.[1]

[1] At my request, during a short absence from the Observatory in August 1908, Professor D. W. Morehouse kept a lookout for auroras. The following record was received from him too late for insertion in the proper place. They are not taken into account in the table on p. 232:

1908 August 23: Faint glow. No streamers.
August 24: Strong glow. No streamers. Night very hazy.
August 25: A long, faint streamer a little west of north; no glow near horizon.
August 26: No evidence of aurora.

September 4. $7^h 30^m$: Bright aurora with nearly full moon. $9^h 0^m$: The arch was quite distinct. Its summit was about 20° to the right of the north point. $10^h 40^m$: The arch was very bright in spite of a bright moon. It seemed to have risen higher. There were fragments of a second arch above, three-fourths as high as the pole. Fragments of this ran over the *Pleiades*. There were some streamers to the left of the pole.

September 12. Mr. Lee reports that there was a large aurora after midnight last night (the 11th) through a very dense sky and nearly full moonlight. It was bright in the north with streamers and with fluctuating bright clouds overhead. It must have been a very large aurora, as the sky was too thick to see anything except the brightest stars. It appears to have been most active from 12^h to 13^h. There had been no aurora as late as 10 o'clock that night.

September 28. Mr. Sullivan reports that during a short period of clear sky at $14^h 30^m$ there was a brilliant aurora with some streamers.

September 29. $7^h 10^m$: Brilliant aurora seen through breaks in clouds, all over the north, northeast, and northwest. Some effort at streamers. $11^h 10^m$: The aurora had been very active with streamers and fluctuating patches of light. At about $8^h 30^m$ it was very brilliant. The sky in the north looked like daylight. The illumination extended a great distance east and west and the whole northern sky as high as the pole was bright, but there was not much activity. It got less bright and broke up into patches and became very active with streamers that reached higher than the pole. If any arch existed it was lost in the broken clouds which covered the sky more or less all the time. Patches or areas of light would ascend to great altitudes and then die out. At this time ($11^h 10^m$) the light had almost died out, but there were some streamers. The sky was pretty well covered with clouds. $12^h 0^m$: Some streamers and broken masses. Clouds. $13^h 0^m$: Less bright. Clouds.

September 30. Cloudy until 10^h. The clouds, disappearing, revealed a strong auroral glow. There was also a very small segment of an arch at the north by east horizon, which was not more than 1° high. The aurora was not active. By $10^h 30^m$ the arch had gone, but the glow continued. At 14^h there was some activity but no arch—only the glow and a few broad sheets of light shooting up from the horizon. The sky was very luminous all night—not haze, but like moonlight. I could read my watch in the Bruce dome at $14^h 0^m$ without artificial light.

October 1. At moonset a feeble auroral glow was seen in the north. This remained essentially all night without any activity. After midnight until dawn there was a pulsating, bright cloud about 25° north of east and 4° or 5° altitude. This would die out entirely for a while and then brighten up rapidly. About 16^h a brilliant bluish-white cloud 25° long by 6°–8° broad appeared midway between ζ and η *Ursae Majoris*. It was approximately horizontal. It would die out entirely and then in half a minute or so would rapidly brighten again, very bright, and remaining thus for a few seconds would die out again. This was a remarkably striking object.

October 2. The only evidence of an aurora was a fluctuating cloud 30° north of west and 5° high. This remained only a short time at about 14^h.

October 3. $11^h 30^m$: A strong auroral glow very low in the north. This remained more or less distinct until daylight. It was not active.

October 4. At moonset a feeble glow in the north. Then at about $15^h 30^m$ a large mass of pulsating light appeared due east at an altitude of about 30°. It was 15° long by 10° wide. It would almost fade out, then suddenly brighten. At times it appeared to be nearly double. It moved very slowly to the south and rather suddenly disappeared at $16^h 0^m$. It moved over about 10° in the three-fourths of an hour of its visibility. During the existence of this cloud the aurora in the north got very much brighter, but not active, and was fairly noticeable at daylight.

October 5. $14^h 0^m$: From this time on till daylight there was a feeble auroral glow. It was a little brighter at $15^h 0^m$.

October 12. $7^h 0^m$: Auroral glow began in low north—not there before this time. It got brighter rapidly and at $7^h 30^m$ was quite strong in moonlight and appeared to be forming an arch. $8^h 50^m$: Very active. Arch low and unfinished. Bright rays were moving rapidly to the left. It was conspicuous in spite of moonlight. $10^h 15^m$: It was apparently dead and had been so for half an hour or more.

October 30. Nothing in early part of night—had looked for aurora at moonset. First saw it at $13^h 20^m$. Not visible half an hour earlier. Very bright arch, almost like daylight. Then a dark under part came up, the whole rising slowly. At $14^h 0^m$ the arch was $6° \pm$ high and intensely bright. It soon broke and several streamers appeared. At $14^h 9^m$ the dark part disappeared and short streamers were visible all along the arch. These all moved to the right (east)—even those at the west end of the arch. $14^h 20^m$: Quiet—had faded much. $14^h 30^m$: Brighter with a great region of diffused light low in northwest. $15^h 0^m$: Arch formed again very bright but very low. Not active—dark below. $16^h 5^m$: Very bright again—active but no long streamers. The illumination was very wide east and west. $16^h 15^m$: Short streamers all along the arch. $16^h 45^m$: Still bright and active. There were no long streamers, only short ones and diffused masses of light moving east along the dark arch. The dark part would form very low, then rise to twice or more the original height. The light at times was so bright that it cast a shadow. Though the brightest aurora in a long time this was not a specially active one.

November 7. $10^h 0^m$: Full moon. Slender, bright auroral streamers shooting up from northern horizon to one-third or half-way to pole. No arch. The streamers came up from behind a low bank of cumulous clouds on the northern horizon. Ten minutes later no trace of the aurora could be seen nor was there any during the rest of the night. None was visible earlier than 10^h.

November 8. $10^h 0^m$: Full moon. Slender bright streamers shooting up from white cumulous clouds on the northern horizon. The aurora must have been of short duration, for it was not seen before this time nor after it as late as $15^h 0^m$.

November 16. $8^h 30^m$: Strong auroral glow, very low, extending 5°–6° high. Ten or twenty minutes before this there was no trace of aurora. $9^h 20^m$: Very bright and active, with bright short streamers. The streamers to the west of the summit moved to the left; those to the east moved to the right. $9^h 50^m$: Very little left of it. $10^h 10^m$: Only a glow left. $13^h 0^m$: A slight glow still visible.

November 17. $6^h 30^m$: Bright aurora. $10^h\pm$: Still visible with feeble arch 4° or 5° high—dark underneath and well defined. At $10^h 20^m$ ϵ *Ursae Majoris* was exactly at the summit of the dark part—exactly on its highest edge. The aurora died out at $14^h 0^m$.

November 18. $9^h 0^m$: A rather large pulsating cloud in the low northwest, close to the horizon, and some light under the pole. (None there at $8^h 0^m$.)

November 19. $12^h 50^m$: Strong aurora behind clouds in the north. There was none in the earlier part of the night. $13^h 45^m$: Still strong but not active.

November 28. An auroral glow low in the north for a short time about 12^h or 13^h.

December 4 (R). Aurora in evening and from 12^h to 14^h.

December 26. Apparently no aurora (and none for a long time). The sky, however, was very luminous. I have often noticed this luminosity of the sky. Though it appeared to be perfectly clear on this night it was whitish all over, as if the sky were phosphorescent.

1909

January 1. $15^h 40^m$: Streamers from behind clouds in the north—the first aurora I had seen for a long time. It seemed to be quite active.

January 24. $12^h 0^m$: A bright aurora all along the northern horizon among broken clouds. Not active. First saw it at $11^h 45^m$—it was not visible an hour earlier. $13^h 0^m$: Aurora visible as a bright streak along the horizon under the clouds. $14^h 0^m$: Same as before. $15^h 0^m$: Was still visible under the clouds.

January 25. $10^h 0^m$: Auroral glow in the low north. $10^h 30^m$: Only feebly seen. $12^h 40^m$: It was active. If any arch existed it was below the horizon. There was not even a glow along the horizon but the streamers and sheets of light rose nearly half-way to the pole. They were not bright.

January 26. $15^h 0^m$: Faint aurora, with very low arch. There was none an hour earlier. $16^h 45^m$: Arch very low, 3° high. No streamers. Bright changing patches in the arch.

February 21. $9^h 30^m$: Very low arch. Dark part 3° high; top of bright arch 5°–6°. $10^h 30^m$: There was an auroral arch at the horizon, the dark part having sunk below. $16^h 0^m$: The aurora had died down. There were attempts at streamers for a while. $16^h 30^m$: It was very bright again. Arch almost on the horizon—so low that there was scarcely any dark beneath.

March 19. There did not seem to be any aurora until $8^h 45^m$, when a long streamer appeared east of north which moved west. At this time there was only the feeblest glow and no arch. It became more active later and the glow was

stronger. $9^h 15^m$: Many streamers but no arch. $9^h 45^m$: Great deal of glow and streamers some of which reached nearly to the pole. $11^h 0^m$: The aurora was dead. $12^h 30^m$: Nothing visible. If there was any arch on this night it was below the horizon.

March 20. $12^h 45^m$: Considerable auroral light and a few streamers. (R) says "all night."

March 21. $7^h 50^m$: A feeble auroral glow.

March 28. $11^h 30^m$: Splendid aurora; curtains; ascending pulsations. The curtain waves went east. At first the narrow streams went west. $12^h 20^m$: It was not active at this time but the arch was very bright. It extended very far east. $12^h 25^m$: It was almost dead. Same at $13^h 20^m$. This aurora was the finest example of curtain effects that I have seen here. They were white and some of them bluish white. These curtains moved rather rapidly east while the slender streamers that made their appearance previous to the curtains moved west all along the arch. There were rapid ascending pulsations, and a broken second arch above the main arch. $15^h 40^m$: Aurora active again. No definite arch, but masses resembling bright cumulous clouds from which sheets of faint light were going up. The aurora was more or less active until daylight. It would frequently almost die out, then brighten up again. Its most active period was before midnight—about $11^h 30^m$.

Observer absent from April 19 to April 30. Mr. Lee reported that there was an aurora with low arch and some streamers at about 15^h on April 25. It was not bright.

May 15. Cloudy in first part of night. $11^h 30^m$: A brilliant fluctuating cloud in about α $9^h 0^m$, $\delta + 47°$. This would entirely disappear and brilliantly reappear again. It was irregular and extended easterly for 20° or 25°. At the same time a very small cloud was at the horizon in the northwest. This would flare up very bright and then fade again. In 10^m or 15^m these fluctuating clouds went out entirely and disappeared. There was no auroral effect elsewhere, except possibly a very uncertain and feeble glow in the north. This glow became stronger, but no arch. $12^h 35^m$: The brilliant pulsating cloud appeared again, or another like it. It passed through the bowl of the Great Dipper and extended through β *Ursae Minoris*. The small cloud was very bright at the northwest horizon. It was 5° high (just under the tip of the sickle of *Leo*). At $13^h 5^m$ they were fluctuating brilliantly. The large elongated mass passed through η *Ursae Majoris* and α *Draconis* and beyond, involving *Vega* on its south side. The whole mass moved east and south. $13^h 35^m$: A small cloud 3° or 4° in diameter appeared in the southeast at an altitude of 5° vertically under ϵ *Pegasi*. It would get intensely bright for a few seconds and then fade out. The large mass moved through the zenith and over γ *Cygni*. Another mass 45° high in the east. They all would flare up intensely bright. $14^h 40^m$: The small cloud in the east was rising and moving slightly north. A magnificent mass extending east and west near the zenith with its east end north of the small cloud by 5° or 6°. It was very long and near the middle jutted out to the south. It would become

intensely brilliant. The large one lay approximately east and west. $14^h 20^m$: The east ones had disappeared while an intensely brilliant one, 45° altitude, was fluctuating in the west. It was very large and brilliant, though seen through haze. Sky in north luminous but no streamers or arch. This was one of the finest displays of these fluctuating clouds that I have seen.

May 17. $11^h 25^m$: A small spot of fluctuating light in the east half-way from the Dolphin to the horizon. It was not bright and did not last long. A large but faint glow in the north, but no arch. $11^h 53^m$: One of the fluctuating clouds appeared in the west. It was low, and about 4° in diameter. It was 4° to the right of ϵ *Leonis*. By this time the glow in the north was much stronger and the bright mass in the east had disappeared. $12^h 0^m$: Arch very low and flat and very extended, close to northern horizon with momentary efforts to produce a second arch above and some effort at streamers. A great, long, fluctuating, bright cloud passing through η *Ursae Majoris* and overhead. Though there were no streamers the aurora seemed to be getting active. $12^h 5^m$: A large fluctuating cloud in the Milky Way in *Cepheus*—between *Cygnus* and *Cassiopeia*. It would become very bright for moments. A fainter one 20° to the left of the polar star. $12^h 30^m$: Very bright everywhere under the pole, with some attempts at streamers. Bright arch, very low and very extended.

May 18. $9^h 30^m$: Strong aurora in the north below the clouds. It kept bright more or less all night, though partly covered with clouds, but less bright after midnight.

June 22. $11^h 10^m$: Slight aurora—the first in quite a long time. Watched every clear night but there was none. $14^h 0^m$: Very bright through break in clouds.

July 23. $9^h 30^m$: A rather feeble arch, which sent up streamers earlier. $10^h 20^m$: Much stronger. $10^h 38^m$: Arch gone. $11^h 50^m$: Arch very strong. $13^h 0^m$: Very strong glow but no arch. $13^h 35^m$: Glow still strong, no activity. $14^h 20^m$: Pretty dense glow. $14^h 40^m$: Aurora dead.

August 19. $12^h 0^m$: Small aurora which soon ended.

August 28. $14^h 30^m$: Strong auroral arch (moon near setting) but not very definite. $14^h 50^m$: Arch very strong and thin and definite. 5° high, very dark below it. η *Ursae Majoris* seen in the dark region. No streamers. $15^h 10^m$: The definite black beneath the arch had disappeared. There was at this time a broad bright band 7° high. Later on there was a very little dark space close to the horizon. The band became irregular—broken along its length—when daylight came on.

September 5. $8^h 15^m$: Slight aurora low in the north beneath the clouds. It did not last long.

September 14. $13^h 0^m$: A small aurora.

September 25. $15^h 30^m$ to $16^h 0^m$: There was a rather strong auroral arch low in the north. No streamers.

October 4. $8^h 30^m$: A bright pulsating cloud in the Milky Way, α $18^h 20^m$, $\delta - 18°$. It would become very bright for a few seconds, then die out. It remained visible for about 10 minutes. No other evidence of an aurora.

October 7. $10^h 25^m$: A strong low arch very dark beneath. It was not present, say 15 minutes before. $11^h 0^m$: The summit of the arch was in the same vertical with β *Ursae Majoris.* The top of the dark arch was half-way from the horizon to this star. The top of the bright arch extended three-quarters of the way from horizon to β. $11^h 20^m$: Arch strong, but no streamers. $11^h 30^m$: No change.

October 8. $6^h 30^m$: Feeble aurora. By $9^h 0^m$ it was dead.

October 18. A magnificent aurora was visible at 7^h. I first saw it at $7^h 45^m$. At this time all the northern sky below the pole was covered with great masses of light, with apparently a double arch. It was very active at $8^h 0^m$, with streamers and broken masses of light. At $8^h 25^m$ there were great masses of light far in the northeast which drifted slowly to the west and rose higher until the sky to the zenith and as far as *Saturn* was covered with them. At $8^h 30^m$ there were rapid ascending pulsations. These ascending waves seemed to bring out and illuminate large areas of matter all over the north, that were drifting westward. At $8^h 35^m$ the arch had again formed and streamers (with dark intervals between them) all along its periphery. A very fine sight. Beautiful slender streamers ascended as high as *Polaris.* Several of those to the left moved rapidly to the right or eastward. These apparently passed others going west. To the west the streamers were reddish, but the general display was of a bright yellow. Many of the slender streamers shot higher than *Polaris.* At $8^h 45^m$ all the northern sky was covered with tufty masses of light which were elongated vertically. At $9^h 5^m$ a thin bright arch extended to an altitude of 10°. At $9^h 45^m$ there was a fine and bright arch at an altitude of 14°, with bright masses far toward the east apparently separated from the rest of the aurora. α *Ursae Majoris* was in the upper edge of the bright arch, which was black beneath only toward the east, the rest being luminous below the arch. The arch was very perfect but not active. At $9^h 58^m$ rapid waves were ascending all along the arch and pulsating diffusions of luminosity all over the heavens north of the zenith, but the arch was not active, no streamers. At $10^h 15^m$ the arch became active to the west. It had risen slowly and was then 17° high, but it was not dark underneath. Waves of faint light were ascending everywhere in the north, to the zenith. α *Ursae Majoris* was at this time in the lower part of the arch. There were masses of light in the east free of the arch which was very large and wide, extending along the horizon for 100°. At $10^h 22^m$ there were beautiful curtains in the west, springing from the arch and extending to great altitudes. These moved rapidly to the east. Their bases were bluish white with strong prismatic colors. It was a splendid sight! Beginning at the west these curtain-forming streamers would burst out along the summit of the arch, which was very perfectly formed, one at a time in rapid succession ascending to great altitudes. Thus, the arch was broken to pieces. The whole system of moving curtains moved bodily to the east. This greatest display lasted only a minute or two, and was one of the grandest I have ever seen. Then everything broke up into masses of light all over the north to the zenith. During this display there was a great curtain-like mass below the middle of the

arch which nearly touched the horizon and moved to the left. $10^h\ 30^m$: Great streamers everywhere in the north reaching nearly to the zenith; very active. Great masses of light and fragments of streamers all over the north, but no arch. At $10^h\ 35^m$ rapidly ascending waves were rising everywhere in the north with great masses of light all over the northern sky extending to the east point. $10^h\ 47^m$: Fantastic masses dancing all over the northern heavens to the zenith, which seemed to be due to rapid pulsations of light, which, as they passed, momentarily illuminated irregular masses of matter that were changing form slowly and moving to the west. $10^h\ 52^m$: The rapid pulsations seemed to have ceased, but there were still great masses of light all over the north to the zenith, which would slowly brighten and fade. At this time the arch was again forming. At $10^h\ 55^m$ the arch was very black below and only half as high as before. Ascending pulsations and bright masses all over the north to the zenith. The sky was more or less luminous everywhere except in the far south. The pulsations were still rising at $11^h\ 5^m$ and the arch was broken, but at $11^h\ 35^m$ it was very strong again and very dark underneath. Ten minutes later it was broken again by masses of light and some streamers. At $13^h\ 0^m$ the arch was still pretty strong, but not active. At $16^h\ 20^m$ it was all dead except some pulsating masses of light near the northern horizon. $17^h\ 0^m$: The same.

This was one of the finest auroras I have seen here. Especially was the display of curtains splendid at $10^h\ 22^m$. I have never before seen the brilliant prismatic color effects which burst out on the forming of the curtains. I was strongly impressed with the resemblance of some of the streamers to comets' tails, etc. They would shoot up, sometimes very slender, and then diffuse into broad wavy masses moving west. The general motion was west except the curtains mentioned and the few streamers that moved east, and the pulsations which were vertical. At times great areas of feeble illumination would appear for a second or so all over the north. Sometimes these illuminations were very bright. They appeared like great areas of bright haze. Altogether the display was one of the most brilliant I have ever seen.

October 19. $8^h\ 45^m$: A long bright mass in the Great Dipper which was $20°\pm$ in length. West of this was a smaller mass which would brighten and fade. In the east was a long luminous region 20° high and south of the east point. The entire sky was luminous as if a considerable moon were shining. No arch. The long mass in the Dipper seemed to be a part of an arch. $10^h\ 30^m$: It was sending up quite a number of streamers, but not bright. There was no arch. The streamers came up from the horizon. The pulsating clouds were gone. $11^h\ 20^m$: It was dead. $12^h\ 0^m$: The sky was luminous as if a bright moon were shining, but no trace of aurora.

October 26. $17^h\ 40^m$: After moonset there were two pulsating clouds, one under the pole $10°\pm$ altitude, the other farther to the east.

November 14. $14^h\ 30^m$: Very low arch, 2° high—rather faint, thin, and not over 25° in extent. Not there half an hour before. $14^h\ 50^m$: Arch very much brighter and had risen somewhat. It lasted until dawn but was not active.

November 19. $10^h 30^m$: After moonset a strong auroral glow in the north. No arch. $14^h 0^m$: Glow was still present among clouds.

December 9. $15^h 0^m$: Part of an auroral arch to the left of the north point.

REMARKS ON THE RESULTS OF THE OBSERVATIONS

In reading the accounts given in *Nature* and the *Astronomische Nachrichten* of the luminous nights seen in England and on the continent about the first of July 1908, by Denning and others, I have thought that the phenomenon of the luminous night of May 26, 1908 (and of other dates), was of a similar nature to those described by the various observers in July.

In *Nature* for September 30, 1909 (**81**, 395), Dr. Chree gives an account of a great magnetic storm, recorded at Kew, which "commenced suddenly at about $11^h 43^m$ A.M." on September 25: "The storm was of comparatively short duration, no movements of any great size being recorded after $8^h 30^m$ P.M. on September 25, and by 1 A.M. on September 26 little trace of the disturbance was left."

This same storm badly interfered with telegraphic operations all over the United States on September 25. According to the newspapers it was very active between 6 and 9 A.M. I observed all night on the 24th, closing observations just before 17^h or 5 A.M. of the 25th, or 11 A.M. at Greenwich. I looked out for any aurora just before daylight but there was none, nor had there been any during the night which would be bright enough to be seen in moonlight. As recorded above in my notes, there was an aurora on the 25th at $15^h 30^m$. I had suspected one earlier in the night but it was uncertain on account of the moon.

For a valuable account of the condition of the solar activities about the period of this great disturbance, see an article by Professor Frederick Slocum in the *Astrophysical Journal* for January 1910.

For some years, I have had in my mind a scheme for systematic observations of the aurora, but for various reasons it has not been possible to carry it out, as it would require the permanent residence of one observer some miles north of here. The scheme would be to have someone, say a resident of some place ten or twenty miles north of the Yerkes Observatory, who would be familiar enough with the

stars to locate an object with the naked eye by them, or who had an instrument for the determination of the altitude and azimuth of such objects. Suppose such an observer to be connected with the Yerkes Observatory by telephone. Upon the appearance of any striking auroral phenomena, such as the moving, fluctuating clouds, simultaneous observations could be made of these that would give their true altitude above the earth. As they sometimes remain visible for upward of an hour, these observations would show if they varied their height and would also give accurately their real velocity with respect to the earth. The actual elevation of the arch could thus be determined also, and various other phenomena of importance. A simple instrument made of wood, such as I have described in the *Astrophysical Journal*, **16**, 144, 1902, would give the position of the object with sufficient exactness for the purpose—especially if pointings were also made on some known star at the time. Observations of this kind would quite definitely show whether different observers at a distance really see the same thing or that each person sees his own aurora, as in the case of the rainbow, as has been suggested by some writers. The fluctuating clouds and their motion would alone perhaps contradict this theory.

In the present paper, I have gone somewhat into detail in the accounts of the auroral displays here. This has been done because it has appeared to me that in general the references to an aurora usually are wanting in details and leave doubt as to what kind of aurora had been present. It would seem that there may be different kinds of auroras due to different causes or to modifications of some one cause. If so, these should be distinguished from each other. For instance, sometimes the fluctuating clouds have appeared when there was no other evidence of an aurora. At other times they have been present during an ordinary aurora. These clouds therefore would seem to depend on different conditions from the regular aurora. At times these conditions alone are present, at other times they combine with those necessary for the production of the ordinary aurora. Certainly the conditions must have been different that produced the great band of May 26, 1908. So also must those that produced the luminous nights mentioned in these observations. A classification of the various kinds of auroras would therefore be valu-

able. It is with this idea in view that I have dealt more extensively with some of the displays of the past ten years.

The recorded times are important, especially in case of the spasms of activity in the large auroras. It would be interesting to compare these with the magnetic records to see if any special disturbances show at these times on the instrumental records.

I will not here go into any discussion of the connection between solar disturbances (as indicated alone by great sun-spots) and the aurora. This does not lie within my province. It may not be out of the way to state, however, that such a connection does not at present seem to be clearly established in all cases. I have within the past ten years or so frequently noted solar spots so large as to be visible to the unaided eye. These have not always been closely associated with auroral displays. A most striking instance of this kind was shown in the case of a large naked-eye sun-spot on and about December 29, 1909. A careful record on every clear night about this time failed to show any evidence of aurora. Indeed this prolonged absence of auroras (up to the latter part of January 1910) would have been noticeable without the incentive of the large sun-spot to look for them.

I have thought that it might be interesting to see if any months are more abundant in auroras here than others. For this purpose Miss Calvert has prepared from my observations the following table, which is interesting from several points of view.

TABLE OF AURORAS OBSERVED FROM 1897 TO 1909 INCLUSIVE

	1897	1898	1899	1900	1901	1902	1903	1904	1905	1906	1907	1908	1909	No. by Months
January . . .	.	2	1	. .	. .	. .	. .	. .	. .	. .	. .	2	4	9
February . .	. .	. .	2	. .	. .	. .	. .	. .	2	2	12	. .	1	19
March	1	6	. .	. .	. .	. .	. .	. .	2	1	3	2	4	19
April	1	. .	. .	1	. .	1	. .	2	. .	. .	2	1	1	9
May	. .	1	3	1	. .	1	. .	. .	. .	1	1	5	3	16
June	. .	. .	3	. .	. .	. .	. .	2	. .	1	. .	2	1	9
July	1	. .	1	. .	. .	. .	. .	. .	. .	. .	. .	2	1	5
August	2	1	1	. .	. .	. .	1	. .	. .	1	1	7	3	17
September	. .	4	4	1	. .	. .	4	. .	1	2	1	5	3	25
October . . .	1	1	. .	. .	. .	. .	1	3	. .	1	3	7	. .	17
November	. .	2	1	. .	1	. .	4	3	1	. .	1	6	. .	19
December .	2	1	. .	. .	. .	. .	1	. .	. .	. .	1	1	. .	6
No. by yrs.	8	18	16	3	1	2	11	10	6	9	25	40	21	170 = total

This table gives the number of auroras seen in each month from 1897 to the end of 1909. The last vertical column gives the entire number for each month during this interval. The horizontal column at the bottom of the table gives the frequency for the various years. September has been especially prolific, but September is a season of clear skies. February also stands high, but this high grade is dependent on the year 1907. July and December stand especially low. There seems to have been a minimum of auroras from 1900 to 1902. The observer was absent on the eclipse expedition to Sumatra from the first of January to the last of July of 1901. The year 1908 gives the highest record of all—auroras on 40 nights. But the summer and fall of 1908 were remarkably free from clouds. The combination of 1907, 1908, and 1909 would seem, however, to indicate a maximum in which we are now placed or which we have just passed through.

It is evident, however, that to determine accurately the frequency of auroras here in the past thirteen years it would be necessary to compare the observations with the various meteorological conditions that have prevailed in that time. I have not the leisure at present to undertake this work and so the results must stand as they are.

YERKES OBSERVATORY
January 1, 1910

STAR COLORS. A STUDY IN PHYSIOLOGICAL OPTICS

By LOUIS BELL

The following investigation is an attempt to discover the relation between the visible colors of the stars with the known facts regarding their spectra, and the varied and bizarre array of tints reported by many of the observers of double stars. The difference between the sober evidence of the spectrograph and visual observations of isolated stars is very small compared with the recorded observations of double stars made even by experienced observers with entirely adequate apparatus. There is sufficient kinship in the various estimates of color thus made to indicate dominant common factors, whether objective or subjective in their nature, and yet the differences and discrepancies are very singular and are sufficient to have led to somewhat startling hypotheses as to great accidental or periodical variations in color.

To begin with, a most casual study of the heavens convinces the observer that among the lucid stars at least there are no extraordinary colors. Broadly speaking, they range from a faintly bluish white, like *α Lyrae*, to a fiery orange, like *α Scorpii*. It is well known that there are at least no isolated stars which can fairly be called blue or green, or in fact which are characterized by any strong dominant hue in the more refrangible part of the spectrum. Even *β Librae*, occasionally reputed to be green, presents to the normal eye only a very faint and faded tint. The spectra of the stars with respect to the corresponding colors have been thoroughly investigated in the Potsdam *Durchmusterung*, as discussed by Pickering,[1] as well as earlier by Vogel,[2] and by Krüger.[3] The results obtained in these investigations are entirely consistent with the evidence of the eye so far as isolated stars are concerned. The stars of the Potsdam list are recorded in a systematic scale ranging from white through the various shades to yellowish red. There is no record of colors materially outside this

[1] *Harvard Annals*, **64**, 125 ff.

[2] *Publicationen des Astroph. Obs. zu Potsdam*, **3**, 127, 1883.

[3] *Publicationen der Sternwarte in Kiel*, **8**, 1893.

simple and commonplace range. The careful analysis of the Potsdam colors by Pickering shows how closely the visible appearance and the spectra correspond. Using the spectral classification of the Draper catalogue it proved possible in the case of the Potsdam observations to make a singularly orderly classification of the spectra with respect to color. The Potsdam list, to magnitude 6.5, included 4206 stars. From the analysis of the data relating to these it plainly appears that the visible colors can be fairly well predicted from the photographic spectra. For example, the chances are 15 to 1 that a class A star is either white or yellowish white, and the chances are 150 to 1 that it will not be called a full yellow. Likewise the late solar stars of classes G and K are predominantly whitish yellow or yellow. The chances are 8 to 1 that a star of these classes is not either white or yellowish white, but rather of a deeper tint, and the chances are 200 to 1 that a star of these classes does nor fall into the white category, these figures being derived from the color estimates of the stars of different spectral classes in the Potsdam list. Moreover, Pickering[1] has shown that there appears to be no evidence that stars of the same class of spectrum differ in color in different parts of the sky.

In considering the fainter stars of the Potsdam list, as in other similar records, there is an evident increase of the apparently white stars of the fainter magnitudes, say from magnitude 6 to 9.6, and of the total number of stars classified as white to magnitude 6.5, 94 per cent. are of classes B or A. The increase in the number of apparently white stars among the lesser lights is not without physiological significance, as Müller and Kempf[2] have shown. From the spectroscopic evidence it is perfectly clear that, so far as isolated stars at least are concerned, the spectra correspond to the visible colors with remarkable closeness, and neither the eye nor the spectroscope gives evidence of unusual colors in isolated stars, especially noteworthy being the complete absence of anything approaching strong blue or green. On the other hand, the reputed colors attributed to many double stars are of the most eccentric and extraordinary character compared with anything revealed by the spectroscope or to the eye in ordinary stellar observations.

[1] *Loc. cit.*

[2] *Astronomische Nachrichten*, **180**, 16, 1909.

The following is a brief list taken quite casually from Webb's *Celestial Objects*, exhibiting the vagaries of nature or of vision in double star chromatics: Indigo, grayish white, olive, bluish gray, bluish red, violet, tawny, ruddy or dusky, brownish, pale rose, fawn color, olive blue, greenish blue, pale green, yellowish green, green, rosy, gray, lilac, mauve, bluish green, ashy yellow, bright green, greenish blue, pale tawny, sombre, cool gray green, greenish red, ruddy olive, dusky red, garnet. These thirty reputed hues are only a part of the eccentricities of color nomenclature which appear in the literature of double stars. A complete list of such fanciful epithets would run to nearly a hundred, but those given are sufficiently characteristic and so obviously outside the range of normal star colors as to demand investigation. From the physical and spectroscopic standpoint such a list is quite inexplicable. From the physiological standpoint it turns out to be quite in accordance with the well-known phenomena of physiological optics.

The very striking fact of the predominance of blue, violet, and similarly colored *comites* among double stars in which apparent color differences exist becomes thus comparatively easy of explanation, together with the remarkable fact that where one star of a pair is blue it is invariably the smaller one. The elder Struve[1] made a careful examination of the colors of double stars and among 596 comparatively bright pairs found 120 blue *comites*, these being all the *comites* in which there was strongly marked color differing from that of the primary. These doubles included both binary stars and optical doubles without discrimination. Later lists give a still greater number of blue *comites* to be accounted for. Levander,[2] for example, in his analysis of various star lists gives 394 *comites* of blue, green, lilac, purple, and violet tints. As his list was compiled very largely from Webb's *Celestial Objects*, the other sources being mainly catalogues of red stars, these 394 are all, or nearly all, the smaller components of doubles. Holden[3] states that stars of such colors are, so far as he knows, invariably associated with larger stars, the colored ones in general being small. Of Levander's list, just cited, 369 of the

[1] *Mensurae Micrometricae.*

[2] *Monthly Notices*, **50**, 33, 1889.

[3] *American Journal of Science*, **19**, 467, 1880.

394 are of the seventh magnitude and below. One is confronted then with the extraordinary fact that of nearly 400 blue and similar stars all are the smaller components of doubles, most, or all, of such colors ascribed to isolated stars being either exceedingly pale, as in the case of *a Lyrae*, or erroneously described, as indicated by Holden.[1] Now the chance of all blue stars being associated, as they are within an average distance of one-half a minute of arc or less, with brighter stars of different color is on the doctrine of chances so small as to be practically negligible. Inasmuch as there are only about 300,000 stars in the northern hemisphere of magnitudes including the larger members of the colored pairs, and these are scattered over 20,626 square degrees, the chance of a fortuitous distribution of 300 or 400 colored stars all forming doubles with brighter primaries needs no discussion. One is therefore driven to an explanation of the situation dependent in some way or other upon propinquity. It has been a hypothesis many times considered that the association of a blue *comes* with a differently colored primary is due in some unexplained way to the physical connection which may exist between them, and so little is known of the actual physical conditions existing in binary stars that such a hypothesis is on its face readily admissible. If such be the explanation, that is, that the color difference depends upon physical connection, then one should find it a peculiarity of stars which are gravitationally related, that is, of the binary class, and it should not exist among stars known not to have a binary relation but to be merely optically double. Holden[2] examined from this point of view and with great care a list prepared by Burnham of all the known binaries at the date of his paper. This list, after excluding two or three dubious cases, contained 162 known binaries of which 40 showed color differences practically all of the nature here considered, or 24.7 per cent. of the whole number. Now Flammarion[3] had a few years before made a similar investigation regarding double stars known not to be binaries, but showing from their proper motions that the connection between them was merely optical. If the color difference is due to physical connection only, it should

[1] *Loc. cit.*

[2] *Loc. cit.*

[3] *Comptes Rendus*, **87**, 835, 872, 1878.

not appear among the stars of Flammarion's list. On the contrary, it does appear in 32 doubles out of 131, or 24.4 per cent. In other words, the proportion of colored doubles is substantially the same whether one considers binaries or optical doubles, and this proportion also holds very closely for Struve's list made up of a considerably larger number of doubles without regard to their physical connection. The lists of Holden and Flammarion contain a few doubles of which the *comites*, while different in color, differ very slightly, and casting out these so as to leave those distinctly of bluish or similar hue, the percentages in the lists of Holden and Flammarion fall respectively to 21 and 20.9 per cent. as compared with Struve's 20.1 per cent. So close agreement is doubtless accidental, but the figures are sufficient to show that there is absolutely no material difference between doubles in general, binaries, and known optical doubles, with respect to the frequency of occurrence of bluish *comites*. Flammarion suggested that his paper gave evidence that isolated stars were sometimes colored, blue and green, but clearly the theory of probabilities renders the chance of Flammarion's colored *comites* having been associated with their primaries by purely fortuitous propinquity, in the absence of any other isolated colored stars, one that is quite negligible. It therefore appears that the existence of colored *comites* must be referred to causes connected with optical propinquity and not dependent on physical connection.

A somewhat significant fact in this connection is the very small proportion of doubles with bluish *comites*, of which the primaries are white. In Holden's list of binaries but five out of forty colored pairs have white primaries. In Flammarion's list of optical doubles the proportion is but 6 to 32, while in Struve's list there are but 53 white primaries out of 173. Primaries of bluish *comites* run predominantly to various shades of yellow and orange in spite of the fact that in general, and especially in the fainter magnitudes, among which most doubles are found, the proportion of distinctly yellowish stars is decidedly small. Fig. 1 shows graphically the variations of apparent star color with magnitude as derived from Müller and Kempf's discussion of the colors of the Potsdam *Durchmusterung*.[1] The colors used here for a general classification are white, yellowish

[1] *Loc. cit.*

white, whitish yellow, and yellow, being a condensation of the general Potsdam color scheme. Müller and Kempf call attention to the difficulty of recognizing colors under faint illumination and to the difference between large and small telescopes in the general color effects. This is a natural result of the theory of color vision, to be referred to later on, and it in fact shows very plainly in the curves, the blanching of the smaller stars, below magnitude 6.5, being very marked. The increase in the number of whitish stars is very apparently at the expense of the deeper-colored classes. The subjective

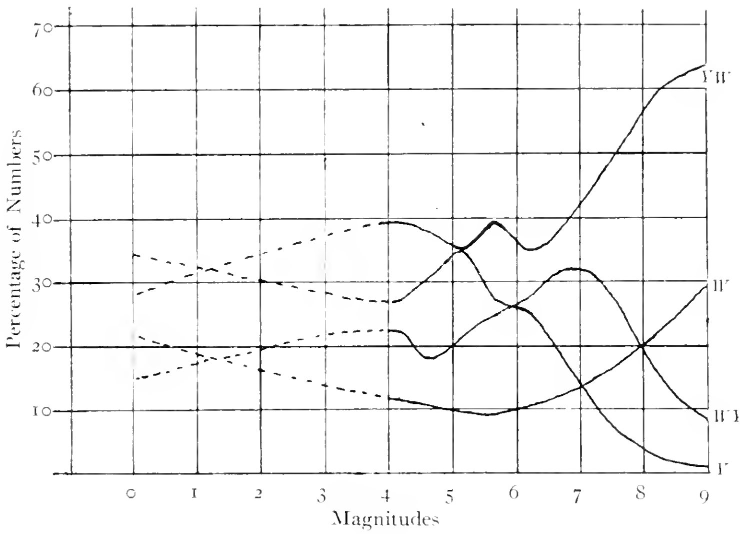

FIG. 1

nature of the phenomenon is borne out by the fact that in the vicinity of the ninth magnitude nearly 90 per cent. of the stars are whitish, at least as seen with moderate apertures, while so far as the spectra of the fainter stars have been investigated, this whitening would not be sufficiently accounted for by increase in the proportion of the type I stars observed. The subjective element in these estimates of star color, or rather the physiological element depending on both instruments and personal equation, is rather conspicuous in spite of the fact that the observations of a single skilful observer, or a group of observers working under uniform conditions, are sometimes remarkably

consistent. Table I shows the percentages of stars of various colors to magnitude 6.5, as shown in a very interesting paper of Franks,[1] and in the Potsdam colors as analyzed by Pickering,[2] respectively.

TABLE I

Franks Color	Per cent.	Potsdam Color	Per cent
White	31	White	13.2
Yellowish white	20.8	Yellowish white	40.2
Pale yellow	22.4	Whitish yellow	23.0
Yellow	11.8	Yellow	20.6
Pale orange	7.7	Reddish yellow	2.98
Orange	5.6	Yellowish red	0.02
Orange red	0.7		
	100.0		100.00

The Potsdam observers saw fewer white and fewer reddish stars than did Franks and evidently classified as yellowish white many stars which Franks called white. Franks's table is based on 3497 stars of his own manuscript catalogue and the Potsdam observations on 4206 of the Potsdam colors. In spite of such differences the infrequency of white or whitish stars and the frequency of those strongly yellowish in the lists of colored doubles are very striking.

The relation of color to difference in magnitude in doubles is also striking. It was noted by Struve[3] many years ago in his analysis of his own catalogue of doubles. He found an average difference of about one-half a magnitude for doubles of exactly the same colors, of something over a magnitude of those possessing slightly different colors, and of over two magnitudes of those showing distinctly different colors. Holden's list of binaries discloses a difference of 0.53 magnitude for binaries of the same color and 2.44 for binaries of different colors. Flammarion's list of optical doubles is somewhat difficult to compare, owing to the fact that the optical doubles are very generally much wider apart than the binaries, including some separated by several minutes of arc. Flammarion's list of colored optical doubles shows an average difference of magnitude of about 2.5, excluding very faint and wide *comites*, a few tenths less including everything. His list of optical doubles of the same color, or nearly

[1] *Monthly Notices*, 68, 672, 1908.

[2] *Loc. cit.*

[3] *Loc. cit.*

the same color, shows difference of magnitude of 1.5, excluding very wide doubles (several minutes of arc apart) and those with *comites* of tenth magnitude or below. Including everything the proportion in the two classes does not differ widely. The fact that nearly all stars of Holden's list are also in Struve's catalogue, which also includes many of Flammarion's list, indicates the same general relation of difference of magnitude to difference of color both in stars strictly binary and in those at large.

The upshot of the statistics in the matter is that most doubles having bluish *comites* have a conspicuously greater difference in magnitude than doubles of the same color, that the former have very notably a large proportion of yellowish primaries, and that binary stars show no greater tendency toward such coloration of the *comites* than do double stars at large. The cause of such coloration must be sought therefore outside of matters involving physical connection as already noted, and this investigation is directed toward assigning the definite physiological causes, which produce, in stars in optical propinquity and differing in magnitude, the particular kinds of apparent coloration that have been persistently attributed to colored doubles. If there were available for a long list of double stars the spectra of both components, one could easily evaluate the objective factor in these colorations so far as it exists. The number of stars thus completely investigated is, however, very small, some important factor in the case being often absent even when careful spectrographic records have been obtained. The writer has been able to find 25 complete records, however, among the stars of the Harvard Observatory catalogue, which are shown in Table II. Ten of these show doubles having *comites* of the same spectral type as the primary, in 9 cases of type I. The striking thing about the list is the very small color difference which exists in any of the cases cited. In all the cases where there is anything that might be called a noticeable difference of color, the colors themselves have been noted as somewhat variable, and in no instance is there a really considerable difference of magnitude. The particular significance of this table will appear later in the discussion of artificial stars. The second section of the table also contains 10 stars, all of which exhibit violent contrast, and each star of this list shows its *comes* of earlier spectral type than the

TABLE II

Star	Color A	Spectrum A	Color B	Spectrum B	Mag. A	Mag. B	Remarks
Doubles with *Comites* of the Same Spectral Types							
α Geminorum	Greenish white	I	Greenish white inclining to yellowish	I	1.99	2.85	B of slightly later class
ν Scorpii	Bright white	I	Pale lilac	I	4.29	6.49	Some variation as to colors
θ Serpentis	Yellowish white	I	Yellowish white	I	4.50	5.37	A5, A5
α Piscium	Whitish	I	Bluish cast	I	4.33	5.23	
κ Bootis	Greenish	I	Bluish	I	4.60	6.61	A5, A5 ?
ζ Ursae Majoris	White	I	White	I	2.31	3.85	
100 Herculis	Greenish white	I	Greenish white	I	5.92	6.00	A3, A3
ζ Lyrae	Greenish [Yellow]	I	White [Greenish]	I	4.29	5.87	A3, A3
19 Lyncis	White	I	White	I	5.61	6.53	
γ Leonis	Gold	II	Greenish red ?	II	2.61	3.80	Both solar stars
Doubles with *Comites* of Earlier Spectral Types							
β Cygni	Yellow	II	Blue	I	3.0	5.3	Early solar, late I type
γ Andromedae	Yellow	II	Blue	I	2.3	5.0	Late solar, late I
12 Canum Venaticorum	Yellowish	II	Bluish	I	3.2	5.7	Early solar, [early I
ι Cancri	Yellow	II	Blue	I	4.2	6.6	
ο′ Cygni	Yellow	II	Blue	I	4.0	5.0	
δ Cephei	Very yellow	II	Blue	I	4.2	5.3	Late solar, I
α Scorpii	Fiery orange	III	Blue green	I–II	1.2	7.	
ε Bootis	Very yellow	II	Very blue	I	2.7	5.1	Late II like *Arcturus, comes* late I. K, A
32 Eridani	Yellow	II	Blue	I	4.9	6.3	Late II like *Capella, comes* I. G5, B ?
α Herculis	Very yellow	III	Very blue	I	3.0	6.1	*Comes* early I
Doubles with *Comites* of Later Spectral Types							
59 Serpentis	Yellow	I	Blue	II	5.5	7.8	A, G
ξ Cephei	Yellow	I	Blue	II	4.6	6.5	A3, G
	White		Tawny or ruddy				
	White		Yellowish				Sestini 1844
μ′ Bootis	Flushed white	I–II	Greenish white	II	4.5	6.5	F, K
	Yellow		Yellowish azure				Sestini 1844
30 Arietis	Yellowish	I–II	Bluish cast	II	6.1	7.1	F, F5
95 Herculis	Greenish	I	Yellowish	II	5.1	5.2	A3, G5, or K

primary, substantially always of type I. The primaries are all of type II or type III, and the differences in magnitude are generally much greater than in the list just considered. There is in all the stars of this list an undoubted slight difference in color between the primary and the *comes*, but it is not at all such a difference as is indicated in the colors given. For example, in the first star of the list, *β Cygni*, the apparent colors are notoriously very yellow and very blue. On the other hand, it is questionable whether any star of the earlier solar type can reasonably be classified as very yellow, and it is entirely certain that no star of the later first type can, save by a prodigious stretch of the imagination, have attributed to it the blue color of the *comes* of *β Cygni*. Franks[1] notes, and very properly, that some stars of the same visual color are of different spectral types, and, conversely, some stars of the same spectral type may differ perceptibly in color, but neither he nor any other experienced observer has detected an isolated late first-type star of the color of this *comes*, or has made any provision in the tabulation of star colors for any such extraordinary phenomenon. In fact, in an earlier paper Franks[2] himself notes *β Capricorni* as having a *comes* which is probably white, being of the first type, and merely blue by contrast. The same general considerations hold for the rest of this list. Finally, there is a brief list containing five doubles with *comites* of later spectral types than the primaries. These have primaries of type I and *comites* of type II. The first on the list, *59 Serpentis*, is typical. It is a distinctly yellow and blue combination in which the yellow star is of type I. Now a type I star may well enough be yellow, but certainly a type II star, bright blue in color, is something for which one may search the heavens in vain. The difference in magnitude between the two components is here over two magnitudes. The other stars of the list have somewhat less striking coloration, and the epithets applied to the *comites* will be shown later to have much significance. The last star of the list is a famous apparent color variable, the vagaries of which are worth separate discussion. In brief it appears from Table I that a double having a primary of type II or type III with *comes* of type I will uniformly

[1] *Loc. cit.*

[2] *Ibid.*, **67**, 539, 1907.

show a contrasted coloration of its *comes*, quite distinct from anything possibly justified by the spectral type. Even β *Cygni*, which, when investigated visually many years ago by Sir William Huggins, appeared to show possible cause for a considerable color difference, displayed, when the same distinguished observer actually got the spectrograms on a large scale, characteristics that indicate a mildly yellow primary and a probably whitish *comes*. Inasmuch as the stars of Table I are all the doubles for which the complete data are available from the Harvard publications, they may be fairly considered typical of the conditions represented.

It is not putting the case too strongly to state that all the colors observed in double stars, and not pertaining to stars in general, are purely of subjective origin and due to causes readily assignable from the data of physiological optics.

The effect of contrast in double star colors has often been recognized, but it has been very generally dismissed from serious consideration as the actual cause of all such colors, by reason of two arguments. First, that the striking colors of many of the *comites* must be real because they persist when the primary is hidden by an occulting bar, and second, the argument advanced by Struve that if such colors were contrast colors they must be complementary, which they are not accurately, save in a comparatively small number of instances. Both arguments are in view of present knowledge of the theory of vision utterly fallacious. In the first place, colors due to subjective causes, which will presently be assigned to them, should not and do not disappear by the use of the occulting bar as ordinarily employed, and second, the observed colors, considering the physiological causes of their origin, can be accurately complementary only in especial cases and generally must vary materially from such character. The fallacy in the two stock arguments is due in the main to an entirely unwarrantable assumption that apparent color variations in such cases should be charged to simultaneous contrast.

In point of fact the subjective colorations observed are chargeable to four separate causes, each of them now well known and among which simultaneous contrast is only of minor importance. Simultaneous contrast, which involves the shifting of contrast colors toward complementary effect, is quite probably mainly a psychological

rather than a physiological phenomenon, inasmuch as it may occur to a certain extent in very brief glimpses (during which there is reason to expect it), as shown by Mayer, and once introduced sometimes persists, as Helmholtz has noted, until the eye gets rid of its bias and takes a fresh start. A far more important source of subjective coloration is fatigue color, whereby fatigue of a color sensation dulls that particular hue and brings out with relative intensity its complementary.

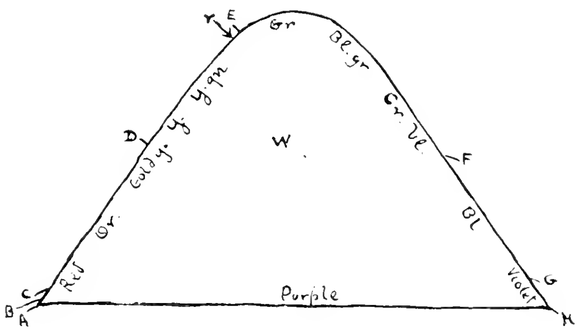

FIG. 2

Fig. 2 shows Maxwell's color diagram as given by Fick, and displays the ordinary range of complementary colors seen in simultaneous contrast and fatigue. A line passed through *W* from the stimulus color shows at its intersection with the boundary beyond *W* the complementary tint. Experiments on complementary colors in this ordinary sense are familiar and need not here be inserted, but both in the case of simultaneous contrast and of fatigue color the subjective hue conferred by the stimulus color is only truly complementary in the sense of this color diagram when the ordinary hue is unmixed with any coloration of the background, subjective or objective. Hence in applying such a color diagram to the case of double stars one would find the true complementary produced only in the absence of any subjective or objective color in the *comes*. The primary would excite, it is true, its complementary tint, but the blending of this with faint actual coloration of the *comes*, or with spurious colorations now to be considered, will throw the visible tint in many, and probably a large majority of cases, wide of that rigorously complementary to the pri-

mary. A third cause operating to produce subjective color is the shifting of the retinal color sensitiveness away from the red in faint illumination, following Purkinje's phenomenon. The facts regarding this are now well known and are shown in Fig. 3, reduced to the normal spectrum from the researches of Sir William Abney.[1] Here curve *A* shows the relative luminosities from the various parts of the spectrum for the normal eye and ordinary intensities of illumination, say a few meter-candles. Curve *B* shows the luminosity value

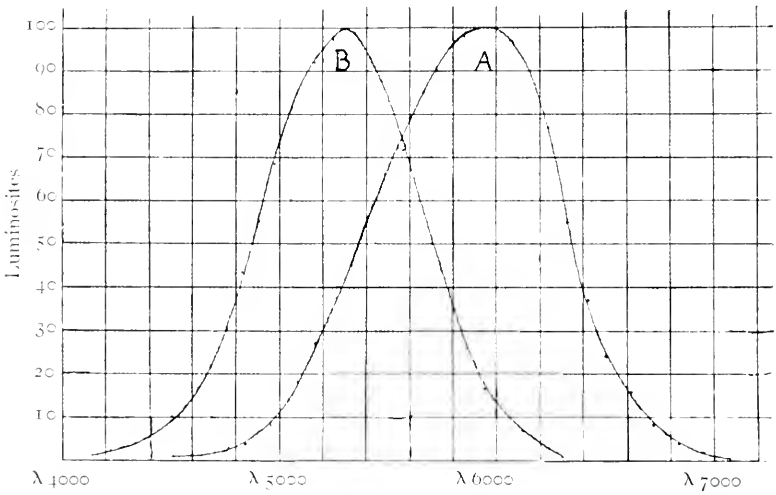

FIG. 3

0.0066 + c = 0.06 meter-candle

for the normal eye at very low intensity of illumination, about one-twentieth of a meter-candle or thereabouts. The maximum of luminosity has here been shifted from near the D lines almost to E. A white object at such intensity of illumination therefore has, as will be readily seen from inspection of the curve, a predominant greenish-blue hue, and this hue may be anything between the limits set by curve *A* and those set by curve *B*, according to the intensity of the stimulus. At a still further reduction of intensity to near the vanishing point even the bluish-green tinge fades, when the eye is in its ordinary stages of adaptation, into an indeterminate sort of whitish. The point to which the maximum luminosity shifts in curve *B*,

[1] *Colour Vision*, p. 103.

which corresponds to what is now well recognized as "rod vision" as distinguished from "cone vision," is marked *r* in the color diagram of Fig. 2. It becomes obvious, therefore, that in observing any very faint objects, irrespective of their real colors, the tendency is to see them of faded blue green merging into a whitish hue, or at least to rob them in very large measure of red and yellow tints, the latter condition marking the transition stage between cone and rod vision. This fact is sufficient cause for the progressive blanching of faint stars and also for the bluish tints which Smyth attributed to some of them. Inasmuch as from Pickering's comparisons[1] a star even as bright as the fifth magnitude would be matched by a candle at a distance of about 5.26 km, it is obvious that even the light-gathering power of comparatively large telescopes leaves relatively small illumination to serve as stimulus, and in the observation of stars like faint doubles one is continuously working near the limits of vision, a fact which is especially significant when one realizes that most of the color observations upon doubles have actually been made with small instruments, say from 12 to 16 cm in aperture. A fatigue color impressed by a primary on a considerably fainter *comes* of which the color has been thus much modified cannot be expected to be, and is not, accurately complementary. Finally, as a powerful and very interesting modifying influence in observed colors one must reckon with the "dazzle tints," that is, the subjective colorations corresponding to positive after-images. These, which may and do occur even in faint illumination when the eye is properly adapted, may produce very extraordinary modifications of color and are chargeable with some of the most remarkable phenomena noted in double star observations. The whole subject of fatigue colors and dazzle tints is set forth in a masterly paper by Burch.[2] The sequence of dazzle tints runs through reddish hues to green and then to blue and violet. Such tints are disclosed after a brief glance at the sun, but the sequence as stated holds for very weak illuminations, as Burch showed in his later paper,[3] and terminates with violet, while abolition of the reddish dazzle tint under such circumstances takes of itself a longer time than

[1] *Harvard Annals*, **61**, 69, 1908.

[2] *Philosophical Transactions*, B, **191**, 1, 1900.

[3] *Proceedings of the Royal Society*, B, **76**, 199, 1905.

is generally allowed for resting the eye in any ordinary observations and the sequence finally and definitely terminates with the fading of the violet subjective hue after a period approaching two hours. Under extreme conditions Burch found extraordinary phases of temporary color-blindness produced by dazzling, so that, for instance, the *b* lines of the solar spectrum appeared in a field of bright red, bright green, or bright blue, as the case might be. Tiring the retina by light from the vicinity of the D lines affected both the red and green sensations, the effect of the positive dazzle tint being to give a purplish cast to the violet. This red dazzle tint, by the way, is well seen in the positive after-image of a yellow "flaming arc." As the reddish dazzle tint died out it was succeeded by a greenish one and the red end of the spectrum began to reappear. Under some circumstances the fatigue effects produced apparent weakening in portions of the spectrum, such as would look in very faint light like absorption bands, showing the extreme caution which must be exercised in the visual observation of very weak spectra when stray light is not absolutely excluded from the eye and the eye itself long adapted by complete darkness.

With these four causes of subjective coloration in coincident operation it is evident that truly complementary hues are likely to be the exception; and the predominance of the quasi-permanent effects due to fatigue and to dazzle tints, effects which disappear only after a few or many minutes of complete relief from the primary stimulus, shows the improbability of any disappearance of such causes of subjective color in the ordinary use of the occulting bar or in the course of lunar occultations. These facts also make it evident that in observing objects like double stars, assuming the colors to be subjective, brilliant colors normally appear only when the difference in magnitude between the components is considerable, so that the primary can impose its fatigue and dazzle tints upon the faded light of the *comes*. This condition is exactly what is found in actual double star work, that is, in most cases the colored *comes* is very much fainter than its primary; and where the difference is relatively small, say only a fraction of a magnitude, the colors are peculiarly shifty and uncertain, varying both in intensity and in hue as will be noted later.

In order to test the efficiency of these physiological causes in producing the colors of double stars, the writer proceeded to the investigation of these phenomena by means of artificial stars. To this end the apparatus of Fig. 4 was constructed. It consisted of a light-tight blackened wooden box, a meter long and about 10 by 13 cm in cross-section, shown at *A* of the figure. Midway of the box and extending to half its height inside was the hard rubber diaphragm *s*. The farther end of the box carried a photographic plate-holder *P*, in the upper part of which the septum had been cut through in a space of about 2 by 3 cm. Diffused light was thrown upon this plate-holder by a white matt paper surface *R*, about 20 cm from the plate-holder, illuminated by a gas burner at *L*, which could be varied

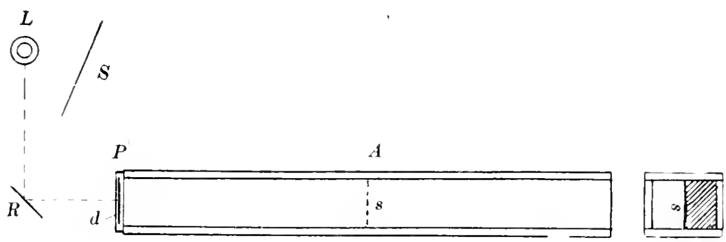

FIG. 4

in distance so as to make the complete distance to the plate-holder from 0.5 m to 1 m. The screen *S* cut off the light from the observer. In one of the slots of the plate-holder *P* were placed the necessary perforated diaphragms to produce the artificial stars, and across the upper half of the aperture in the septum there was provision for sliding a photographic wedge so as to cover one of the double star apertures in the diaphragm. Provision was also made for covering either or both of the apertures by tinted glasses of the same or different colors. The color of the light itself could also be modified by using a tinted diffuser at *R*. The purpose of the diaphragm *s* was to enable one of the stars to be occulted merely by dropping the eye a trifle in observing it. The apertures employed were holes through the hard rubber slide of the plate-holder of various sizes placed in various relations. In the preliminary experiments four sets of apertures were used: *A* and *B* 3.25 mm and 2.1 mm in diameter, respectively; apertures *C* and *D* each 3.25 mm in diameter; apertures *E* and *F*

each 1.6 mm in diameter, and apertures G and H each 0.8 mm. The uniform distance between the centers of these artificial stars was 9.8 mm, corresponding in angular separation therefore to a double star with a separation of 10″, as observed under a power of about 200. The writer should here state that he is possessed of absolutely normal color vision so far as pretty thorough testing in spectroscopic and other work has disclosed, and of about a normal degree of acuity. The photographic wedge used for cutting down the light of one of the components was made at the Harvard Observatory and is of the type regularly used in stellar photometric work there and elsewhere. These photographic wedges are well known from the investigations at the observatory to be free from selective color absorption. Two such wedges were used in these experiments, one ranging up to 18 stellar magnitudes of difference, that is, to practically complete opacity, and the other on a wider scale ranging up to 7 magnitudes. As a check upon these a double-image prism, and a thick block of Iceland spar producing the same effect, were used in conjunction with a Nicol prism as analyzer for cutting down the light of one of the components. The same color phenomena were noted with the polarizing devices as with the wedge, and the latter were used on account of their greater convenience and especially on account of the facility with which doubles differing in apparent size as well as intensity could be produced. Observations were conducted in a darkened room, lighted only by stray light from the working burner. A small slitless spectroscope by Hilger, used with or without a weak cylindrical eye lens, enabled the apparent spectra of the artificial doubles to be investigated. The prisms of this instrument were those customarily supplied by Hilger for small ocular spectroscopes for stellar work.

To summarize briefly the results of these experiments, they showed that merely cutting down the intensity of one of the elements of the artificial double invariably shifted its color progressively from the initial color through a variety of transition hues, sometimes complicated by the appropriate dazzle tints, to a distinctly greenish, bluish-green, or bluish color varying somewhat with the intensity of the illumination and with the whitish or yellowish casts given the stars, and sometimes reaching the glittering contrast found in doubles

like β *Cygni* and ϵ *Bootis*. When the *comes* is reduced to bare visibility, practically all color may be lost from complete failure of color sensation. Moreover, these subjective colors in the *comites* do not disappear on occulting the primaries, unless after a protracted rest of the eye in darkness. When such rest was taken, the colors were dim only for a brief period and came back quickly from simultaneous contrast and increasing fatigue.

Shifting the observation from the tired eye to the fresh eye changed the hue somewhat but did not abolish the subjective colors, which, perhaps from having become firmly impressed on the mind, were the more readily picked up by simultaneous contrast. On starting a set of observations with an absolutely fresh eye rested by darkness, the colors observed were usually weak, but by the time the eye had fairly settled down to close observation they were rapidly enhanced toward the usual results. A little exposure to light before beginning brought on the tints more quickly and sometimes they appeared almost at the first glance even when the eye was in the condition of rest, particularly if there was a slight initial color difference between the elements of the double, such as might exist between stars of type I and type II. Table III, culled from the original observations, gives a good idea of the normal effects.

In the case of the first set of readings it will be observed that the artificial stars were both weak in the blue on account of the yellowish source of light, and the colors were less conspicuous than in the later sets. In particular there were observed curious flickering effects of the smaller yellowish star, with an uncertain blue-green tint washed over it. This effect is common during the transition tints prior to the final contrast at the end of the experiment, and it was very noticeable here as always that occulting the primary produced no change in the color of the *comes* unless the eye was rested for some minutes. After this the length of time required to bring the *comes* to a different tint varied considerably, but it never came clear back to the color of the primary, since when very weak it would necessarily appear somewhat blue green on account of the shift in luminosity in weak light. Occulting the primary in such cases merely slowly weakens the fatigue and dazzle tints and eliminates whatever effect may be obtained from simultaneous contrast. With the spectroscope

TABLE III

NOVEMBER 14, 1909. SOURCE, GAS JET GIVING A CLEAR LIGHT YELLOW
APERTURES *E* AND *F*

Primary	Color	Δm	*Comes*	Color
E	Yellowish	1	F	Same a little dulled
E	Yellowish	2	F	Much paler tint
E	Yellowish	3	F	Yellowish with distinct greenish cast
E	Yellowish	4	F	Stronger cast, flickering and dubious
E	Yellowish	5	F	Distinct blue-green cast, shifty

Colors do not change in *comes* by occulting primary.

DECEMBER 5, 1909. SOURCE, FRESH WELSBACH MANTLE, WHITISH
APERTURES *E* AND *F*

E	Yellowish white	1	F	Paler and faintly greenish white
E	Yellowish white	2	F	Faintly bluish green
E	Yellowish white	3	F	Contrast same but stronger
E	Yellowish white	4	F	Bluish green, contrast striking
E	Yellowish white	5	F	Rather vivid blue green

E paled a little from fatigue toward the end and contrast then came out strongly with the other less fatigued eye. Colors remained on occultation.

DECEMBER 1, 1909. WELSBACH WITH LIGHT AMBER GLASS OVER PRIMARY
APERTURES *A* AND *B*

A	Yellowish	1	B	Slightly pale in hue
A	Yellowish	2	B	Distinctly whiter than A
A	Yellowish	3	B	Faint bluish cast evident
A	Yellowish	4	B	Stronger bluish, A looks yellower
A	Yellowish	5	B	Still stronger, decidedly blue
A	Yellowish	6	B	Contrast strong as in β *Cygni*

Color of *comes* has an odd flickering quality, now caerulean now merely dull violet, sometimes almost purplish. No change on occultation of A.

DECEMBER 15, 1909. WELSBACH WITH LIGHT BLUE GLASS, GOOD WHITE
APERTURES *E* AND *F*, *G* AND *H*

E	White	1	F	*Comes* has faint tinge of bluish
E	White	2	F	Same a little stronger
E	White	3	F	Almost violet blue
E	White	4	F	Stronger blue but duller
E	White	5	F	Very strong bluish
G	White	1	H	Faded greenish blue, washed effect, shifty color
G	White	3	H	Greenish blue, very distinct
G	Greenish white ?	5	H	Vivid blue spark

None of these colors changed by occulting primary.

the spectrum of the *comes* was very faint in the red end from weakening of the red sensation.

The second set of readings with a fresh Welsbach mantle as a source of light gave decidedly stronger effects, as might be expected. The subjective colors came on at a less difference of magnitude and were altogether more striking. The fact of fatigue was plainly shown in the primary by apparent paling in color. In the third set of readings the primary star was given a strong yellowish tint by a colored glass screen, and here the contrast in colors became ultimately very striking, quite as brilliant as in the case of *β Cygni*. A bright yellow primary easily turns a whiter *comes* blue, whether with real or artificial stars. During the transition period when the colors were beginning to appear, the usual curious uncertain quality of the color was very conspicuous and especially casts of violet and purplish, the necessary result of the dazzle tint of the brighter star. In the fourth set of readings both stars were screened with a very light blue glass, bringing up the color of the Welsbach to a good white. The effects were very brilliant and during the transition period there was the usual tendency to shifting color. These observations of Table III are typical of many, all leading to the same general results, which have been checked by five observers besides the writer. Color difference as a rule sets in at about $\Delta m = 1$, and increases progressively. As between one observer and another, differences in the names of colors may be noted, while the phenomena described are obviously the same. For example, one observer will call a dull yellow in the beginning of a series of observations fawn color and another brownish, or later in the series the *comes* may be called bluish or greenish, while the dazzle tints also come in with varying effect, so that lilac, violet, and purplish are about equally likely to be reported. But a series always begins at small difference of magnitude with a slightly varying tint of the primary color at the start when the eye is fresh, passes through at medium differences of magnitude the various transition tints, and progresses to green, blue, or sometimes violet of a rather striking character. Some of the most curious effects are obtained when a dazzle tint is superimposed on the dulled tint of the *comes*, producing shades of ruddy brown, purplish grey, violet grey, and lilac. These colors are extremely suggestive of

Struve's remarkable epithet, "olivacea subrubicunda," applied to the *comes* of ξ *Orionis*, which has been variously described by others as light purple and azure.

In fact the whole curious list of colors given at the beginning of this paper may be picked up in these observations on artificial stars before the difference in magnitude has become great enough and the eye sufficiently settled into its fatigued state to produce the final color contrast. A very little experience in observing these artificial doubles makes comprehensible the differences in color assigned to certain stars by various observers, the color seen depending merely on the condition of the retina as regards the effect of fatigue and dazzle tints, and the difference in magnitude between the components. For example, γ *Ceti*, with a difference of magnitude of 3.8 between primary and *comes*, has uniformly had its primary classified as yellowish, while the *comes* has been called "ashy," "tawny," "olive green," and "ruddy or dusky," by various observers. "Ashy" is a term confined in double star observations mostly to cases where there is considerable difference in magnitude and the *comes* is relatively very faint and apparently of a dull washed-out bluish color. "Tawny," "olive green," and "ruddy or dusky" are tints entirely typical of the varying superposition of the fatigue and dazzle tints which one readily gets with the artificial stars. The exact effect of these cannot be predicted, but it is always of the general type here described. All observers seem to get similar results, but in very varying degree according to the condition of the eye at the beginning of the experiments and its sensitiveness to slight color changes. The color contrasts being in this way a function of the condition of the eye, it is not to be wondered at that various observers of double stars have reported a great variety of tints, the exact thing seen in any given observation being dependent on what the eye has been doing previously; for instance on whether the observer has been reading his position circles by too bright a light or has been observing some brilliant pair just before shifting to a fainter one. A careful study of *Antares* and its companion would certainly show effect in other observations for some minutes.

Following all these observations on artificial doubles the writer extended his observations next to artificial triples with rather striking

results. To this end a slide was given a combination of three apertures, *A*, *B*, and *E*, of the previous list, *B* and *E* being about 1 cm above *A* and about 7 mm apart, so that when a photographic wedge covered these two apertures the difference in magnitude produced was about 1.1. Thus appeared a primary star below, and two *comites* differing in magnitude above. Observation of this artificial triple star gave some rather beautiful color effects, the *E* aperture farthest along on the photographic wedge taking on its full bluish cast, while the *B* aperture was still in a transition stage and showing a variable wash of color. For instance, when the primary was a good yellow white, *B*, two magnitudes fainter, would show the merest trace of a blue wash, while *E*, 1.1 magnitudes fainter still, had come to a bluish washed with purple. The exact hue observed varied from experiment to experiment, but always ended with *E* a strong blue or blue violet, while *B* was weakly bluish, or bluish green, or faint purplish. The effect varied a little as between observer and observer, but the general results remained the same, and occulting the primary changed neither of the others in color until the eye had had a liberal time in which to recover. Another set of apertures differing slightly in arrangement and size gave entirely concordant results.

Finally an experiment was devised to see the possible effect of subjective coloration in the observation of star clusters. For example, Sir John Herschel described the loose cluster κ *Crucis* as like a gorgeous piece of jewelry in the various colorations of its components, blue, green, and reddish tints being most conspicuous. If one must consider the colors in double and triple stars as almost wholly subjective and involving in reality only such small color differences as exist between first-type and the second- and third-type stars, then there is a strong inherent probability that colors thus reported in clusters are also largely subjective, and in fact subsequent observers have been seldom able to see them as Herschel described them. An artificial cluster was therefore constructed consisting of 16 apertures of various sizes in a space of a little less then 2 cm square. The apertures were merely pin holes of various sizes. They were covered to reduce the light by a bit of thin white tissue paper and three large and centrally located apertures were then carried through the tissue

paper. The cluster thus presented three bright stars and 13 fainter ones. The result when this combination was put in the plate-holder of the star-box was absolutely startling; the large apertures were yellow white while the smaller ones varied greatly in color, running from dull yellow through greenish, violet, and purplish tinges according to the condition of their illumination and the state of the retina. A second similar cluster containing 19 apertures varying from 0.15 to 0.75 mm in diameter was then constructed, the apertures being covered with white tissue paper as before, with three of the largest apertures continued clear through the tissue. Over one of these large apertures was placed a shred of orange tissue paper to give a genuine initial tinge to one of the stars in the cluster. With this addition the effects were even more brilliant than before, the single orange star acting as a sort of *agent provocateur* to start the rest of the group into parti-colored activity. Viewed in the star-box the two brightest apertures were a good yellowish white, a third, the fine orange given by the tissue paper, and the others varied according to conditions through yellowish, purplish, bluish, and greenish hues, the small stars being as a rule conspicuously blue or green and with weakened light growing almost colorless. This result was checked by three observers besides the writer and while no two would have described the individual stars in exactly the same terms, save for the three large ones, each saw substantially the same thing, that is, the small stars were of some shade of blue or green, and the medium-sized stars yellowish occasionally running to purple or ruddy tints. From these experiments it appears that not only in the case of doubles, but in the case of multiple stars and in clusters, subjective colors play the chief, if not the only, rôle, in determining the apparent tints observed. The colors seen in double and multiple artificial stars of known equality or approximate equality of colors are quite sufficient in degree and in kind to account for the double star colors which have been reported in astronomical literature. Some initial color differences there certainly are, as the spectra show, but the actual colors do not vary enough to account in any material degree for the strange hues which have been reported. The subjective colors arising from the causes here set forth are, however, fully adequate to account not only for the extremely great differences in color reported, but

for the curious and evanescent tints which have so put to the test the descriptive powers of those who have noted them. The rôle of the dazzle tints is particularly noteworthy as bearing on the roseate, lilac, and purplish hues never observed in isolated stars and very far from affording complementary tints to their primaries. An extension of these results to the remarkable cases of apparent variations in the colors of double stars is now under way and will be reported later. The writer's particular thanks are due to Professor E. C. Pickering for friendly interest in this investigation and for the resources he has kindly placed at the writer's disposal.

Boston
February 3, 1910

ON CERTAIN STATISTICAL DATA WHICH MAY BE VALUABLE IN THE CLASSIFICATION OF THE STARS IN THE ORDER OF THEIR EVOLUTION[1]

By J. C. KAPTEYN[2]

Frost and Adams have brought to light the very significant fact that the helium stars have exceptionally low peculiar[3] linear velocities. This property of the bodies which are generally considered as representing the earliest stage in stellar evolution seems to be particularly promising as an aid in fixing the successive stages in the life-history of the stars.

For it seems to indicate that, as a rule, the velocity of the star's motion increases with age and, this being so, we conclude that the helium stars must owe their origin to heavenly bodies having still lower velocities. Here then we have at once a test for the theory which assumes that the stars are evolved from nebulae. We know little about the velocities of the nebulae; but, as will be seen below, what we do know concerning them points to a very considerable linear motion instead of a vanishing one.

Have we to conclude that the nebulae are *not* the parents of the stars? With one exception the nebulae whose radial velocities have been measured belong to the class of planetary nebulae. For the one exception, the *Orion* nebula, the peculiar velocity is vanishing. We should conclude, therefore, that the planetary nebulae do not stand at the beginning of the series, but rather at the end. The fact that the changes in the light of the new stars finally make their spectra identical with those of the planetary nebulae seems not unfavorable to such a conclusion.

As will be seen below, the radial velocity of these objects exceeds that of the helium stars to such an extent that, even notwithstanding the very restricted number of objects measured, our conclusion seems

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 45.

[2] Research Associate of the Mount Wilson Solar Observatory.

[3] In accordance with custom we call *peculiar* velocity the velocity freed from that part which is due to the motion of the solar system through space.

already pretty well established. We cannot invoke, for the explanation of the displacement of the spectral lines, causes other than the Doppler effect, because the magnitude and the sign of these displacements are altogether different for the different objects.

On the other hand, the irregular nebulae *may* prove to be the birthplace of the stars. The case of the *Orion* nebula is decidedly favorable to this view, which harmonizes well with what we know from independent evidence. Still, it is clear how important must be the attempt to collect sufficient data for this and for other classes of nebulae—the white nebulae, the spirals, and the nebulous stars.

The importance of radial velocities for purposes of classification must not be restricted to the beginning of the series. Using all the data that I found it possible to collect, I obtained the following table of average values for the radial peculiar velocities, these velocities being all taken with the positive sign. The main value of the table lies in the fact that the number of *Orion* stars is relatively so great. This is owing to the courtesy of Professor Frost, who kindly placed the results obtained at the Yerkes Observatory at my disposal, so far as they rest on two or, with a few exceptions, on three or more plates.

Stars whose velocities were measured or published only because the astronomical or radial proper motion was exceptionally large were excluded. Such cases must vitiate the results. Spectroscopic doubles for which the velocity of the center of mass is well determined were included. The correction for the sun's motion was made on the assumption that the co-ordinates of the apex are

$$\alpha=269^\circ\!.7, \qquad \delta=+30^\circ\!.8 \ (1875.0)$$

and the linear velocity of the sun 20.0 km per second.[1] The spectra are given in the well-known Harvard notation.

If we combine the results of all the second-type stars (F to K5), we find a regular increase in the peculiar radial velocity when we pass from the *Orion* (B) to the Sirian (A) stars, from these to the second-type stars (F to K), and, finally, to those of the third type (M); that is, if we follow what, according to our present knowledge, must be

[1] For the *Orion* stars the velocity given is computed with the sun's velocity equal to 23.7 km, a value derived from these stars themselves. With the sun's velocity equal to 20 km we should have found 6.9 km for the average radial velocity.

considered as the most probable succession of the different classes. The difference between the Sirian and the second-type stars is small. We have, however, further evidence that it must be real. It is true that Campbell found rather the reverse from his 280 stars.[1] But his classification is avowedly a very crude one. In fact he did not use any direct data relating to the spectra, but arranged his stars according to the amount of the difference, *visual magnitude – photographic magnitude*. These results need not be given any considerable weight, therefore, in the present instance.

Spectrum or Object	Peculiar Radial Velocity	Number
B to B9	6.5 km	64
A to A5	12.6 (11.2)	18
F to F8	14.5	17
G to G5	12.6 } 14.5	26
K to K5	15.4	55
Ma	19.3	6
Planetary nebula	26.8	13
Orion nebula	0.1	1
N	13.1	8
L	3.7	2
Total		210

On the other hand, we have: First. all the stars, not spectroscopic binaries, the velocities of which were measured or published only because the motion is exceptionally large, belong to the classes G and K—not a single one either to class B or to class A. The total number of these is only nine, but I think that later observations have corroborated the fact. Second, we have pretty strong evidence furnished by the astronomical proper motions. From these I have derived[2] the linear velocities projected on a plane, with the following results:

Stars of first type and unclassified stars	$1.429\,h$
Stars of second type	$1.494\,h$

where h is the linear velocity of the solar system. The difference, though in the expected direction, is small. At a later period, however, I separated the stars with unknown spectra from the rest and at the

[1] *Astrophysical Journal*, **13**, 84, 1901.

[2] *Astronomische Nachrichten*, **146**, 97, 1898.

same time followed a somewhat safer method in forming averages. I then found:

Type II (F to K)	1.46 *h*	(1093 stars)
Type I (B, A)	1.02 *h*	(1144 stars)
Type unknown	1.45 *h*	(381 stars)

A considerable number of the nonclassified stars are probably of the first type. It is likely, therefore, that if we knew the spectra of these stars the difference between the first two values would be diminished somewhat. We have to consider further that among the type I stars have been included a few helium stars. For the A stars separately we would thus have found a somewhat greater value. As, however, the number of B stars is only one-eighteenth part of that of the A stars, the change must be small. On the other hand, the influence of the errors of observation, which are much greater for the A stars owing to the smallness of their proper motions, must have been to diminish the difference between the two types.

In conclusion, I think that the evidence of the astronomical proper motions leads us to the belief that the ratio

$$\frac{\text{average linear velocity of the F, G, K stars}}{\text{average linear velocity of the A stars}}$$

cannot be smaller than 1.3. If we accept this value from the radial velocity 14.5 km for the stars of the second type (see table), which must be fairly well determined, we derive for the Sirian stars a velocity of 11.2 km. Thus there seems to be good reason for admitting that the velocities increase regularly if we pass from the B stars to the A stars, and from these to the stars of the second type. Does the velocity still increase when we pass from the second-type stars to those of the third? The six stars of this latter type for which I have been able to find radial velocities give indeed the considerably greater value 19.3 km. But of course the number of stars is too small to allow of any safe conclusion.

The above table is both incomplete and provisional. The time cannot be far distant, however, when we shall have the means of establishing it on an adequate number of observations. When this has become the case may we not hope that such questions as the following may be settled, or at least be brought much nearer to their

solution? Do the fourth-type stars (N) represent a later or an earlier stage than those of the third? Do the Wolf-Rayet stars represent an early or a late stage? Do they stand at all near the planetary nebulae, as Pickering thinks, or are they more nearly related to the helium stars? Are the *novae* in their last stage really to be considered as ordinary planetary nebulae? What is the place of the helium stars showing bright lines? And such more general questions, as the following: Is the order of evolution, O, M, (G), A, (G), N,[1] as proposed by Lockyer, tenable?

The phenomenon of the increase of velocity with the evolutional stage of the stars must give rise to speculation as to its cause. The observational results contained in our table naturally lead us to conclude that the matter from which the stars originate must have little or no velocity. How is this possible under the influence of the combined attraction of the rest of the system? Is it not *as if*[2] gravitation had no effect on the cosmical matter in its primordial state? If this be so, then no relative motions of its several parts will arise: they will remain at rest relatively to their common center of gravity, that is, relative to the same center to which the velocities in our table are referred. As soon as matter changes from this state to another in which gravity begins to act, or to act freely, motion will arise; and it is evident that, as a rule, this motion must be accelerated, at least during immense periods, so that, the longer the period elapsed since the birth of the stars, the greater must be their average velocity, which is just what we find to be the case.

It may be well to direct attention to other phenomena by which

[1] Some of the G stars would be on the ascending, others on the descending branch of the curve. The data already available seem hardly reconcilable with Lockyer's series.

[2] We need not necessarily make the hypothesis that really primordial matter is not subject to gravitation. If, for instance, as was suggested to me by a friend, the tenuity of this matter were such that it were very materially hindered in its motion by the matter which we must assume as filling the universe in order to explain the phenomenon of selective absorption of light recently found, the velocity of this matter could not exceed the value for which the resistance is equal to the total attraction. We have only to assume that this is the case for a relatively low value of the velocity. Other suppositions may probably be made of forces which, in the primordial state of matter, counteract gravity. But it is evident that in such cases, where gravity is just counterbalanced by another force, things happen *as if* there were no force at all.

we are driven to a similar conclusion. In the *Hyades*, the *Pleiades*, *Ursa Major*, etc., we have physical groups of stars, the motions of which are parallel and equal in amount. It is certain that, under the influence of the mutual attraction of the members of the groups, and in part also under the influence of the attraction of other stars, this parallelism and equality cannot continue to exist indefinitely. The time must inevitably come when they will be so thoroughly destroyed that no appreciable trace of a community of motion will be left.

But if it be true that, under existing circumstances, this community of proper motion can exist only temporarily, how does it happen that it exists at all, that it was not destroyed long ago? The only answer to this question at all satisfactory that I can find is, in accordance with the conclusion to which we were led a moment ago, that there has been a time when the matter now composing such groups was in a state in which things happened *as if* gravitation had no effect on it. For a group such as that of the *Hyades*, for which the parallax is now known with fair accuracy, and in which, according to a private communication[1] from Professor Frost, the number of spectroscopic binaries is particularly great, so that data concerning the matter will soon be available, we shall be able to determine roughly the time necessary to produce internal motions in the group of an amount equal to that which the observations allow us to assume as possibly now existing. This interval will be the maximum interval during which the system can have existed abandoned to the normal and unchecked action of mutual gravitation.[2]

Again, the same question is presented to us, on a larger scale, by the stellar system as a whole. The members of the two star-streams, which seem largely to constitute the system, must, by their mutual attraction, destroy in the long run every trace of the now existing regularity of the motions. That the stream motion is still recognizable at the present time must be due to the fact that the perturbing forces have not effectually worked for an indefinite time. The star-

[1] See also *Astrophysical Journal*, **29**, 237, 1909, and **31**, 178, 1910.

[2] I have already made, some time since, an estimate based on what seemed to me a plausible hypothesis as to the matter. Now that we have the prospect of getting better data soon, I prefer to suppress my provisional results.

streams, too, therefore point to a time when gravitation apparently or really had no effect. Another phenomenon might finally be mentioned in this connection, to which several astronomers have already called attention, namely, the fact that the Milky Way has not long since been dispersed. But I think that enough has already been said to justify our point of view, and I prefer to devote the rest of this paper to the consideration of elements other than the linear velocities of the stars, which may render service in finding the position of certain classes of stars in the order of evolution.

It seems probable that the average *absolute* magnitudes are different for stars of different spectral classes, and that they will fall into a smooth curve when the stars are arranged in the proper order. Attempts have already been made to derive such average absolute magnitudes. I think, however, that such determinations must be illusory, at least as long as we have to deal with classes of stars, part of which our observations cannot reach. The intrinsically faint stars of every class must, as a rule, be found among the stars of faint *apparent* magnitude, and the spectra of these have not yet been classified. It seems that there is no hope of a really good determination of the average absolute magnitude of any spectral class before we can include stars of every class of apparent magnitude that furnishes specimens of the spectral class in question. There is, besides, the difficulty of the parallaxes; and the use of average parallaxes, which are more easily obtained, is highly objectionable for the present purpose. The outlook in this direction is therefore not promising.

As long as our data concerning radial velocities are so scanty, some help may be obtained from the astronomical proper motions.[1] In the main, however, the varying amount of these motions must be due to differences in distance, so that what they can teach us for the present question cannot be compared to what we may hope to learn, in the near future, from the radial motions.

There are other elements which seem more promising, though they need confirmation by further observations. In the first place: The observations at the Yerkes Observatory have brought to light an unexpectedly large number of spectroscopic binaries among the *Orion* stars. The percentage of the stars known to be spectroscopically

[1] *Astrophysical Journal*, 30, 173, 1909.

double is far higher for this class than for any other type. Now it is difficult to imagine how binaries in their evolution from the earlier *Orion* stage to the later Sirian . . , solar . . types can, in great part, become single stars. It seems much more satisfactory to assume that the higher proportion of binaries among the *Orion* stars must be apparent and due to some circumstance which makes the discovery easier for this class of objects. Such a circumstance would be given should it appear that the periods are longer for the older types. For the longer the period, the longer the binary character of the star may escape notice. The data at present available indeed show such an increase in the period very clearly. From Campbell's list of spectroscopic binaries,[1] to which I have added 15 binaries whose orbits have subsequently been determined,[2] I find:

Period	Number of Stars			Percentages		
	B, A	F, G	K, M	B, A	F, G	K, M
0^d to 10^d	22	14	0	65	41	0
$>10^d$	12	10	4	35	59	100

For a great number of stars, for which the period cannot yet be determined accurately, we can still judge from the observations as to whether it will turn out to be long or short. According to Campbell's catalogue we thus have in addition to the above:

Period	Number of Stars			Percentages		
	B, A	F, G	K, M	B, A	F, G	K, M
Short	18	3	1	90	33	17
Long	2	6	5	10	67	83

We thus see, as a simple result of the observation of a very moderate number of stars, that there can be no reasonable doubt as to the reality of the phenomenon. On the other hand, we know that theory demands a lengthening of the period under the influence of tidal attraction. Thus, I think, we may confidently look for a confirmation of our conclusion by further observations and computations.

There is another fact which may very naturally be explained by the present phenomenon, viz., the fact that, so far as we know, all the *Algol* variables belong to the first type. For it is evident that the chances of an eclipse are enormously diminished as soon as the period, consequently the distance, of the components increases.

[1] *Lick Observatory Bulletin*, No. 79, 1905.

[2] Visual binaries have been excluded.

In the second place, we have the above-mentioned gradual dissipation of the star-streams. According to the views here put forward, the cause of obliteration of the streams must have been longer at work for the stars of the later types, so that, for the younger spectral types, the phenomenon of star-streaming must show itself in greater purity than for the later ones.

We have already observational evidence that this is so. In his second paper on the systematic motion of the stars[1] Dyson finds that the stars of type I "diverge less from the general drift of the streams than the other stars." From Dyson's numbers I find for the average deviation of the first-type stars only three-quarters of that for all stars together.[2] Not only this. Observation shows further that for the *Orion* stars the stream velocity is small. This is already implied by the fact that the total motions of the *Orion* stars are small. But as the point seems important I have derived, on the basis of Eddington's theory, the values of both the cloud velocity and the average internal velocity[3] as far as they are obtainable from the radial velocities alone. For the apex of the sun's motion were assumed the co-ordinates

$$\alpha = 269^\circ.7, \qquad \delta = +30^\circ.8 \quad (1875.0).$$

The supposition that the sun's velocity is 23.7 km per second leads to a value of the average radial motion ($\bar{\rho}$) which near the vertex is actually slightly smaller than that at a greater distance (see footnote). This is incompatible with the two star-stream theory. The difference, however, is far within the limits of the errors of observation, and in making the computation I took the two separate averages both equal to the total average ($\bar{\rho}=6.63$). I thus found for the *Orion* stars:

[1] *Proceedings of the Royal Society of Edinburgh*, **29**, Part IV, 390.

[2] Deviations exceeding 60° being excluded in both cases.

[3] As is well known, Eddington's theory of the star-streams assumes two star-clouds in each of which the velocities are distributed according to Maxwell's law. I here simply assume the number of stars in the two clouds to be the same: the velocity of the center of gravity of the two clouds in respect to the center of gravity of the whole of the two clouds together will then be equal and opposite. This direction will meet the celestial sphere in what I called the true vertices; the velocity itself may be called the *cloud* or *stream velocity*, whereas the velocity of the individual stars with respect to the center of gravity of the cloud to which they belong may be called the *internal velocity*.

Let ω = cloud velocity;

$Ae^{-h^2(t^2+u^2+v^2)}dt\,du\,dv$ = number of stars having components of internal velocity between t, u, v and $t+dt$, $u+du$, $v+dv$, respectively;

Sun's Velocity	Cloud Velocity	Average Internal Velocity
20.0 km	5.8 km	12.1 km
23.7 km	0.0 km	13.3 km

These numbers must be compared with those obtained from all the stars, without distinction as to spectral type. These I derived from the astronomical proper motions in Eddington's paper. I find, again assuming an equal number of stars in the two clouds, and the sun's velocity as 20.0 km:[1]

Cloud velocity = 25.6 km,
Average internal velocity = 31.2 km.

λ' = angular distance of star from vertex;

$\bar{\rho}$ = average radial velocity, freed from the sun's motion through space, all the velocities being counted positive.

Then I find

$$\bar{\rho}=\frac{2h}{\sqrt{\pi}}\,\omega\cos\lambda'\int_0^{\omega\cos\lambda'}e^{-h^2x^2}dx+\frac{1}{h\sqrt{\pi}}e^{-h^2\omega^2\cos^2\lambda'}\qquad(a)$$

If $\bar{\rho}$ is known by observation for two widely different values of λ', this equation will furnish the values of ω and h. From this we get the average internal velocity Ω by the formula (given by Eddington):

$$\Omega=\frac{2}{h\sqrt{\pi}}\qquad(b)$$

If from the *Orion* stars used for the table I exclude the two whose spectra are peculiar, I get:

$\cos\lambda'$	Average $\cos\lambda'$	Average $\cos^2\lambda'$	$\bar{\rho}_1$	$\bar{\rho}_2$	No. of Stars
0.00 to 0.71	0.367	0.186	6.20	6.76	31
0.71 to 1.00	0.895	0.808	7.40	6.50	31

where $\bar{\rho}_1$ is computed on the supposition that the sun's velocity is 20 km per second; $\bar{\rho}_2$ on the supposition that the velocity is 23.7 km, as found from the *Orion* stars themselves. From the values $\bar{\rho}_1$ and $\bar{\rho}_2$ the values of ω and Ω were then obtained by the formulae (*a*) and (*b*). I take occasion to state in passing that in my own theory (not yet published) the formula which takes the place of formula (*a*) is

$$\bar{\rho}=\frac{\pi}{2h^4}(2a-b\sin^2\lambda')\qquad(c)$$

which with the values derived for the constants h, a, b, and the sun's velocity as 20.0 km per second becomes:

$$\bar{\rho}=25.07-12.38\sin^2\lambda'\qquad(d)$$

It was with this formula that the theoretical velocities were computed in my paper in the *British Association Report* for 1905.

[1] In accordance with Eddington's data (*Monthly Notices*, **67**, 34–63, 1907) I took
Co-ordinates of point Q_1, $6^h\ 0^m, -14°$; drift velocity, 1.65;
Co-ordinates of point Q_2, $19^h\ 20^m, -58°$; drift velocity, 0.50;
from which I derive
Co-ordinates of apex of sun's motion, $17^h\ 43^m$, $+31°.5$;
Co-ordinates of true vertices, $6^h\ 11^m$, $+1°$, and $18^h\ 11^m$, $-1°$;
Sun's velocity $= 0.705/h$, which if equal to 20.0 km gives $h = 0.0361$;
Cloud velocity $= 0.922/h = 25.6$ km.

Though the uncertainty in the amount of the cloud motion for the *Orion* stars is considerable, there cannot be the slightest doubt but that both the cloud motion and the internal motion must be very small as compared with the corresponding quantities for the rest of the stars.

Apart from the advantages that we may derive from this result for the classification of the stars in the order of their evolution, it has, I think, a great importance in its bearing upon the question of the generation of the star-streams themselves. For it proves that the streaming motion, too, is not an initial motion, but one generated at an epoch which, for the stars of any one type, must be placed at a time relatively but little preceding the time when they passed through the *Orion*-type stage.

In a lecture delivered before the Holland Society of Science on May 19, 1906,[1] the following questions were put:

> Have we to imagine that originally space was traversed by two independent star-clouds, similar to those clouds which we know, on a smaller scale, in the meteoric streams; that these clouds have met at some time in the remote past and have penetrated each other; and finally that the divergence in the directions and velocities have been caused by mutual attractions, both between the members of one and the same cloud and by the members of the two streams on each other?
>
> Or have we to deal with clouds which were not originally independent, so that the observed motions are to be attributed simply to the original form and the distribution of the star-density in the nebula that we call the stellar system?

In the light of the present discussion I think that the first of these two suppositions will have to be given up, so that the second thus gains immensely in probability. Howsoever this be, a more thorough investigation of the star-streams, *separately for stars of different spectral class*, seems highly desirable and promising, and we have placed such an investigation on the working program of the Groningen Astronomical Laboratory.

Leaving aside the more or less theoretical considerations, we may sum up what has been said as follows:

For the classification of the stars in the order of their evolution

[1] *Archives Neerlandaises*, Series II, Tome XI, pp. liii and liv.

the following, to be determined separately for the stars of each of the spectral classes, may be of great help:

1. The average amount of the radial velocity.
2. The average period of the spectroscopic binaries.
3. The average amount of the divergence of the astronomical proper motions from the general stream motions.
4. The quantity of the stream motions.

GRONINGEN
January 1910

THE ABSORPTION OF STAR LIGHT CONSIDERED WITH RELATION TO THE GALAXY

By GEORGE C. COMSTOCK

It appears to be well established at the present time that the major part of the lucid and brighter telescopic stars constitute two great groups, interpenetrating and flowing one through the other, and there is evidence indicating that the space occupied by these star groups is in some measure permeated by sparsely diffused meteoric matter, whose individual particles are of very small size and mass, but whose combined effect is to render less transparent the region that they occupy. If such be the case the motion of the two star groups must from time to time bring particles of the meteoric matter within the sphere of attraction of some particular star and impart to each such particle motion in a conic section having the star at its focus. The orbits thus formed must be predominantly hyperbolas and if either star group be conceived as having finite dimensions, the motion thus imparted to the meteoric particle must ultimately carry it outside this group without appreciably disturbing the arrangement of the group. There is thus a continuing tendency to sweep clear the space along the line of motion of the star groups and to produce in this direction a region of maximum transparency and therefore of greater apparent richness in stars. If one of the groups, e.g., the smaller one, be conceived to have the form of a widely extended and relatively thin stratum, the space swept clear will have a similar form and will mark upon the sky, as seen from any point near the center of the group, a luminous great circle whose plane passes through the direction of relative motion of the two groups. The galaxy is such a great circle and I have elsewhere indicated[1] that many of its features, not otherwise explained, result immediately from this conception of its nature. It is, however, fundamental to such a concept of the Milky Way that the absorption of star light should here be a minimum and the concept loses in probability if

[1] *Popular Astronomy*, **17**, 339, 1909.

such is not the case, although the star stratum would of itself tend to produce the appearance of a galactic ring, even though no absorption effect were present in sensible amount.

Directly pertinent to this matter is Kapteyn's extremely interesting research on "The Absorption of Light in Space," appearing in the *Astrophysical Journal* for November 1909, in which (**30**, 306) it is set forth that: "There is thus no reason for the supposition that the selective loss of light is different for galactic and extra-galactic regions. In particular we cannot explain the greater richness in stars of the Milky Way by a smaller space absorption for, if anything, this absorption is greater there than elsewhere." This result is obtained from a discussion of the discordances between the Harvard star magnitudes photographically and visually derived, and rests upon the assumption that if there be a sensible space absorption its effect must be apparent in a difference between the photographic and visual magnitudes, which difference will systematically increase with increasing distance of the stars observed. Classifying his material with respect to the source from which it is derived (Miss Cannon, Miss Maury), with respect to type of spectrum and to position near to or remote from the galaxy, Kapteyn makes a considerable number of determinations of the value of the coefficient *d* that serves as an index to the amount of absorption suffered by star light in transmission. In the mean these furnish a slightly greater value of *d* for galactic than for extra-galactic stars and this disparity is the basis for the conclusion above quoted.

To this treatment of the data it may be objected that if the galaxy is a stratum relatively free from absorbing matter, evidence of its character in this respect can be found only by comparing stars lying within the stratum with those outside it. Galactic latitude, which is the criterion used by Kapteyn to discriminate between galactic and extra-galactic stars, suffices for this purpose only when the stars are known to be very remote, and this condition does not generally obtain in Kapteyn's data since they relate for the most part to lucid stars. Thus, an average fifth-magnitude star situated in any galactic latitude may, and in many cases will, lie within the galactic stratum itself and its light will suffer as little absorption as does that of a star in the median line of the galaxy. Whatever view may be enter-

tained as to the nature of the galaxy, it seems evident that the effects of absorption of light will be more readily apparent in the case of distant stars than in those nearer to the earth, and in order that due weight may be given to this consideration, I have arranged Kapteyn's results in the following table, in which the first column shows the authority for the data, the second the spectral type of the stars employed, the third the mean magnitude of the stars, the fourth and fifth the mean centennial proper motion of the extra-galactic and galactic stars respectively, and the sixth the difference between the absorption coefficient found by Kapteyn for galactic and for extra-galactic stars of the given spectral type. These differences are so taken that a positive sign indicates an excess of absorption outside the galaxy. The last column shows the relative weights of the several determinations of $d_e - d_g$ computed from Kapteyn's weights for the individual quantities, depending chiefly upon the number of stars included in each group.

Authority	Spectrum	m	μ_e	μ_g	$d_e - d_g$	p
C.	B_3	5.1	2″.2	2″.0	+0.0052	1.6
C.	B_5	5.0	2.5	2.3	+ 29	0.9
C.	B_8	5.2	3.1	2.6	+ 131	2.7
C.	B_9	5.3	3.6	2.4	+ 10	1.0
M.	A	4.3	7.6	4.5	+ 16	1.5
C.	A	5.2	5.7	4.6	+ 35	7.5
M.	A	4.9	9.2	5.9	+ 126	1.2
C.	A_2	5.4	6.2	4.6	− 5	8.2
M.	A_2	4.7	10.3	5.4	+ 28	0.6
C.	A_3	5.0	6.8	6.0	− 447	1.2
C.	A_5	5.0	9.8	6.6	+ 30	2.5
C.	F	5.2	14.3	8.4	+ 292	1.3
C.	G_5	5.0	34.7	30.5	− 98	2.7
C.	K	5.0	13.3	11.9	− 132*	13.9
M.	K	4.2	17.8	22.3	− 36	3.3

* Kapteyn questions the legitimacy of the data upon which this quantity depends.

If the average proper motions, μ, can be considered as an index of relative distance, the beginning of the table corresponds to remote stars, e.g., from five to ten times as distant as those represented at the end of the table, and contains therefore that part of the data best adapted to a determination of the quantity in question. This fact is not taken into account in the weights, p, assigned to the several results, and in fact Kapteyn, in deriving mean values of d, arbitrarily

diminishes the weight assigned to the quantities at the head of the table, thus in part rejecting the best evidence and relying upon the worst. This appears inadmissible, and taking the data as they stand, they seem better interpreted as follows: Wherever the material is at all well adapted to show a possibly existing difference between galactic and extra-galactic absorption (e.g., the first half of the table), it consistently shows by the sequence of plus signs an excess of absorption outside the galaxy. Where it is less well adapted to the end in view (e.g., the last half of the table), the results are conflicting and subject to such wide accidental variation that little reliance can be placed upon their mean value.

One might expect, a priori, that the results obtained would present this general character in respect of internal consistency, if the absorption outside the galactic stratum were really greater than within it, but the unbroken sequence of positive values of this difference presented by the more distant stars appears somewhat remarkable in view of the character of the data employed. To adopt this sequence of values as substantial proof of a diminished absorption of light within the galaxy is perhaps premature, but to ignore it and to draw the converse conclusion from the discordant testimony of the nearer stars is surely even less warranted. Whatever probative force the data may possess tends away from rather than toward Kapteyn's conclusions above quoted, and appears to render that conclusion entirely untenable, so far, at least, as the present data are concerned.

WASHBURN OBSERVATORY
MADISON, WISCONSIN
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REVIEWS

Temperaturbestimmung von 109 helleren Sternen aus spectralphotometrischen Beobachtungen. Von J. WILSING und J. SCHEINER. Publikationen des Astrophysikalischen Observatoriums zu Potsdam, Nr. 56, Bd. XIX, I. Leipzig: W. Engelmann, 1909. M. 15.

This notable publication includes spectral-photometric measurements of the sun and of 109 stars of magnitudes 4.2 and brighter, classified as of Vogel's types Ia1, Ia2, Ia3–IIa, Ib, IIa, IIa–IIIa, IIIa. The stellar spectra were compared in brightness at wave-lengths 0.448, 0.480, 0.513, 0.584, and 0.638 μ with the spectrum of an electric glow-lamp, itself compared with the "black" radiation of an electrically heated Heraeus oven of about 1500° absolute temperature. Assuming the applicability to the stars of Planck's formula for the distribution of radiation in the spectrum of a "black" body, the authors have computed from these data the temperatures of the sun and stars investigated.

Before discussing the results a few words may be devoted to the method of observation. A spectrometer with a flint glass prism was attached to the 80-cm refractor, thus giving a dispersion of 2°7 between $H\alpha$ and $H\gamma$. A corrector lens was used to reduce the chromatic aberration of the less refrangible rays. The stellar spectra were not broadened out for surface photometry, but diaphragms were introduced so as to select patches of spectrum at the desired wave-length, and these were treated as if they were stars to be compared in brightness by a Zöllner photometer with similar starlike patches of comparison spectrum. The latter spectrum was adjusted to equality in brightness with the stellar spectrum by means of a fixed and a rotating Nicol prism. For the brighter stars it was necessary to use means of reducing the intensity of the stellar spectrum. Numerous corrections were applied, among others for the absorption and reflection of the objective and corrector lenses; for the variation of focus; for atmospheric extinction as determined by G. Müller; for the current-strength in the glow-lamp; and for personal equation of the observers. The authors decide that the probable error for each wave-length of a final result of a spectrum comparison between a star and the glow-lamp, obtained as the mean of four observations by two observers on two separate nights, is

5.7 per cent. It is an interesting thing that there should have been no difference in probable error for the different wave-lengths, considering the great difference in sensitiveness of the eye in different parts of the spectrum and also the wide range of ratios of brightness in the stellar spectra. Numerous comparisons of the spectrum of the glow-lamp with that of the "black" body at determined temperatures showed a satisfactory degree of constancy of the glow-lamp, and yielded the relative intensities of its spectrum for the five wave-lengths observed, within a probable error estimated as not much exceeding 2.4 per cent. Hence, according to the conclusions of the authors, the probable error of a final determination of the brightness of a stellar spectrum for a given wave-length relative to its brightness at other wave-lengths would be about 6.2 per cent. In their observations of the sun the refractor was not used, but it was assumed in accordance with unpublished results that the sunlight reflected from a surface of chalk was unchanged in quality. Probably on account of the larger number of measurements the solar determination should be regarded as of somewhat greater accuracy than the others. By means of the spectral distributions determined by the five observed values for each star, the authors computed according to least squares by Planck's radiation formula the temperature of a "black" body nearest approximately the spectral distribution. They early found that the deviations of observed and computed distributions had a systematic course dependent on wave-length and not on temperature. This led them to investigate the spectral distribution in several terrestrial sources which might be regarded as approximately "black," and they found similar discrepancies. Accordingly they determined from the stellar observations themselves an empirical correction "Δ" of the following magnitudes for the several wave-lengths:

Wave-Length	$0^{\mu}.448$	$0^{\mu}.480$	$0^{\mu}.513$	$0^{\mu}.584$	$0^{\mu}.638$
Δ in percentages	$+7.9$	$+0.7$	-16.7	$+2.8$	$+4.5$

In the principal table beginning on p. 48 they give in column 3 the logarithms representing the directly observed spectral distribution, and in column 4 the logarithms corrected by the empirical quantity Δ. It is a great pity that the source of this supposed systematic error was not discovered, and its magnitude determined by laboratory experiments; for in the process of correction which involves the assumption that the spectra of the stars can be represented by Planck's formula, with temperatures of course determined from the same observations as the corrections, there is reasoning in a circle.

It is of interest to compare their mean result for the spectral distribution of the sun (S_λ) with that recently obtained by Abbot and Fowle. Unfortunately Wilsing and Scheiner do not give directly the solar spectrum intensities, but only their ratios (a_λ) to the intensity of the spectrum of glow-lamp No. 2 (E_λ) (glow-lamp No. 1, which was used for the stellar work, burned out before the solar work was done). From their statements on pp. 36, 38, 47, and 62, I take the following as the logarithms of the quantities to use to obtain the solar distribution:

Wave-Length	0.448	0.480	0.513	0.584	0.638
Log $\left(a_\lambda = \frac{S_\lambda \text{ sun}}{E_\lambda \text{ lamp}}\right)$....	0.696	0.398	0.081	9.577	9.251
Log E_λ (No. 2)..........	9.269	9.623	9.889	0.457	0.777
Log Δ_λ..................	9.967	9.997	0.067	9.988	9.981
I: Log $a_\lambda E_\lambda$..........	9.965	0.021	9.970	0.034	0.028
II: Log. $a_\lambda E_\lambda \Delta_\lambda$.......	9.932	0.018	0.037	0.022	0.009

On another hypothesis which the authors themselves tentatively adopted (see p. 45), I have considered the values on p. 38 from the standpoint that the work at $\lambda=0.513$ should be neglected. On this basis I have made two calculations, first, adopting the values just given for E_λ; and second, assuming that glow-lamp No. 2 was in fact just like glow-lamp No. 1 (see p. 33), but for some reason its comparison with the "black" body was in error. On this basis I find:

Wave-Length	0.448	0.480	0.584	0.638
Log a_λ...........................	0.696	0.398	9.577	9.251
Log E_λ (No. 2).....................	9.269	9.623	0.457	0.777
Log E'_λ (No. 1)....................	9.347	9.652	0.414	0.707
III: Log $a_\lambda E_\lambda$....................	0.965	0.021	0.034	0.028
IV: Log $a_\lambda E'_\lambda$....................	0.043	0.050	9.991	9.958

The work of Abbot and Fowle I take from unpublished spectrobolometric experiments at Mt. Wilson and Mt. Whitney of 1909, made with an especial view to determine the form of the solar energy-curve outside the atmosphere. In this work the scale of galvanometer deflections was uniform, and the corrections for change of sensitiveness of the bolometric apparatus, formerly so troublesome, were now almost entirely overcome. Various different optical arrangements were used, sometimes including five silvered mirrors and a flint glass or ultra-violet glass prism and sometimes with only two magnalium mirrors (no coelostat) and a quartz prism. The results are reduced to equality with each of the four Wilsing and Scheiner values at wave-length $0.448\,\mu$ as follows:

Wave-Length	0.448	0.480	0.513	0.584	0.638
W. and S. I	1000	1138	1012	1172	1156
W. and S. II	1000	1219	1274	1230	1104
W. and S. III	1000	1138	....	1172	1156
W. and S. IV	1000	1016	....	887	822
A. and F.	1000	1040	1000	893	800

We may suppose that Wilsing and Scheiner have employed substantially what is designated above as "W. and S. II" in their computation of the solar temperature, and they find $T = 5130° \pm 106°$ Absolute. This temperature, according to Wien's displacement law, would give a maximum of energy at $0.571\ \mu$. Abbot and Fowle's mean observed solar curve of 1909 gives its maximum at $0.460\ \mu$. It is clear that if the reviewer is correct in his understanding of Wilsing and Scheiner's solar work, there is a large discrepancy between their results and those of Abbot and Fowle, which, to be sure, would be sufficiently removed if the assumptions embodied in "W. and S. IV" above could be justified. If this last alternative could be accepted, whatever weight Abbot and Fowle's determination may have would go to confirm the accuracy of Wilsing and Scheiner's stellar observations, which were all made with glow-lamp No. 1. Otherwise, we must suppose one pair of observers or the other to be greatly in error.

To the reviewer, the reduction of the stellar spectral results to "black" body temperatures seems a by-product, rather than a principal result of the investigation worthy to have its place in the title; for, in the first place, it seems misleading to compute temperatures from a spectral range of only $0.2\ \mu$, whose distribution is fixed by five observations with probable errors of 6 per cent. each. Moreover, Planck's "black" body formula does not represent the distribution, even of the solar radiation, in all parts of the spectrum; nor ought it to be expected to, because the sun's spectrum is a composite spectrum: (1) the apparent temperature of the photosphere, as indicated by its energy spectrum, is wholly different at the limb from what it is at the center; (2) the apparent temperature at the center is probably a mean of a range of temperatures obtaining at different depths; (3) the violet end of the spectrum, from whatever part of the photosphere, is disproportionately weakened by selective absorption. It seems, therefore, of doubtful value to compute the temperature of the stars, in which these uncertainties are doubly uncertain, though possibly it might be worth while to make such computations as a mere curiosity. The values derived by many observers for solar temperatures seem chiefly interesting as they indicate the improbability of solids or liquids in the photosphere.

The main and highly valuable results of Wilsing and Scheiner's work, to

the reviewer, is contained in column 3 of the table beginning on p. 48. Its value would be powerfully enhanced if the uncertainties represented by the empirical correction and by the discrepancy between Wilsing and Scheiner and Abbot and Fowle could be removed. One regrets that *Capella* was not observed, because its spectrum is almost identical with that of the sun. For the following summary, the reviewer has ventured to omit the values at $\lambda = 0.513\,\mu$, and has computed the mean spectral distributions from column 3 (pp. 48 to 62) for four stars of each of the seven spectral classes investigated.

Type	Stars		Intensity			
			$\lambda\ 0^{\mu}.488$	$\lambda\ 0^{\mu}.480$	$\lambda\ 0^{\mu}.584$	$\lambda\ 0^{\mu}.638$
Ia1	*β Can. Min.*, *α Delphini*,	*12 Can. Ven.* *α Pegasi*	1000	836	579	505
Ia2	*α Androm.*, *γ Ophiuchi*,	*γ Coronae* *γ Lyrae*	1000	796	625	525
Ia3–IIa	*α Trianguli*, *δ Leonis*,	*ξ Geminorum* *δ Aquilae*	1000	948	902	845
Ib	*γ Pegasi*, *ρ Leonis*,	*η Leonis* *ζ Pegasi*	1000	887	578	530
IIa	*η Boötis*, *μ Herculis*,	*β Virginis* *γ Cygni*	1000	998	993	1005
IIa–IIIa	*α Arietis*, *δ Cancri*,	*ο Tauri* *β Ophiuchi*	1000	1205	1766	1897
IIIa	*α Orionis*, *χ Serpentis*,	*δ Virginis* *δ Sagittae*	1000	1368	3296	4406

Whatever may be the justification for the correction called by the authors Δ, or the uncertainty raised by the discrepant solar values, the combined magnitude of these possible systematic errors as compared with the changes between different spectral types as shown in the table just given is too small to nullify the very substantial value of the distinctions which this great piece of work now enables us to make in the spectral energy distribution for the various stars investigated.

C. G. Abbot

Les observations méridiennes. Par F. Boquet. Tome I, "Instruments et méthodes d'observation," pp. 314; figs. 96; Tome II, "Corrections instrumentales et équations personnelles," pp. 342; figs. 76. Paris: Octave Doin et Fils, 1909. Fr. 10.

These two volumes form part of the "Bibliothèque d'astronomie et de physique céleste" of the *Encyclopédie scientifique*, which is being publishedunder the direction of Dr. Toulouse.

The treatment of the subject is both theoretical and practical, and is adapted to the needs of the amateur as well as the professional astronomer.

Tome I deals with the fundamental conceptions of the celestial sphere, meridian instruments and their accessories, clocks, chronographs, and the methods of making observations for the determination of time, right ascension, and declination.

Tome II is devoted to a consideration of the errors which may affect observations made with meridian instruments.

For the professional astronomer this second volume is by far the more important. The meridian observer is no longer satisfied with a precision of one-tenth of a second of arc. He wants to be sure of the hundredth of a second, and this degree of accuracy can be attained only by taking into consideration every known error. It is fitting therefore that a whole volume should be devoted to this subject. Some idea of the detail with which the various errors are discussed may be obtained from the fact that the errors of adjustment cover 115 pages; errors of construction, 95 pages; refraction, 26 pages; and personal equation, 67 pages. No numerical illustrations of any kind are given.

The work is thoroughly up to date. Throughout the text are numerous references to original sources, and, at the end of Tome II, is a 22-page bibliography.

FREDERICK SLOCUM

Spectroscopie astronomique. Par P. SALET. Paris: Octave Doin et Fils, 1909. 12mo, pp. 425, with 44 figures and 1 plate. Fr. 5.

This handy volume is one of the units of a very extensive *Encyclopédie scientifique*, to be comprised in about one thousand volumes. Twenty-nine are to form the subdivision entitled "Bibliothèque d'astronomie et de physique céleste," of which M. J. Mascart, of the Observatory of Paris, is "directeur."

Following an excellent general introduction, the subject is compactly treated in thirteen chapters. The first three chapters consider the instruments of spectroscopy and their adjustment, the fourth discusses measures and standards of wave-lengths, the fifth treats of the physical causes affecting the appearance or position of spectral rays, while the sixth is devoted to the Doppler-Fizeau principle. The remaining chapters have for topics the solar spectrum, and the spectra of the different kinds of celestial objects —sun, planets, comets, stars, and nebulae.

The treatment is generally clear and succinct, with some reference to the historical development in each line. There are no marginal references,

but at the end of each chapter a brief bibliography is given, which is particularly adapted to the convenience of the French readers of the encyclopedia. Recent researches are not neglected, so that the work is up to date. This is important, for there are now hardly any books, in any language, available in the field of celestial spectroscopy which include the progress of the last decade.

The defects of the book are those rather unavoidable in any single part of such an immense cyclopedia: the paper is not as good as might be desired, and many of the illustrations are exceedingly unsatisfactory. Improvement in these respects would, however, have increased the cost—a very significant item for the whole series of books. But it might have been as well to omit some of the reproductions of spectra as to print such inadequate ones.

The author has selected his material from reliable sources, and, as is natural and proper in such a work, gives full prominence to the researches of his countrymen. We note, on page 150, that he accepts as real the large differences between wave-lengths in arc and spark found by Haschek and Mache, but not confirmed by other spectroscopists.

The book is to be commended to astronomers and physicists—astrophysicists will secure it as a matter of course—and we shall await with interest the appearance of the other volumes of the series relating to astronomy and to physics.

E. B. F.

Annuaire pour l'an 1910 publié par le bureau des longitudes. Paris: Gauthier Villars, 1909. Pp. 862. Fr. 1.50.

This valuable annual returns in its usual form. With the scheme of alternation in effect since 1904, the volume for 1910 gives detailed tables of a physical and chemical nature, but omits those concerned with geography and statistics. Similarly, this year the elements of the minor planets are tabulated in full, while the lists of stellar parallaxes, double stars, etc., given last year, are omitted. We note an error on p. 228 in the assignment of Des Moines instead of Williams Bay as the discovery point of Comet 1908 III (Morehouse).

The "notices" appended to the *Annuaire* are: a note upon the meeting in 1909 of the permanent international committee of the astrographic chart, by M. B. Baillaud; an essay of ninety pages on the tides of the earth's crust and the elasticity of the globe, by M. Ch. Lellemand; and indices of the notices published in previous issues of the *Annuaire* from 1804 onward, by M. G. Bigourdan.

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RADIATION AND ABSORPTION

By W. J. HUMPHREYS

INTRODUCTION

The author has often had weary hunts, through the voluminous literature on the subject, for known facts in regard to radiation and absorption; and many others, doubtless, have had the same sort of time-wasting experience. Perhaps, therefore, the following brief discussion of certain of the more important radiation and absorption phenomena, though containing but little that is new, may be of some use.

A similar article, differing decidedly from the present one, both in scope and method of treatment, and with meteorological applications, will, because of these applications, appear in the Mount Weather *Bulletin*, **2**, 109–132. The circulation, however, of this bulletin overlaps but slightly that of the *Astrophysical Journal*, and, as the readers of the latter often are especially interested in the subjects here discussed, it may be worth while to place a modified and more comprehensive article at their service.

GENERAL REMARKS

Everyday experience tells us that a cold object in the neighborhood of a hot one becomes warm, and experiment shows that the same phenomenon can happen when either or both are in as complete a vacuum as can be obtained. The hot object, we say, loses heat by radiation, and the cold one absorbs radiation and becomes warmer; but it is not at all clear how these things are brought about.

We are not yet able to follow, in all its details, the cycle of changes involved in the transformation of molecular kinetic energy into energy of radiation, and this in turn back to molecular kinetic energy. We do not understand, in the sense that we should like to, either cooling by radiation or warming by absorption.

But no matter what the real mechanism of radiation or of absorption, or by what process temperature is modified by these phenomena, they act, in many cases, according to known laws, some of which are general and only qualitative, while others are definite and quantitative. But before outlining the experiments and arguments that led to and justify the formulating of laws, it will be convenient to consider the different known processes of radiation and see to which these laws apply.

CLASSIFICATION

Radiation is often classified, according to its action on the eye, as *luminous* and *non-luminous;* or, according partly to its position in the spectrum as formed by a glass prism, and partly to the sensation aroused in those of normal vision, as *ultra-violet*, *violet*, *blue*, *green*, and so on through the so-called primary colors, and, finally, *ultra-* or *infra-red*.

Both these methods of classification serve useful ends, but are inadequate when minute subdivision is required. For this purpose *wave-lengths* and *wave-frequencies* are used; the former commonly being the more convenient, and the latter often the more valuable of the two.

If now we consider, not the radiation itself, but the processes by which it is produced, we shall find that these fall into two distinct classes, namely:

1. *Incandescence.*—Whenever an object radiates continuously the same way, so long as its temperature is kept constant, we say that the source of the radiant energy is *incandescence*, or that the process by which it is emitted is *pure temperature radiation*. It is excellently represented by a carbon filament when electrically heated *in vacuo*.

2. *Luminescence.*—Whenever the radiation given out by an object changes while the temperature is kept constant, and whenever the intensity of the radiation in any spectral region, however narrow or extended, is greater than that in the same region of an ideally black

body at the same temperature, we call the process by which any excess of radiation is produced *luminescence.* Here the energy flux is not maintained wholly by the heat of the object concerned, but, in part, in some cases almost entirely, by some other of its supplies of energy, and is therefore accompanied by a molecular or other internal change.

Luminescence, in turn, may be subdivided according to its more or less obvious causes. Thus we have:

a) *Chemi-luminescence.*—Illustrated by phosphorescent bacteria, by the firefly, the glow-worm, and the like, where the exciting cause apparently is some sort of chemical change, the nature of which is not understood. Also illustrated by slowly oxidizing phosphorus, where, though the chemical action is well understood, the details of the process by which the object becomes luminescent are still unknown.

b) *Photo-luminescence.*—Divisible into (1) *fluorescence,* as beautifully illustrated by the glow of sodium vapor while excited by radiation of any one or more of certain wave-lengths; (2) *phosphorescence,* as illustrated by Balmain's luminous paint (calcium sulphide), which glows for hours after an exposure to any intense ordinary light. Fluorescence appears to end, like simple reflection, immediately the incident energy is arrested. Still, it is not a reflection phenomenon, and probably does last through an interval of time, however brief. Phosphorescence, on the other hand, continues of itself, like a pendulum set swinging, for a measurable length of time after the initiating disturbance has ceased to act. Both phenomena evidently are due to some effect of absorption other than mere temperature change, perhaps chemical in its nature. Probably, too, they mainly differ from each other only in their durations after the exciting radiation has been shut off.

If X-rays consist, as we believe, of abrupt ether-pulses, then the luminescence excited by them may also be included in this class.

c) *Electro-luminescence.*—Good examples of this are seen in the glows of electrodeless tubes; in barometer stems, sometimes, when the mercury is made to oscillate up and down, and even in electric light bulbs when properly excited. Probably under this head should be classed all radiation due directly to electric discharges, such as the faint glow along the path of cathode rays, the glow excited in many

objects when used as cathode targets, the light of the Geissler tube, of St. Elmo's fire, and of auroras—presumably. Also the steady glow excited in many substances by radium may belong to this class.

d) *Mechanical, or tribo-luminescence.*—Luminescence of this origin usually is faint. It may be produced by the cutting of glass, by the rubbing together of quartz, by the crushing of lump sugar, and by many other processes, such as freezing, crystallization, bombardment by α particles, and the like, all of which, possibly, give but varieties of electro-luminescence.

e) *Thermo-luminescence.*—In this case temperature, apparently, serves only the purpose of setting free some form of stored-up or potential energy. Fluor-spars and diamonds are among the best-known objects that are thermo-luminescent, and these differ greatly in the kind and quantity of the light they emit, and the temperatures at which they become active. Thus the chlorophane (fluor-spar that emits essentially a pure green light) found at Amelia Court House, Va., becomes luminescent on being held for a few moments in the hand, while that from some other localities remains inactive even at the temperature of boiling water.

f) *Cometary luminescence.*—This term is used here to apply to the light phenomena of comets' tails, and, perhaps, also of nebulae. Possibly this too is a case of electro-luminescence, but as yet no one knows, and therefore the new name is suggested as a means of stating briefly, but unequivocally, that we know nothing, or next to nothing, about the real process by which the tails of comets become luminous.

In regard to all classes of luminescence our information is extremely fragmentary. In fact it is only that radiation which is due entirely to incandescence, of whose quantity and spectral distribution we have knowledge sufficiently extensive and exact to formulate into laws. Nor does this knowledge extend, even approximately, to the thermal radiation of all substances, nor are the quantitative laws applicable except to specially devised and artificially constructed bodies, and to substances with ideal properties. Nevertheless, in spite of the excessive difficulty of the subject, a great deal of labor, both on experiment and on theory, has been given to radiation and absorption; and the following few laws are of the greatest value, not only in pure physics, but also in many of the arts as well.

It must not be overlooked, however, that the lights themselves of different luminescent origins obey the same laws as does that due to incandescence. They are indistinguishable, when the examination is confined to any single wave-length, each from any other, and doubtless are all due to internal or ionic disturbances, no matter how different may be the methods, thermal, electrical, chemical, or what not, of initiating and maintaining these disturbances.

CONTINUOUS EXCHANGE—PREVOST'S LAW

More than a century ago Prevost[1] advanced the theory, substantiated by every known test, that any two objects, when there is no intercepting medium between them, continue to radiate and, to some extent, mutually to absorb each other's radiation, whatever their temperatures. According to this well-substantiated theory any object, no matter how cold, supplies some heat to its neighbor, no matter how hot.

This means, (1) that an object continues to radiate to a greater or less extent, whatever its temperature; and (2) that no supply of heat can render it either completely diathermanous, or a perfect reflector. It therefore continues to give out energy by radiation, and at the same time to absorb a part, at least, of any radiant energy that may be incident upon it, so that partial energy exchange must take place between objects unscreened from each other, regardless of either equality or inequality of their temperatures.

A valuable application of this law of exchanges is found in the use of a pyrheliometer. The readings of this instrument depend in part upon its own temperature before exposure, as well as upon that of the object whose radiation and temperature are being determined.

As will be more fully explained later, we can write

$$R=CT^n\ ,$$

in which R is the total radiation per second per unit area of any given object, C a constant, peculiar to the substance of the object in question, T the absolute temperature, and n a number which, in the case of the full radiator, or black body, is 4. Then, if G is the amount of heat gained per second per unit area by the pyrheliometer

[1] *Sur l'équilibre du feu*, Genève, 1792; *Du calorique rayonnant*, Genève, 1809.

receiver, we have, in the case of no intervening absorbing medium, and no other contributing radiator,

$$G=\omega_1 C_1 T_1^{n_1}-\omega_2 C_2 T_2^{n_2},$$

in which T_1 is the absolute temperature of the object under consideration, C_1 and n_1 its radiation constant and exponent; T_2, C_2, and n_2 the corresponding pyrheliometer values, and, finally, ω_1 and ω_2 the solid angles subtended at the receiver by the radiator and the bolometer opening respectively.

This assumes absolutely no loss of heat from the receiver except by radiation through the pyrheliometer opening, an assumption that, in practice, would be difficult to realize even approximately. A more practicable equation is

$$G=\omega_1 C_1 T_1^{n_1}-\omega_2 C_2 T_2^{n_2}-H,$$

in which H is the heat lost by methods other than radiation from the receiver, controllable, by fixing the value of T_2, and accurately measurable.

UPON WHAT THE RADIATION AND ABSORPTION OF AN OBJECT DEPENDS

The amount of energy emitted by incandescence per second per unit area (smooth) of an object depends upon its composition, its temperature, and the refractive index of the surrounding medium, and not in any way upon the temperatures, compositions, or positions of neighboring objects; while the energy absorbed depends upon the composition of the absorbing surface and also upon the amount and quality of the incident energy, that is, upon the compositions, temperatures, and angular sizes, as viewed from the absorber, of surrounding objects.

As surfaces are ordinarily measured it is necessary to consider their physical state. A mat surface radiates and absorbs more than does a smooth one, because of multiple reflections within the minute cavities and its consequent approach to a full radiator, or ideally black body.

TWO RELATIONS, QUALITATIVE AND QUANTITATIVE

It must be clearly kept in mind, as Cotton[1] has emphasized, that there are two distinct relations between radiation and absorption, one

[1] *Astrophysical Journal*, 9, 237, 1899.

merely qualitative, the other quantitative, that often are confused. The qualitative relation deals with the radiation and absorption of a single object under ordinary but not necessarily clearly defined conditions, while the quantitative relation is that which exists between the radiation and absorption, properly defined, of two different bodies at the same temperature, one of them at least being a full radiator and perfect absorber. It shows how this relation, which is the same for all bodies, varies with temperature and is related to wave-length.

Whenever any object is emitting radiations of a given wave-length and polarization, it will absorb identical radiations coming to it from without. But, in many cases at least, absorption is not confined to the kind of radiation emitted. Nearly all objects, even some of the most transparent, such as water and glass, absorb ultra-violet radiations at ordinary temperatures; though at these temperatures they emit them so feebly, if at all, that they have never been detected. Similarly, colored objects absorb radiations, which, at room temperatures, there is no evidence of their emitting. Although carbon dioxide, water vapor, and some other substances, when cold, absorb lines and bands which at higher temperatures they appreciably emit, still there is no certainty that this property is universally true, nor are we aware of any quantitative relations between temperature, absorption, and emission in even the best-known of these cases. All that can safely be said, in this connection, is that good absorbers are good radiators, and that good radiators are good absorbers; but it must be distinctly understood that these are only qualitative rules.

RADIATION WITHIN A CLOSED EQUAL-TEMPERATURE SHELL

Let a shell of any shape and material whatever be at a uniform temperature and have an adiathermanous outer surface so that all its heat must be retained—a polished silver surface would approach these ideal conditions. Now let a flexible tube with parallel walls that are perfectly reflecting within and without be placed within the shell, and let the ends of this exploring tube be located and oriented at random. Radiation will stream through it from both ends, and if it is more intense in one direction than in the other, then the heat energy will assume a new distribution on the shell, and the second

law of thermodynamics will be violated, inasmuch as heat, in excess of mutual exchange, will be transferred, without the aid of external work, from an object at one temperature to another at the same or even a higher temperature.

We conclude, therefore, that the radiation within a closed equal-temperature shell is of the same intensity throughout the inclosed space, and in every direction.

Now let the tube be crossed by a partition such that only radiation of a certain type of polarization, or of a certain limited range of wave-length, or of both, can get through. Again, if the second law of thermodynamics is not to be violated, the energy flux must be the same in both directions, and we add to equality of intensity equality of kind of radiation at every place and direction within an inclosure whose walls are in thermal equilibrium throughout. We say that radiation in such a space is perfectly diffused.

Even if the walls of the shell be made of different materials, of one thing at one place and of something else at another, still, if the second law of thermodynamics is to hold, there must obtain, under all such conditions, equality of intensity and kind of radiation at every point within the inclosure, and in every direction. That is to say, when two objects are in thermal equilibrium, each with the other, the radiation emitted by the first and absorbed by the second agrees in every particular, heating effect, intensity, wave-length, and polarization, with that emitted by the second and absorbed by the first.

To determine just what kind of radiation fills an inclosed cavity let a portion of the inner wall be made of a perfect absorber, or an ideally black body. Such material reflects no radiation of any kind, and therefore radiation directed *away* from it must be entirely its own radiation. Hence a closed shell, whose inner walls are in thermal equilibrium with each other, is filled with radiation that everywhere, and in every direction, is identical, both in intensity and in kind, with that from the surface of an ideally black body at precisely the same temperature and surrounded by a medium of the same refractive index as that which fills the shell. The reason for this last condition, involving the refractive index of the adjacent medium, will be seen later.

Since the inner walls of the shell may consist in part or wholly

of materials selectively reflective, that is, of materials that reflect radiation of certain wave-lengths better than that of others, and since the radiation that fills the shell still is of the black body type, it therefore follows that those spectral regions which a substance reflects well it radiates poorly, while those it absorbs it correspondingly radiates, when at the proper temperature. Similarly, those radiations a substance transmits well it radiates but feebly. Also, inasmuch as an object of any kind within a constant-temperature shell does not disturb the perfectly diffused condition of the radiation, therefore the radiation, e_θ, emitted by any object at the angle θ from the normal is equal to the radiation, a_θ, absorbed by this object at the same angle, or $e_\theta = a_\theta$ for all angles, when the object is in thermal equilibrium with its surroundings.

COSINE LAW—LAW OF LAMBERT

By observation and by experiment we know that radiation moves in straight lines through a medium whose refractive index is constant. Therefore within a shell of constant temperature, where the radiation is perfectly diffused, the amount of radiation parallel to the sides of a straight-walled tube of constant cross-section is the same throughout its length and independent of the direction along which it lies.

Let one side of the shell be flat, and let the tube point first to one and then to another part of this flat portion. Let the angle between the normal to this surface and the axis of the tube be θ_1 in the first position, and θ_2 in the second, and let A be the cross-section of the tube. Also let e be the radiation per second per unit area normal to the flat surface, and $e\psi(\theta)$ its radiation per second per unit area at an angle θ from the normal.

The areas that contribute radiation parallel to the walls of the tube, in its two positions, are $\dfrac{A}{\cos\theta_1}$ and $\dfrac{A}{\cos\theta_2}$, respectively. But the total amount of this radiation is the same in the one case as in the other; therefore

$$\frac{A}{\cos\theta_1} e\psi(\theta_1) = \frac{A}{\cos\theta_2} e\psi(\theta_2)\ ,$$

or

$$\frac{\psi(\theta_1)}{\cos\,\theta_1}=\frac{\psi(\theta_2)}{\cos\,\theta_2}\,.$$

But θ_1 and θ_2 have any values from 0 to $\frac{\pi}{2}$, and, besides, when $\theta_1=0$, $e\psi(\theta_1)=e$, or $\psi(\theta_1)=1$.

Hence; in this case

$$\psi(\theta_2)=\cos\,\theta_2\,,$$

or, in general,

$$\psi(\theta)=\cos\,\theta\,.$$

That is to say, the intensity of the radiation in any direction from a black surface is directly proportional to the cosine of the angle between that direction and the normal to the surface. In the case of objects that are not full radiators the same law, while not exact, applies approximately, since the color and brilliancy of objects remain substantially the same, no matter at what angle they are viewed.

TOTAL RADIATION FROM A BLACK SURFACE

Let $e_\lambda d\lambda$ be the radiation per unit area per second, between λ and $\lambda+d\lambda$, normal to the radiating surface. Then, where θ is the angle between the direction of the radiation and the normal to the surface, we have for the total radiation, of wave-length λ, per second from a flat black surface of area S,

$$R_\lambda=2\pi e_\lambda d\lambda S\int_0^{\frac{\pi}{2}}\cos\,\theta\,\sin\,\theta d\theta=\pi e_\lambda d\lambda S\,,$$

or $\pi e_\lambda d\lambda$ per unit flat area.

This value differs greatly with different wave-lengths. Hence for the total radiation per second per unit flat black area we have

$$R=\pi\int_0^\infty e_\lambda d\lambda\,.$$

When the temperature of the radiating surface is 1° C. absolute, then, by definition, its *total* radiation per second per unit area is its *emissive power.*

STEWART-KIRCHHOFF LAW

Consider a unit flat surface of any material surrounded by walls at the same temperature as that of the inclosed object. As just seen, the total radiation per second of this surface, if black, is

$$R=\pi\int_0^\infty e_\lambda d\lambda\,.$$

But the temperature of this object remains constant (to change would be to violate the second law of thermodynamics) and as it is black it absorbs all incident radiation. Let H be the amount of radiation incident per second on the given unit area, then $(H)_T = (R)_T$, in which T is the constant temperature under consideration.

If, however, the surface is not black then it will absorb only A units of the total, H, incident, and, during the same time, emit E units. But, as the temperature remains constant, $(E)_T = (A)_T$.

Hence

$$(E)_T = \left(\frac{A}{H}R\right)_T,$$

or

$$\left(\frac{E}{\frac{A}{H}}\right)_T = (R)_T.$$

This means that the ratio of the emissivity of any object to its coefficient of absorption (energy absorbed divided by energy incident) depends upon its temperature only; and, numerically, is equal to the emissivity of the full radiator at the same temperature.

By the aid of an inclosing screen, perfectly reflecting to all wave-lengths except one, and fully transparent to this, the interchange of radiant energy between the inclosed object and the outer shell can be restricted to a single wave-length, λ, but still the temperature of the inclosed object must remain constant, since otherwise an object at one temperature would be able to warm another to a higher, a result contrary to all experience. Therefore the radiation, $(A_\lambda)_T$, absorbed, must equal the radiation, $(E_\lambda)_T$, simultaneously emitted, and

$$\left(\frac{E}{\frac{A}{H}}\right)_{\lambda,\,T} = (R)_{\lambda,\,T}.$$

By the introduction of a polarizer in the path of the energy flux, the radiation exchanged can be restricted to any particular azimuth, ϕ, of polarization, so that we can more specifically write

$$\left(\frac{E}{\frac{A}{H}}\right)_{\lambda,\,\phi,\,T} = (R)_{\lambda,\,\phi,\,T}.$$

ENERGY OF RADIATION

Let the motion of a vibrating particle be expressed by the equation

$$y=a \sin(\omega t-\alpha) ,$$

in which y is the displacement at the time t from the point of rest, a the amplitude, or maximum displacement, $\omega=\frac{2\pi}{\tau}$, in which τ is the time of a complete vibration, α the epoch, or phase at the instant observations began.

The velocity, s, of the displaced particle is given by the equation

$$s=\frac{dy}{dt}=a\omega \cos(\omega t-\alpha) .$$

The time of a complete vibration is τ, and therefore the average energy of a vibrating particle of mass m, is

$$\frac{1}{\tau}\int_0^\tau \tfrac{1}{2}ms^2dt=\frac{ma^2\omega^2}{4\tau}\int_0^\tau 2\cos^2(\omega t-\alpha)dt ,$$
$$=\frac{ma^2\omega^2}{4}=m\pi^2\frac{a^2}{\tau^2} .$$

Therefore the average energy is just half the maximum, since the maximum velocity is $a\omega$. Also, on the whole, the energy of the vibrating particle is half potential and half kinetic.

Where the velocity of the wave disturbance is independent of wave-length, as is the case with radiation *in vacuo*, we can write $\frac{\tau'}{\tau}=\frac{\lambda'}{\lambda}$, and therefore, if E_λ is the kinetic energy per unit volume, due to radiations of wave-length λ, we can write $E_\lambda=\rho\pi^2\frac{a^2}{\lambda^2}c^2$, where ρ is the ether density, and c the velocity of light *in vacuo*.

DOPPLER EFFECT OF A MOVING MIRROR ON RADIATION

Following the method used by Larmor,[1] let radiation, moving with velocity c, meet normally a perfect mirror approaching it with the velocity u. The reflected radiation will be changed in several particulars, among them wave-length. This, in regions where the refractive index is greater than unity, would lead to some complication, so that, in what follows, it will be assumed that $\mu=1$ for all wave-lengths, or that we are dealing with radiation phenomena

[1] *Encyclopaedia Britannica*, **32**, 121.

in vacuo. Let the path of the incident radiation and of the mirror be along the axis x. Let the initial position of the mirror be at $x=0$, and let it move in the positive direction.

The displacement, y, due to the incoming radiation, at any place x and time t is given by the equation

$$y=a \sin\frac{2\pi}{\lambda}(ct+x) ,$$

in which a is the amplitude and λ the wave-length.

The corresponding disturbance due to the reflected radiation, since c is independent of both wave-length and amplitude, is represented as follows:

$$y'=a' \sin\frac{2\pi}{\lambda'}(ct-x) .$$

Since the mirror is a perfect reflector, at its surface $y+y'=0$ at all times, and therefore also at the point x when $x=ut$. Hence

$$a \sin\frac{2\pi}{\lambda}(ct+ut)+a' \sin\frac{2\pi}{\lambda'}(ct-ut)=0 ,$$

whatever the value of u.

Therefore

$$a=-a'$$

and

$$\frac{\lambda'}{\lambda}=\frac{c-u}{c+u} .$$

This means that the reflected and the incident radiations have the same amplitudes, but, at the surface of the reflector, opposite phases; while their wave-lengths are to each other as the difference to the sum of the two velocities concerned.

RADIATION PRESSURE—MAXWELL-BARTOLI EFFECT

That radiation exerts a pressure was deduced theoretically first by Maxwell,[1] and later in a different manner by Bartoli.[2] Subsequently it was proved experimentally by Lebedew[3] and by Nichols and Hull.[4]

[1] *Treatise on Electricity and Magnetism* (first ed.), **2**, 391, 1873.

[2] *Nuovo Cimento,* **15**, 193, 1884.

[3] *Rapports, Congrès International de Physique,* **2**, 133. Paris, 1900.

[4] *Physical Review,* **13**, 293, 1901; *Proc. Am. Acad.,* **38**, 559, 1903.

Consider a straight cylinder of unit cross-section with perfectly reflecting walls and ends. Let this be filled with monometric, or single wave-length, radiation, and let one of the ends move in slowly with the velocity u. Let a wave start from the fixed end with the velocity c and, at the end of the time t, reach, at the distance ct, the moving end which, during the same time, has come in a distance ut. It is now reflected, and when it has returned to its starting-point, or completed a cycle, the movable end will have come in an additional distance ut. Therefore the length of the closed cylinder, when the radiation left the fixed end, is to its length at the completion of the cycle as $ct+ut$ is to $ct-ut$. But this, as we have seen, is the ratio of the old to the new wave-lengths, or of λ to λ'.

As the walls are perfectly reflecting no radiation energy can get out, and as amplitude is not changed by reflection, therefore the energy density is increased in the ratio of λ^2 to λ'^2, or of $(c+u)^2$ to $(c-u)^2$. Therefore, after, and because of, the compression the energy in the inclosure is greater than before, which, from the conservation of energy, proves that work has been done by virtue of volume change. Hence there must be a radiation pressure, such that $dQ=pdV$, where dQ is the change in the total energy of the volume V, and p the average radiation pressure.

The following discussion of radiation pressure on a mirror is essentially that given by Larmor.[1]

Let e be the energy density of the normally incident radiation, then, as above seen, the energy density of the reflected radiation is

$$e\left(\frac{c+u}{c-u}\right)^2.$$

If e_λ and $e_{\lambda'}$ are the energies respectively per incident and per reflected wave-lengths, then

$$\frac{e_{\lambda'}}{e_\lambda}=\frac{c+u}{c-u}.$$

The number of waves incident per unit of time on the reflector is $\frac{c+u}{\lambda}$, and therefore the radiation energy added in the same time,

[1] *Encyclopaedia Britannica*, **32**, 121.

by virtue of the moving mirror, is

$$(e_{\lambda'} - e_\lambda)\frac{c+u}{\lambda} = pu ,$$

the corresponding work done against pressure. Substituting for $e_{\lambda'}$ its value in terms of e_λ, we get

$$\left(\frac{2e_\lambda u}{c-u}\right)\frac{c+u}{\lambda} = pu .$$

But $\frac{e_\lambda}{\lambda} = e$, the energy density of the incident radiation. Therefore we get

$$p = 2e\frac{c+u}{c-u} .$$

The energy density in front of the mirror, including both incident and reflected radiation, is

$$e\left[1 + \left(\frac{c+u}{c-u}\right)^2\right],$$

or

$$2e\frac{c^2+u^2}{(c-u)^2} .$$

If we let E represent this total energy density, then

$$p = E\frac{c^2-u^2}{c^2+u^2} .$$

The above concerns radiation pressure when the mirror is moving *against* the radiation. If it is moving in the same direction, so as to increase its distance from the source, the signs are reversed. Therefore in general

$$p = 2e\frac{c \pm u}{c \mp u} = E\frac{c^2 \mp u^2}{c^2 \pm u^2} ,$$

in which the upper signs refer to motion toward the source of radiation, and the lower to motion away from it. When $u = 0$, $p = 2e = E$.

It must not be overlooked that the above concerns only that radiation which is to and fro along a path at *right angles* to a *mirror* that sustains the pressure.

In the case of perfectly diffused radiation, filling a cube, say, whose inner walls are perfectly reflecting, we can resolve the pressure due to the radiation in each direction, normal and parallel to the walls. This will be equivalent to what would be obtained by similarly

resolving the radiation itself, if such a thing were possible. Consequently, in the case of diffuse radiation, acting on a mirror,

$$p_d = \tfrac{1}{3}E\frac{c \pm u}{c \mp u},$$

E being total radiation density.

When $u=0$, $p_d = \frac{1}{3}E$, in the case of a mirror. If a black surface is used instead of the reflecting mirror, energy is absorbed in it at the rate of $e(c \pm u)$. By moving the absorbing surface a distance l along the path of radiation the amount of energy absorbed, or the amount of energy transferred—work done—is increased, if toward the radiator, or decreased, if in the opposite direction, by el. Therefore, in the case of a perfect absorber, whatever its velocity along the path of radiation, $pl = el$, or $p = e$, in which e is the energy density of the singly directed radiation. Hence radiation pressure on a perfect absorber, being independent of velocity, is analogous to sliding friction. If the radiation is diffused and the absorber is in thermal equilibrium with it, $p = \frac{1}{3}E$, in which E is the energy density of the total radiation. Here half the pressure is the reaction due to the emission by the radiator.

As no known substance is either a perfect absorber or a perfect mirror, radiation pressure, where the distance to the source is constant and the receiver adiathermanous, must be greater numerically than once and less than twice the density of the incident radiation. If the receiver is in part diathermanous, additional complications are introduced.

RELATION OF TOTAL RADIATION OF A BLACK BODY TO ITS TEMPERATURE—STEFAN-BOLTZMANN LAW

This relation was first determined as an empirical law by Stefan[1] from the experiments of others, and later deduced by Boltzmann[2] from thermodynamic considerations similar to those used by Bartoli in his work on radiation pressure. In what follows, however, Larmor's[3] method will be closely followed.

Let $E_\lambda d\lambda$, which we will briefly designate a, be the energy density of diffuse radiation of wave-lengths λ to $\lambda + d\lambda$ within a space whose

1 *Wien. Akad. Ber.*, **79**, 391, 1879.

2 *Wied. Ann.*, **22**, 291, 1884.

3 *Encyclopaedia Britannica*, **32**, 122.

volume is V. Let it be surrounded by perfectly reflecting walls, so that none of the energy can escape, and let the walls be pushed in slightly against the radiation pressure.

Now, remembering that the change in volume is a decrease, that dV is negative, we get, from the conservation of energy,

$$aV - \tfrac{1}{3}adV = (a+da)(V+dV)\ .$$

Therefore

$$Vda = -\tfrac{4}{3}adV$$

and a is proportional to $V^{-\frac{4}{3}}$.

If the original radiation density a is in equilibrium with a thermal source at the absolute temperature T, and the new density $a+da$, with some other temperature $T+dT$, then, by Carnot's principle,

$$\frac{aV - \frac{1}{3}adV}{aV} = \frac{T+dT}{T}\ ,$$

or T is proportional to $V^{-\frac{1}{3}}$, and T^4 varies as $V^{-\frac{4}{3}}$. But a varies as $V^{-\frac{4}{3}}$.

Therefore $a = KT^4$, K being a constant. That is, $E_\lambda d\lambda$, for every value of λ, is directly proportional to T^4, or the total radiation, R, per unit of time and surface of a black body at temperature T, may be written as $R = CT^4$.

This furnishes an excellent practical means for determining the temperature of one full radiator in terms of another.

An imperfect radiator, even when losing energy through incandescence only, radiates according to some power of its temperature greater than the fourth, but one that cannot be calculated, or easily determined experimentally, since, in many cases at least, it changes—approaches four, as the temperature increases.

One of the most carefully investigated radiators is polished platinum, which from 400° C. to 1600° C. gives off total radiation at a rate approximately proportional to T^5.

It must not be supposed, however, that polished platinum or anything else, for that matter, radiates more at a given temperature than does the full radiator whose temperature exponent is only 4. As a matter of fact all such objects radiate less, and thus have smaller values for their temperature coefficients.

RELATION OF WAVE-LENGTH TO TEMPERATURE—WIEN'S DISPLACEMENT LAW

As just shown, T varies as $V^{-\frac{1}{3}}$. But as compressing the volume inclosed by perfectly reflecting walls decreases the wave-lengths in proportion to the change in their paths, that is to say, changes the wave-lengths without altering the number of inclosed waves, therefore

$$\frac{\lambda_1^3}{\lambda_2^3}=\frac{V_1}{V_2},$$

or

$$\lambda=\lambda_0 V^{\frac{1}{3}},$$

in which λ_0 is the wave-length corresponding to unit volume.

Therefore λT is a constant, or the higher the temperature the shorter the corresponding wave-length.

The equation $\lambda_{max}T=$ a constant, in which λ_{max} is the wave-length of maximum radiation, furnishes a reliable means of determining the temperature of a full radiator.

RELATION OF THE SPECTRUM OF A BLACK BODY TO ITS TEMPERATURE

As already explained, $E_\lambda d\lambda=KT^4$, in which K is a constant.

Also $T\lambda=$ a constant.

Now let λ_1 and λ_2, of which $\lambda_2>\lambda_1$, be two wave-lengths of nearly the same value at the temperature T, and λ'_1 and λ'_2 their corresponding values at the temperature T', then

$$\frac{T(\lambda_2-\lambda_1)}{T'(\lambda'_2-\lambda'_1)}=\frac{T\delta\lambda}{T'\delta\lambda'}=1,$$

or $T\delta\lambda=$ a constant, and therefore E_λ is proportional to T^5.

If we consider the radiation energy confined to the region between λ and $\lambda+d\lambda$ at any given temperature T, this energy will remain proportional to T^4, while the width, $d\lambda$, of the spectral region it covers, and its wave-length λ both will remain inversely proportional to T.

If then we plot wave-length against radiation intensity, or λ against E_λ, we can pass from the curve for the temperature T of any body whose radiation is purely thermal to its corresponding curve at the temperature T', by changing all the wave-lengths in the proportion of T to T', and the value of the shifted E_λ in the proportion of T'^5 to T^5.

The position of maximum radiation is the easiest to determine on a radiation curve. Calling this λ_m, we have the fact that $E_{\lambda m}$ varies directly as T^5. This too furnishes a method for comparing the temperatures of full radiators.

COMPLETE RADIATION EQUATION

In the foregoing there is nothing to show what is the actual distribution of the energy in the spectrum of a black body at any given temperature. The equations only show how the total energy, the energy of any given region, and the wave-length are related to the absolute temperature.

Several attempts have been made to find an expression for the distribution of energy in the spectrum of a full radiator at any given temperature, but some of the assumptions and methods of treatment have been questioned, and no effort will be made to give their substance here.

From electromagnetic considerations Planck[1] obtained the following equation:

$$E_\lambda = C_1 \frac{\lambda^{-5}}{e^{\frac{C_2}{\lambda T}} - 1}$$

in which C_1 and C_2 are constants, T the absolute temperature, E_λ the intensity of the energy of the wave-length λ, or the ratio of q to $d\lambda$, where q is the energy between λ and $\lambda + d\lambda$, and e the Naperian base.

Larmor[2] has also derived this equation, and by a different method.

While the reasoning that led to this equation is not so obvious, and may not be so certainly correct as that that has given the Stewart-Kirchhoff, the Stefan-Boltzmann, and certain other radiation and absorption laws, it probably is the best general radiation equation that has been devised, and agrees almost exactly with the observations from 85° C. to 1773° C. by Rubens and Kurlbaum.[3]

[1] *Annalen der Physik*, **4**, 553, 1901.

[2] *Proc. R. S.*, A, **83**, 81, 1909.

[3] *Annalen der Physik*, **4**, 649, 1904.

RADIATION AND REFRACTIVE INDEX OF ADJACENT MEDIUM
KIRCHOFF-CLAUSIUS LAW

Let single wave-length radiation have its origin at O_2, Fig. 1, where the refractive index for this radiation is μ_2. Let this radiation, filling an infinitesimal solid angle, pass across the interface AB into a medium of smaller refractive index, μ_1, and let the index-change be so gradual that there is no reflection.

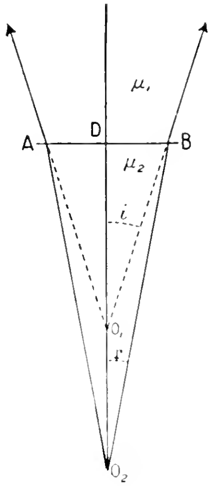


FIG. 1

The radiation that leaves O_2 will, to an observer in the outer medium, appear to come from O_1, and therefore

$$\frac{O_2D}{O_1D}=\frac{\sin i}{\sin r}=\frac{\mu_2}{\mu_1}.$$

Let ω_1 and ω_2 be the corresponding small solid angles filled by the radiation at O_1 and O_2 respectively.

Then

$$\frac{\omega_2}{\omega_1}=\left(\frac{O_1D}{O_2D}\right)^2=\left(\frac{\mu_1}{\mu_2}\right)^2.$$

Therefore, if R_1 and R_2 represent the flux of energy through unit solid angles from O_1 and O_2 respectively, then

$$\frac{R_1}{R_2}=\frac{\omega_2}{\omega_1}=\left(\frac{\mu_1}{\mu_2}\right)^2.$$

This, of course, holds for every direction from O_2, or for the solid angle 4π.

The total radiation, therefore, from a given object varies directly as the square of the refractive index of the surrounding medium.

This law, presumably, applies equally to both luminescence and incandescence, and advantage might well be taken of it in certain delicate radiation experiments.

DISCONTINUOUS RADIATION

Solids and liquids, with but few exceptions, yield continuous, though not strictly black body, or full radiator spectra; while gases give radiations that differ so greatly in intensity from one wave-

length to another that, except when the gas layer has considerable depth or density, they may be regarded as absolutely discontinuous. Possibly the radiations given out by an isolated luminous particle are strictly discontinuous.

Each element yields its own discontinuous, or line, spectrum. Many of the elements, in addition to, so far as known, wholly unrelated lines, give series of lines whose wave frequencies are very approximately connected together by simple mathematical relations; though the reason for this, the underlying physical cause, is not known. Most of the elements, however, yield only a bewildering multitude of lines that have no known orderly, or series, relation to each other.

The method of rendering the gases luminous, whether by mere heating, by electric arc or spark, or otherwise, has great influence on their resulting spectra; but chiefly, if not entirely, on the number, and on the absolute and relative intensities of the lines.

INFLUENCE OF MAGNETIC FIELD ON SPECTRAL LINES
ZEEMAN EFFECT

When the strength of the magnetic field in which a luminous gas is situated is changed, its spectrum is correspondingly modified. Many lines are changed in a complex and most puzzling manner, but whatever modification applies to one line of any given series applies equally to all the other lines of the same series; but not, in general, to the non-series lines nor to the lines of some other series.

In the simplest cases a single non-polarized line becomes a polarized triplet, when viewed across the lines of force. The middle component retains the same wave-length as the original line while the others are displaced equally, or nearly so, to either side. All three components are plane polarized, the middle line being polarized in one plane, and the side lines in a plane at right angles to this. When viewed along the lines of force two oppositely circularly polarized spectral lines are seen, displaced equally to either side of the original.

Inasmuch as the luminous particle is acted upon by a magnetic field, as this Zeeman phenomenon shows, it is clear that, for the time being at least, it must possess a magnetic field of its own, due, conceivably, to orbital rotations of electrons.

If the frequency of the vibration is a direct function of the orbital rotation of the electron, and this seems probable, then the magnetic field of the particle, due to the rotating electron, must also be proportional to the frequency, or inversely proportional to the wavelength. But changing the magnetic field within the orbit of such a rotating electron will simply alter proportionately its period of rotation.[1] Therefore, with N for the magnetic field due to the rotating electron, we can write

$$\frac{\delta\lambda}{\lambda}=\frac{\delta N}{N}\ .$$

But $\delta N=H$, the strength of the disturbing external magnetic field. Therefore, remembering that N is inversely proportional to λ, we get $\frac{\delta\lambda}{H\lambda^2}=$ a constant. And this, regardless of whether the above conception even remotely agrees or not with the real structure and behavior of luminous particles, is the fundamental Zeeman equation, as established by experiment.

PRESSURE DISPLACEMENT

As a luminous particle is more and more crowded by the presence of other particles, their nature apparently making but little difference, the frequency of its vibrations is gradually decreased. The experimental evidence, extending over a pressure range of 200 atmospheres, indicates that for any line we can write, at least approximately,

$$\delta\lambda=K(p_2-p_1);\ p_2>p_1\ ,$$

in which K is a constant, different however for different lines, and p the pressure. For any given group of lines, $\delta\lambda$ increases with λ, but for the non-series lines the irregularities in this particular are very great, though, in general, the same law holds for them also.

When confined to the *same* Mendeléjeff *group* of elements, $\delta\lambda$, in the case of *analogous* lines, increases with atomic weight.

EMISSION AND ABSORPTION OF RADIATION BY GASES

When a high-temperature flame, such as that of a Bunsen burner, contains the vapor of a metal or some compound of it, it generally

[1] Langevin, *Journal de Physik* (4) **4**, 678, 1905.

gives out radiations characteristic of this metal. And when radiations of these particular frequencies are incident upon the flame they are absorbed to a greater or less extent. Such radiations, however, as the hot gas does not emit it does not absorb. Let a layer of gas of constant composition and density, and of unit thickness, emit normally to its bounding planes a given radiation of intensity R_1, and let it so absorb radiation of this same wave-length that only the nth part of that incident normally on one side gets through at the other. Then its coefficient of absorption in this case, the fraction of the incident given radiation absorbed by a layer of the gas of unit thickness, is $1-\frac{1}{n}$, or $\frac{n-1}{n}$, a value independent, apparently, of intensity. The maximum intensity, therefore, of this radiation that this gas could give would be from a layer of infinite thickness, and is expressible as

$$R_\infty = R_1 + \frac{R_1}{n} + \frac{R_1}{n^2} + \text{etc.} = \frac{nR_1}{n-1}.$$

In other words, the maximum radiation intensity for a mass of gas, however deep, is given by the radiation intensity of a layer of any thickness whatever divided by the coefficient of absorption of that same layer.

When the spectrum of an incandescent solid is taken through a colored flame the appearance will depend upon the relative brilliance at that place of the solid to the flame. Let the depth of the flame be such as to give R_∞, then, calling S the intensity of the radiation of the solid over a narrow spectral region, we have,

where $S > \frac{nR_1}{n-1}$, a dark line on a bright background;

where $S = \frac{nR_1}{n-1}$, uniform brilliancy—no line visible; and

where $S < \frac{nR_1}{n-1}$, a bright line on a relatively dark background.

The first case gives the phenomenon of true reversal, so conspicuously seen in the solar spectrum.

In the case of many metallic lines, of certain hydrogen lines, of the bands of cyanogen, and of the bands due to the fluorides of cal-

cium, barium, and strontium, there often occurs what is known as spontaneous reversal. Here the cooler vapors surrounding the flame absorb greatly the radiations from the hotter vapors nearer its center, radiations which the cooler vapors themselves would send out at higher temperatures, and thereby produce lines with bright centers and dark sides.

TRANSMISSION OF SINGLE WAVE-LENGTH RADIATION
LAW OF BOUGUER

When an observer is on that side of an absorbing medium away from the source of radiation, it is easy for him, by varying the thickness of the absorbing layer, to determine the coefficient of transmission for a given wave-length and intensity of the incident radiation, provided that there is no disturbance due to internal reflections.

Let the absorbing medium be a uniform one and let the intensity of the incident radiation be I_0. Let a be the fraction of the incident radiation transmitted a unit distance, then for distances m and n we obtain, respectively, $I_m = I_o a^m$, and $I_n = I_o a^n$ (equation of Bouguer) in which I_o and a are unknown, but the other terms known. By combining the two equations we get

$$a = \left(\frac{I_m}{I_n}\right)^{\frac{1}{m-n}}, \text{ and } I_o = I_m \left(\frac{I_m}{I_n}\right)^{\frac{m}{n-m}}.$$

For convenience of computation, if practical, let $n = 2m$. Then

$$a_m = \frac{I_n}{I_m},$$

and

$$I_o = \frac{I_m^2}{I_n}.$$

These equations are used in determining the transmissibility of the atmosphere to various solar radiations, and to finding their intensities just outside the absorbing medium.

TRANSMISSION OF MULTIPLE WAVE-LENGTH RADIATION
LANGLEY'S LAW

It might seem at first that the above absorption equation for single wave-length radiation would apply to radiation of any complexity whatsoever. But, as Langley has shown, it does not.

Let the intensities of the incident single wave-length radiations be A_o, B_o, C_o, etc., and their respective coefficients of transmission a, b, c, etc. Then their combined intensity, through the thicknesses m and $2m$, will be

$$A_o a^m + B_o b^m + C_o c^m + \text{etc.} = R_m ,$$

and

$$A_o a^{2m} + B_o b^{2m} + C_o c^{2m} + \text{etc.} = R_{2m} ,$$

respectively.

The intensity of the incident radiation, according to the Bouguer formula, is

$$R_o = \frac{R_m^2}{R_{2m}} .$$

The difference between the real and this calculated value of the intensity of the incident radiation is

$$A_o + B_o + C_o + \text{etc.} - R_o = A_o + B_o + C_o + \text{etc.} - \frac{(A_o a^m + B_o b^m + C_o c^m + \text{etc.})^2}{A_o a^{2m} + B_o b^{2m} + C_o c^{2m} + \text{etc.}} =$$
$$\frac{A_o B_o (a^m - b^m)^2 + A_o C_o (a^m - c^m)^2 + \ldots\ldots\ldots + B_o C_o (b^m - c^m)^2 + \text{etc.}}{A_o a^{2m} + B_o b^{2m} + C_o c^{2m} + \text{etc.}} .$$

An occasional term in the numerator may reduce to zero, since possibly $a = k$, $c = l$, etc., but, in general, no two of the coefficients a, b, c, etc., are equal to each other. Therefore every term, except the few zero ones, if such exist, and consequently the whole fraction, is real and positive, and the value of the intensity of the radiation, calculated according to the Bouguer formula, too small. The error in using this equation consists in the assumption that the coefficients of transmission are the same for radiations of all wave-lengths.

Integrating pyrheliometers, therefore, give values of the solar constant that are too small.

THE ÅNGSTRÖM PRESSURE EFFECT

Ångström[1] has shown that the absorption of radiant energy by carbon dioxide increases with the density as well as with the quantity of the absorbing gas; and also[2] that the effect is the same, both qualitatively and quantitatively, whether the given increase in pressure be secured by compression into a shorter column of the pure carbon dioxide or by the addition of an inert gas.

[1] *Annalen der Physik*, **6**, 163, 1901.

[2] *Arkiv för Matematik, Astronomi och Fysik*, **4**, No. 30, 1908.

A column of carbon dioxide ten centimeters long gave a certain absorption, but when it was expanded into a column of thirty centimeters length, the cross-section and the temperature remaining the same, the absorption was found to be decidedly less. However, on pumping an inert gas into this new volume until a pressure was obtained equal to the original pressure in the ten-centimeter column the absorption was increased up to its first value.

This interesting investigation recently has been extended[1] to carbon monoxide, water vapor, and a number of other gases, all of which show the same phenomenon, but to unequal extents. In all cases so far examined it is most pronounced at pressures below one atmosphere.

This property of gases introduces complications into the absorption of radiation as it passes through atmospheres of varying density, such as that surrounding the earth.

SUMMARY

The following are the principal laws of radiation and absorption.

1. *Prevost's theory of exchanges.*—In the case of full radiators we have

$$G=KC(T_1^4-T_2^4) ,$$

in which G is the energy gain per second by the black body at the absolute temperature T_2 from some other black body at the absolute temperature T_1, C the absolute emissive power of a unit area of a black surface, or its total radiation per unit time when at the absolute temperature 1° C., and K a constant depending upon the distance between the two objects and their sizes. If this distance is r centimeters, and their cross-sections at right angles to r, A and A' square centimeters respectively, then

$$K=\frac{AA'}{r^2} .$$

Also, with one square centimeter for the unit area,

$C=5.32\times10^{-5}$ ergs per second, or 5.32×10^{-12} watts (Kurlbaum).

Therefore,

$$G=\frac{AA'}{r^2}5.32\times10^{-5}(T_1^4-T_2^4)\frac{\text{ergs}}{\text{second}} .$$

[1] E. v. Bahr, *Annalen der Physik*, 29, 780, 1909.

2. *Lambert's cosine law.*—This is expressed by the equation

$$R_\theta = R_n \cos\theta\ ,$$

in which R_n is the radiation per unit area normal to the full radiator, or black surface, and R_θ its corresponding radiation in a direction θ degrees from the normal.

3. *Stewart-Kirchhoff law.*—The most complete expression of this law is given by the equation

$$\left(\frac{\dfrac{E}{A}}{H}\right)_{\lambda,\,\phi,\,T} = (R)_{\lambda,\,\phi,\,T}\ ,$$

in which E is the radiation per unit area and time of any object, H the radiant energy incident during the same time upon this area, A the energy it absorbs, and R the corresponding radiation of an equal black body surface—all at the same temperature T, and restricted to the same state of polarization ϕ, and the same spectral region λ, which, however, may cover any range from single wave-length to full radiation.

4. *Energy of radiation.*—The energy per (centimeter)^3, say, along a train of uniform radiation, is half kinetic and half potential, and the kinetic proportional to $\left(\frac{a}{\lambda}\right)^2$, in which a is the amplitude and λ the wave-length. If we restrict the consideration to a (centimeter)^2 and a single wave-length in depth we will find that the kinetic energy, still equal to the potential, is proportional to $\frac{a^2}{N\lambda^2}$, or to $\frac{a^2}{\lambda}$, in which N is the number of wave-lengths per centimeter.

5. *Light pressure—Maxwell-Bartoli effect.*—When the radiation is unidirectional and the receiver a black body, $p = e$, in which p is the pressure per unit surface at right angles to its direction and e the density of the radiation energy. If the receiver is a perfect mirror

$$p = 2e\frac{c \pm u}{c \mp u}\ ,$$

in which c is the velocity of light and u the velocity of the mirror with reference to the source; the upper signs being used when the mirror is moving toward the radiator. When $u = 0$, $p = 2e$.

If the radiation is perfectly diffused,

$$p=\tfrac{1}{3}E,$$

in the case of all non-diathermanous objects, in which E is the energy density of the diffused radiation.

6. *Stefan-Boltzmann law.*—This is generally expressed by the equation

$$R=CT^4,$$

in which R is the complete radiation, or total radiation in all directions, of a black body at the absolute temperature T, while the constant $C=5.32\times10^{-5}$ ergs per second per square centimeter (Kurlbaum).

7. *Wien's displacement law.*—This law states that

$$\lambda T=\text{a constant},$$

in which λ is the wave-length corresponding to any given point on the radiation-energy curve, and T the absolute temperature. As a special and important case we can write $\lambda_m T=A$, in which λ_m is the wave-length of maximum intensity. If λ is expressed in thousandths of a millimeter or microns, then $A=2940$, according to Lummer; or 2921, according to Paschen.

8. *Temperature effect on the spectrum.*—A simple expression for this effect is $\phi(\lambda)T^{-5}=$a constant, in which $\phi(\lambda)$ is the ordinate in a normal spectrum energy-curve, or a curve in which the abscissae are proportional to wave-lengths, and T, as usual, the absolute temperature. An important special case is $\phi(\lambda)_m T^{-5}=$a constant, in which $\phi(\lambda)_m$ is the ordinate corresponding to the wave-length, λ_m, of maximum intensity.

9. *General radiation equation.*—Presumably the best general equation is

$$E_\lambda=C_1\frac{\lambda^{-5}}{e^{\frac{C_2}{\lambda T}}-1},$$

due to Planck, in which E_λ is the ratio to $d\lambda$ of the energy belonging to the spectral region between λ and $\lambda+d\lambda$, e the Naperian base, $C_2=4.965$, $\lambda_m T=14{,}500$ (Paschen); 14,600 (Lummer), and $C_1=3.073\times10^7$, on the assumption that $E_{\lambda_{max}}=1000$ when $T=1000°$ C.

10. *Radiation and refractive index—Kirchhoff-Clausius law.*—

$$\frac{R_1}{R_2}=\left(\frac{\mu_1}{\mu_2}\right)^2.$$

Here R_1 is the radiation per unit area and time of a black body in a medium whose refractive index is μ_1, while R_2 is its corresponding radiation, when in a medium whose refractive index is μ_2.

11. *The Zeeman effect.*—Radiation that ordinarily appears, when viewed in a spectroscope, as a single line, yields multiple polarized lines when the source is in a strong magnetic field.

The displacement of a component line from the original position, and its general dependence upon the strength of the magnetic field and the wave-length are shown in the equation

$$\frac{\delta\lambda}{H\lambda^2}=A\ .$$

Here $\delta\lambda$ is the displacement of the component, λ the original wave-length, H the strength of the magnetic field, and A a constant for any given line or series of lines, but different for different series.

12. *Pressure shift.*—When the pressure is increased about the source of radiation, the period of vibration is slightly increased, and the position of the line correspondingly shifted toward the region of longer wave-lengths. The amount of this for any given line is closely expressed by the relation

$$\delta\lambda=K(p_2-p_1)\ ,$$

in which p is the pressure and K a constant that depends upon the line and the element to which it is due.

13. *Bouguer's law of absorption.*—This is expressed by

$$I_m=I_oa^m\ ,$$

in which I_o is the intensity of the incident radiation, a the coefficient of transmission, or the ratio of the radiation that gets through a layer of unit thickness of the absorbing medium to the incident radiation, m the thickness traversed, and I_m the intensity after the radiation has passed through a layer of thickness m.

This law applies rigidly only for single wave-length radiation, and to media that are without internal reflection.

14. *Absorption as affected by pressure—Ångström effect.*—The coefficient of absorption of many gases (all that have been examined) increases with increase of pressure; and this increase, while different for different gases, and most pronounced when the pressures are below one atmosphere, is the same, in every particular, whether the given increase of pressure is due to compression of the absorbing gas or to the addition of an inert one.

15. *Type of absorption independent of temperature.*—So long as its physical phase and its chemical composition remain unchanged, a body continues to absorb the same wave-length radiation, independent of change of temperature. This is of universal application, so far as known, and not restricted to full radiators.

16. *Increase of radiation with increase of temperature.*—Every wave-length radiation that an object emits increases in intensity with increase of temperature, so long as its physical phase and chemical composition are unaltered. This, too, so far as we know, is of universal application.

LITERATURE

Among the best summaries of our knowledge of radiation and absorption are the following: A. Cotton: "The Present Status of Kirchhoff's Law," *Astrophysical Journal*, **9**, 237, 1899; *Revue Général des Sciences*, February 15, 1899; W. Wien: "Lois théoriques du rayonnement," *Rapports, Congrès International de Physique*, **2**, 23, Paris, 1900; O. Lummer: "Le rayonnement des corps noir," *Rapports, Congrès International de Physique*, **2**, 41, Paris, 1900; H. Kayser: *Handbuch der Spectroscopie*, **2**, chaps. i, ii, and iii, 1902; J. Larmor: *Encyclopaedia Britannica*, **32**, article "Radiation," 1902.

In closing I wish to thank Professor Henry Crew for his kind criticism and various helpful suggestions.

Mount Weather Observatory, Va.

THE ABSORPTION SPECTRUM OF SULPHUR DIOXIDE

By FRANCES LOWATER

In some earlier work on sulphur dioxide[1] the author obtained an absorption spectrum, which exhibited marked changes with change of pressure and change in the length of the column of gas. The change in the absorption with the reduction in the absorbing medium consisted not merely in a reduction in the width of the region, but also in breaking up very wide absorption bands into narrow bands.

With a column of gas 207 cm in length and at a pressure of from one to three atmospheres the spectrum consisted of:

Narrow bands from λ 3900 to 3333
One wide band from λ 3330 to less than 2100

At a pressure of 1.5 cm of mercury it consisted of:

Narrow bands from λ 3226 to 3146
One wide band from λ 3134 to 2467
Narrow bands from λ 2456 to 2297
One wide band from λ 2290 to less than 2100

At a pressure of 0.13 cm it consisted of:

Narrow bands from λ 3180 to 2970
One wide band from λ 2968 to 2715
Narrow bands from λ 2702 to 2269
One wide band from λ 2250 to less than 2100

When the length of the column of gas was reduced to 20 cm and the pressure was less than 1 cm of mercury, the whole spectrum consisted of narrow bands lying within the region λ 3133 to 2200.

The changes in the spectrum produced by change of pressure suggested that with the gas at a suitable pressure and with spectral apparatus of greater dispersion the bands of the spectrum might be broken up into lines.

OBJECTS OF INVESTIGATION

The present investigation was undertaken to determine whether the bands of absorption could be broken into lines and, if so, whether

[1] *Astrophysical Journal*, **23**, 324, 1906.

a relationship existed between the wave-numbers of lines of similar physical character.

In the case of sulphur dioxide a special value is attached to the absorption spectrum and its resolution into lines, as its emission spectrum is so difficult to obtain, on account of the great tendency of the gas to dissociate, even when a very weak discharge is passed through it. So far its emission spectrum[1] is known only as a band spectrum with heads toward the ultra-violet, and its investigation has not been carried into wave-lengths less than 3270; the author, however, intends to make a further attempt in this direction at the first opportunity. No resemblance between the emission and absorption spectrum has yet appeared. This fact is true also of chlorine, but in the case of iodine, Konen has found that the absorption spectrum corresponds with the band emission spectrum from a vacuum tube.

APPARATUS AND MANIPULATION

The source of light was the spark of an alloy of cadmium and zinc mixed in atomic proportions; this light gives a continuous spectrum down to λ 2100, provided a sufficiently long exposure is given and a suitable capacity is placed in parallel with the spark, across the terminals of the secondary of an induction coil.

The spectral apparatus was a concave Rowland grating, of 180 cm radius of curvature, with 15,028 lines to the inch (592 to the mm, approximately) and the width of the ruled surface 6.2 cm. The photographic plates used were Seed's No. 27 Gilt Edge on lantern-slide glass.

The gas was inclosed in steel tubes which had been thoroughly cleaned and their ends closed with quartz plates. At the beginning of the investigation, a tube 20 cm in length was used, but as the spectrum obtained was not well defined, a tube 80 cm long was adopted and better results were obtained.

The tube was exhausted by a Geissler pump and the pressures read by a McLeod gauge. The gas was obtained from liquid sulphur dioxide which had been redistilled, and care was taken that the gas used in the tube should be free from air; its high temperature of liquefaction ($-10°$ C.) insures its freedom from other gases.

[1] *Astrophysical Journal*, 23, 338, 1906.

The beam of light from the spark was made parallel by a quartz lens before entering the tube; on emergence it was brought by a second quartz lens to a focus on the slit of the grating apparatus. For comparison a photograph of the spectrum of the unabsorbed light of a second spark of the same alloy was taken immediately above or below that of the absorption spectrum of the gas. The beam from this second spark was made parallel by a third quartz lens and when required was brought between the end of the absorption tube and the second quartz lens which focused the light on the slit.

For determination of the wave-lengths of all the lines, measurements of the photographic plates were made in the usual way by means of a dividing engine; on the one used for this purpose readings could be made to 0.0001 mm, that is, to a greater degree of accuracy than that to which settings could be made on the absorption lines. The reduction factor was roughly 9.3 Å to 1 mm.

STANDARD LINES

Metallic lines of cadmium, zinc, lead, and iron were transmitted through the gas, the lead and iron appearing from impurities in the two other metals. Certain cadmium lines were used as standards of reference; intermediate lines of cadmium, zinc, lead, and iron were used for plotting a curve of errors, which was applied in the usual manner for the correction of the calculated values of the unknown lines. By using as lines of reference those transmitted through the gas, one avoids possible errors due to displacement of standard lines by want of perfect adjustment of another source. A further advantage in having the reference lines superposed on the spectrum to be investigated is that the plate can be placed under the microscope of the dividing engine, so that lines stretch continuously across the whole field of view, and this makes easier the exact adjustment of the cross-hair parallel to the lines.

The best values known of the wave-lengths of cadmium and zinc in the region in which this spectrum lies, namely λ 2700 to λ 3200, were those determined by Eder and Valenta;[1] these were based on

[1] *Normal-Spectren einiger Elemente zur Wellenlängenbestimmung im äussersten Ultraviolett*, 1899.

their own values of iron lines which were referred to Rowland's normals.

The last photograph for the present work was taken before the publication of Fabry and Buisson's iron standards[1] reached this country, March 1908; otherwise a photograph of iron lines would have been taken on the same plate as the absorption spectrum and measurements on these lines compared with those made on the lines of reference transmitted through the gas. It was considered that Fabry and Buisson's standards would form a more uniform basis for a relationship among wave-numbers, hence the wave-lengths of the standard lines were referred to these normals by the following plan. A curve was plotted having as abscissae wave-lengths of iron lines and as ordinates the ratios of Rowland's wave-lengths to those of Fabry and Buisson's of the same lines. From this curve was read off the ratio by which Eder and Valenta's wave-lengths must be divided to give their values referred to Fabry and Buisson's normals. The wave-lengths of the lines used for this curve are given in Table I, together with the ratio of their values as determined by Rowland and by Fabry and Buisson.

TABLE I

Rowland's Value*	Fabry and Buisson's Value	$\frac{\text{Rowland's } \lambda}{\text{Fabry and Buisson's } \lambda}$
2435.247	2435.159	1.00003601
2506.994	2506.904	3590
2528.599	2528.516	3322
2679.148	2679.065	3098
2778.340	2778.225	4135
2813.388	2813.290	3483
2851.904	2851.800	3646
2912.275	2912.157	4052
2987.410	2987.293	3916
3075.849	3075.725	4032
3225.907	3225.790	1.00003627

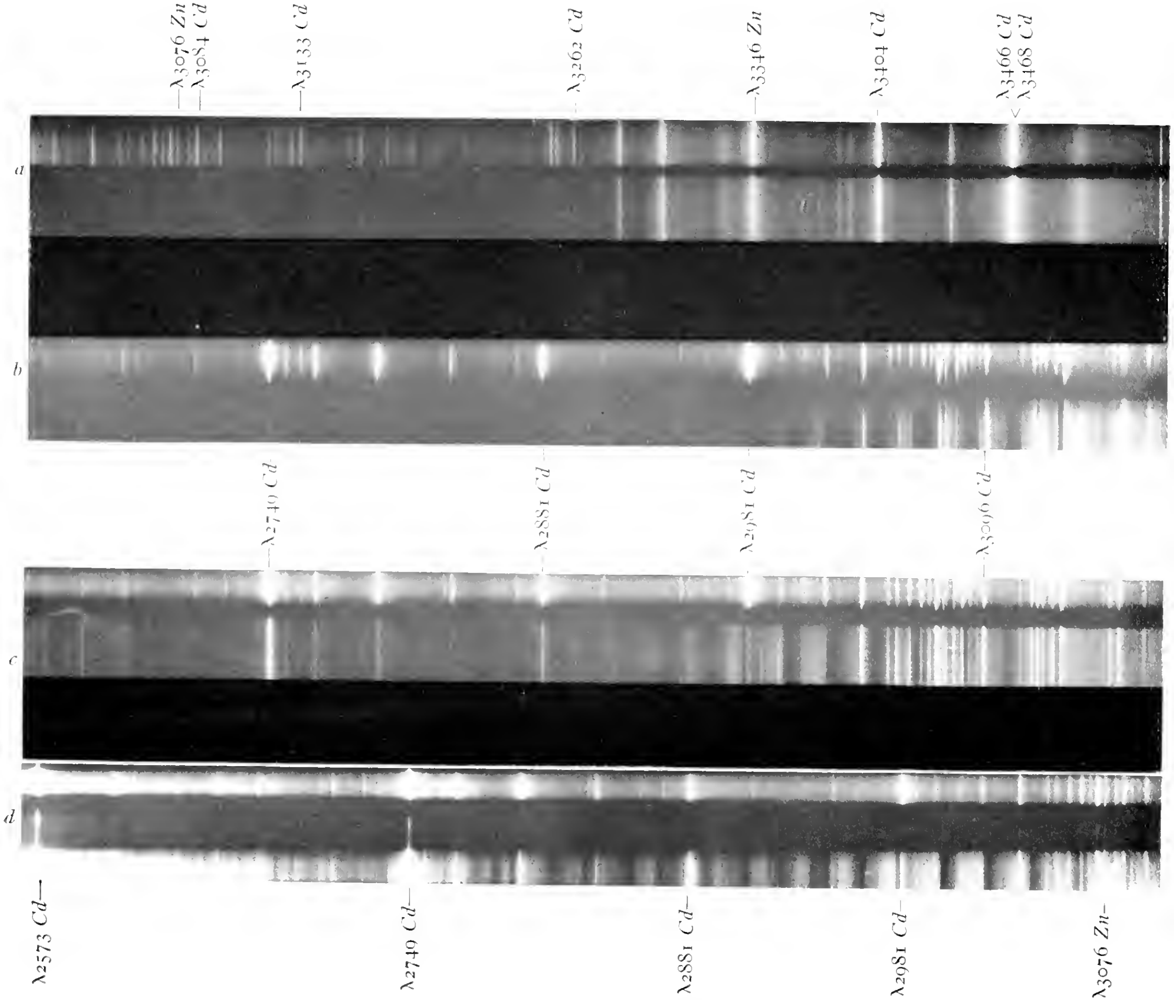
* *Philosophical Magazine* (5), **36**, 49, 1893.

The cadmium standards used for the regions lying between them have the following wave-lengths when referred to Fabry and Buisson's normals:

2707.05	3080.877
2833.073	3133.228
2996.049	

[1] *Journal de Physique* (4), **7**, 169–193, 1908.

PLATE IX



ABSORPTION SPECTRUM OF SULPHUR DIOXIDE
(All the fine lines are lost in the engraving)

DETERMINATION OF EXPERIMENTAL CONDITIONS FOR BEST DEFINITION

Photographs of the spectrum were taken first with a column of gas 20 cm in length and with the gas under various pressures and the plates were given different times of exposure. These experiments showed that the bands obtained in the earlier work could be broken into lines, but so far the lines were not well defined. The experiments were repeated with a column of gas 80 cm in length until conditions were found which gave the best-defined lines. These conditions were a column of gas 80 cm long, a pressure of about 1 mm, and an exposure of four hours. With smaller pressure the absorption was too weak to give clear lines; with greater pressure the bands were not completely broken up into lines. Similar effects were produced with different exposures; with too long an exposure, too much was transmitted; with too short an exposure, not sufficient light was transmitted to give clear lines.

RESULTS

The spectrum under different pressures is shown on Plate IX. The positives from which the reproductions are made are threefold enlargements of the original negatives. Much of the detail of the negatives is lost even in the positives, but for reproduction, enlargement was necessary, as it requires a magnifying glass to show the lines on the negatives.

The spectrum is shown on Plate IX under the following conditions:

Fig.	Length of Column of Gas	Pressure of Gas
a	20 cm	1 atmos.
b	80	3 mm
c	20	5
d	80	0.8

The spectrum was found to consist of about 590 lines lying between λ 2707 and 3120; the absorption extended to shorter wave-lengths but the lines were much less clearly defined. The wave-lengths were determined as described above and expressed in Ångströms; the wave-numbers were then calculated as $\frac{1}{\lambda}\times 10^8$ and therefore represent the number of vibrations executed while the wave is propagated through one centimeter. The wave-lengths, λ, of the lines, with their intensities, I, are given in Table II, the intensities being estimated by eye from 10, the greatest, to 1, the least.

TABLE II

λ	I	λ	I	λ	I	λ	I
2707.6	7	2780.85	7	2807.18	7	2829.79	8
10.28	6	81.43	8	07.56	7	30.38	9
22.33	8	81.96	6	08.03	6	30.85	10
25.84	7	82.50	6	08.41	6	31.38	10
26.22	7	82.98	6	09.30	6	31.83	10
26.73	7	83.47	7	09.66	6	32.24	10
28.04	5	84.01	7	10.07	7	32.78	10
28.56	5	84.52	7	10.3	7	33.47	9
29.03	5	84.93	7	10.73	8	33.86	9
33.42	8	85.22	7	11.03	7	34.27	9
35.66	7	86.38	6	11.51	8	34.79	9
36.61	7	86.69	6	11.88	7	35.15	9
37.48	6	87.02	6	12.28	8	35.58	8
37.86	6	87.30	6	12.70	7	35.91	8
38.61	6	87.72	8	13.20	7	36.23	8
39.2	8	88.24	8	13.60	7	37.46	8
39.92	6	88.66	8	14.21	8	37.85	8
40.46	6	88.96	9	14.57	8	38.19	8
40.8	6	89.50	9	14.85	8	38.63	8
41.18	6	90.06	9	15.35	9	39.02	7
41.65	5	90.58	8	15.80	9	39.33	7
42.0	6	91.06	8	16.1	9	39.79	6
42.3	5	91.49	7	16.5	9	40.29	7
42.84	5	91.82	7	16.93	9	40.55	7
52.94	2	92.14	7	17.22	9	41.00	7
53.43	3	92.68	7	17.59	9	41.61	7
53.91	3	93.07	7	17.99	10	42.06	7
54.4	4	93.37	7	18.46	10	42.46	7
55.02	4	93.86	6	18.71	10	43.15	8
59.12	5	94.32	6	18.97	10	43.60	8
60.02	5	94.74	6	19.26	9	43.99	8
60.69	5	95.13	6	19.65	9	44.40	8
61.32	5	95.58	6	20.13	9	44.74	8
61.97	5	95.93	7	20.48	9	45.27	7
62.80	5	96.65	8	20.79	9	45.60	7
63.39	5	97.13	9	21.12	9	45.91	7
64.29	5	97.62	9	21.53	9	46.31	7
64.75	6	98.08	8	21.78	9	46.81	8
65.24	7	98.91	7	23.42	6	47.27	8
65.75	6	99.31	7	23.76	6	47.68	9
66.16	4	99.70	6	24.16	7	48.16	9
66.52	4	2800.35	5	24.44	7	48.56	8
67.37	3	01.56	5	24.75	7	48.96	8
67.77	3	02.10	5	25.21	7	49.54	8
69.08	4	02.4	5	25.57	7	49.98	8
76.8	5	03.22	5	26.01	7	50.3	8
77.26	8	03.79	5	26.23	8	50.67	9
77.73	7	04.19	5	26.60	7	51.08	9
78.25	7	04.76	6	26.96	6	51.45	9
78.91	7	05.20	6	27.37	6	51.79	10
79.34	7	05.49	6	27.96	6	52.14	10
79.85	7	06.04	6	28.40	6	52.57	10
80.26	7	06.45	6	28.84	6	52.98	9
80.6	8	06.80	7	29.30	7	53.58	9

TABLE II—*Continued*

λ	I	λ	I	λ	I	λ	I
2853.92	9	2886.51	8	2920.2	8	2949.28	5
54.31	9	86.82	9	20.55	8	49.6	5
54.85	9	87.22	9	20.89	8	49.90	5
55.28	9	87.74	10	21.16	9	50.28	5
55.60	9	88.10	10	21.45	9	54.3	3
56.10	8	88.65	10	21.71	9	54.7	3
56.63	8	89.13	9	22.21	9	55.62	3
57.04	8	89.66	9	22.89	8	56.16	4
57.94	8	89.91	9	23.38	9	56.80	5
58.32	8	89.42	9	23.75	9	57.50	7
58.69	8	91.08	8	24.34	9	57.89	7
59.13	8	92.21	7	24.65	9	58.22	8
59.63	7	92.7	6	25.52	9	58.57	8
60.03	7	93.03	6	26.20	8	58.99	8
60.47	7	93.43	5	26.59	8	59.30	9
60.98	7	94.06	6	26.81	8	59.84	9
61.35	7	94.38	7	27.84	8	60.22	10
61.88	7	94.69	7	28.43	7	60.78	10
62.46	6	96.55	7	28.89	7	61.18	10
63.42	6	98.48	6	29.21	7	61.45	10
64.02	5	2900.27	9	29.56	6	61.98	10
64.9	7	01.15	9	30.01	6	62.40	10
65.64	8	01.56	8	30.33	6	62.83	9
65.98	9	01.96	8	30.69	5	63.27	9
66.5	10	02.62	8	31.10	5	63.72	9
66.82	10	03.30	8	31.72	5	64.82	8
67.38	10	04.37	8	32.95	4	66.04	8
67.87	10	04.8	7	33.50	4	67.40	5
68.62	9	05.20	8	34.69	3	68.05	5
68.97	9	05.81	8	35.11	4	68.34	5
69.21	9	06.14	8	35.52	4	69.09	5
69.66	9	06.49	10	36.38	5	69.76	5
70.31	9	07.17	10	37.2	5	70.51	3
70.76	9	07.90	10	38.07	10	71.70	1
71.32	9	08.2	9	38.5	10	72.91	1
72.19	8	08.6	9	39.10	9	73.57	1
72.72	8	08.98	8	39.63	10	74.33	1
73.1	8	09.86	8	40.06	9	74.8	1
73.72	8	10.13	8	40.41	9	75.89	1
74.1	8	12.04	5	40.79	9	78.2	8
74.43	8	12.36	5	41.18	9	78.55	8
74.95	8	13.28	5	41.57	9	79.13	9
75.55	8	13.55	5	41.97	9	79.85	8
76.92	6	14.3	4	42.83	9	81.66	5
77.62	5	15.25	4	43.40	9	81.99	5
78.01	6	16.1	5	43.93	9	83.40	6
78.35	7	16.53	6	44.39	9	84.79	6
79.86	5	16.82	7	44.82	9	86.13	5
82.73	3	17.42	8	45.64	9	88.02	4
83.5	3	17.74	8	46.09	9	88.77	4
83.79	4	18.31	9	46.65	8	89.46	3
84.4	4	18.94	8	47.23	8	90.27	2
85.1	5	19.35	7	48.55	6	93.8	2
85.93	8	19.66	7	49.00	5	98.0	5

TABLE II—*Continued*

λ	I	λ	I	λ	I	λ	I
2998.33	7	3021.18	10	3048.22	6	3083.21	5
99.0	9	21.62	9	49.46	6	84.02	6
99.56	9	22.15	9	49.82	5	85.63	7
99.81	10	22.68	9	50.46	4	86.20	7
3000.41	10	23.25	9	50.9	4	86.54	6
00.94	10	23.87	8	51.54	4	86.88	6
01.22	10	24.29	8	51.79	4	87.4	6
02.00	10	24.83	8	52.43	4	87.99	5
02.37	9	25.40	8	53.7	3	89.49	5
02.93	9	25.65	7	54.03	2	89.79	4
03.53	9	26.10	7	54.76	2	90.30	3
04.15	9	26.51	7	55.31	2	90.62	3
04.50	8	26.87	7	55.7	2	91.24	3
05.43	7	27.15	6	56.0	2	99.27	½
05.8	7	27.64	6	56.62	2	3101.2	3
06.2	5	28.19	6	57.15	2	03.0	½
06.55	4	28.72	6	57.73	2	04.16	1
06.96	4	29.25	5	58.38	2	05.29	2
07.40	4	29.87	5	61.72	7	05.81	2
07.99	3	30.4	5	62.48	8	06.36	5
08.58	3	33.8	3	63.11	8	06.89	5
09.35	3	34.7	3	63.7	8	07.43	4
09.87	3	35.2	3	64.10	7	08.08	2
10.33	2	37.1	2	64.4	8	08.66	1
10.87	2	37.6	2	65.46	8	09.26	1
11.42	2	38.0	2	65.71	7	09.86	3
12.29	2	38.5	2	66.49	7	10.73	3
13.00	2	39.5	3	67.0	6	11.18	1
13.56	2	40.07	4	67.76	6	11.77	1
13.88	2	40.8	3	70.02	2	12.27	2
14.82	1	41.43	10	70.57	4	13.61	2
15.28	1	41.79	10	71.20	4	14.18	2
15.86	1	42.03	10	75.31	3	16.1	2
16.59	2	42.40	9	78.3	3	16.9	1
18.1	2	43.14	9	79.0	3	17.3	1
18.83	8	44.0	8	79.74	3	17.8	1
19.26	8	44.96	8	80.31	3	18.2	1
19.63	9	46.00	8	81.5	3		
19.98	10	47.20	6	82.03	4		

STRUCTURE OF THE SPECTRUM

A curve was plotted having wave-numbers as abscissae and intensities as ordinates (see Fig. 1). The regularity of this curve suggested a relationship between the wave-numbers of lines of relatively equal intensity. With this clue as a guide, it was found that 92 per cent of the lines could be arranged in groups or series of lines, in each of which the *first* differences of consecutive wave-numbers were approximately constant, or that each series of wave-numbers formed an

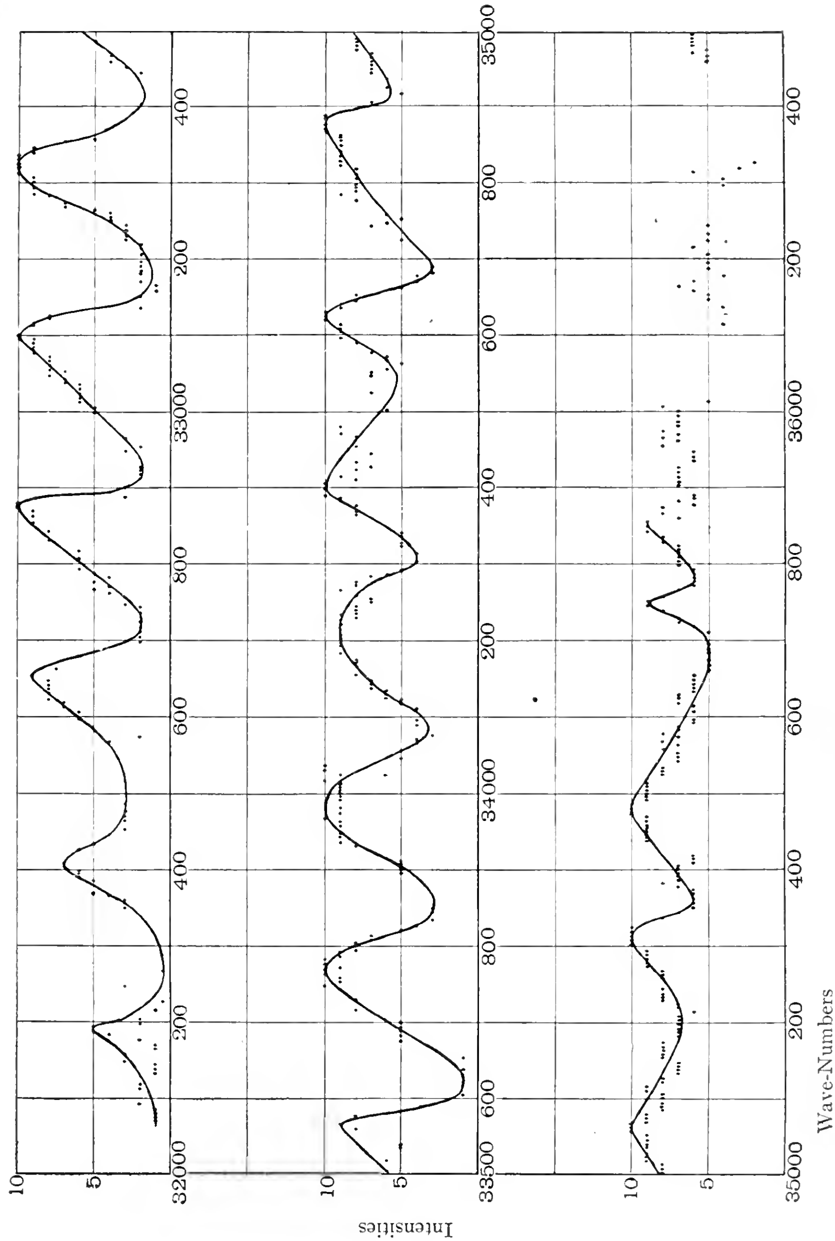


FIG. 1.—Curve of Wave-Numbers and Intensities of Absorption Lines

arithmetical progression; further, it was found that all these progressions, 44 in number, had approximately the same common difference, namely, 223. This equality of first differences is expressed by the simple equation:

$$N = a + bm \tag{1}$$

where

N = wave-number
a = constant
b = the common difference
m = the number of the term in its series,
= 1, 2, 3 . . . . .

It is to be noted that the spectrum of sulphur dioxide does not obey Deslandres' laws of equal *second* differences, between the lines of a band or between the heads of bands—laws which, as is well known, are expressed by a quadratic equation of the form

$$N = a + bm + cm^2 ,$$

where the first differences form arithmetical progressions. In the spectrum of sulphur dioxide the first differences are constant and the wave-numbers themselves form arithmetical progressions. Of the 586 lines in the region measured, only 47 do not appear in these 44 series; five of these 47 lines form a short series with the same mean common difference as the others; it is between series 43 and 44. In each series the greatest deviation of the common differences from their mean value is less than 1 per cent.

The 44 series of wave-numbers, with their first differences and intensities, are given in Table III; for convenience in reference, the wave-length of the line at the head of each column is inclosed in brackets above the column of differences. The wave-numbers which do not appear in this table are collected in Table IV, the five which form a short series being in the first column. In these tables N denotes the wave-number, D the first difference in wave-numbers, m the number of the term in the series, and I the intensity. Of the series found, that of lowest wave-numbers has been called the first; similarly for the terms in the series. Whether this series is really the first has not been determined; Plate IX shows that the spectrum extends into longer wave-lengths when the gas is at higher pressures,

but at these pressures measurable lines have not been obtained; hence it is undetermined whether or not the series extend into the region of longer wave-length. Thus the constant, a, in equation (1) has not been found. Apparently each series should have 23 terms, but none of the series is complete.

TABLE III

	Series 44			Series 43			Series 42		
m	I	N	D	I	N	D	I	N	D
23									
22									
21				7	36680.8	(2726.22)			
						222.2			
20	5	36466	(2742.3)	5	36458.6				
			223						
19	5	36243.4					5	36231.6	(2760.02)
									224.9
18			2×225.3			3×223.9	8	36006.7	
									225.2
17	6	35792.8		6	35786.9		6	35781.5	
			224.7			223.6			223.2
16	8	35568.1		7	35563.3		8	35558.3	
			223.7			225.0			
15	7	35344.4		8	35338.3				2×224.0
			223.0			222			
14	8	35121.4		9	35116		9	35110.4	
			225.2			224			224
13	8	34896.2		9	34892.1		10	34886.	
						223			225
12			2×222.3	4	34669		5	34661	
						225			
11	8	34451.6		7	34443.6				2×224
			222.0			222.9			
10	8	34229.6		9	34220.7		9	34212.7	
			225.1			225.2			221.9
9	9	34004.5		9	33995.5		9	33990.8	
			223.2			225.2			223.6
8	10	33781.3		9	33770.3		10	33767.2	
			222.6						
7	8	33558.7				2×223.7			2×223.8
			223.3						
6	10	33335.4		10	33322.9		10	33319.7	
						223.3			224.9
5				10	33099.6		9	33094.8	
						224.2			222.0
4			3×224.7	10	32875.4		10	32872.8	
						222.1			
3	7	32661.4		9	32653.3				2×223.6
2			2×223.3				6	32425.3	
									222.2
1	1	32214.8					2	32203.1	
	Mean differences		223.7			223.8			223.8

TABLE III—*Continued*

	Series 41			Series 40			Series 39		
m	I	N	D	I	N	D	I	N	D
23									
22	6	36896.6	(2710.28)						
			222.6						
21	7	36674.0							
20			2×225.6						
19	5	36222.8					6	36214.6	(2761.32)
			222.2						
18	7	36000.6		7	35993.9	(2778.25)			2×224.2
			224.1			223.1			
17	6	35776.5		6	35770.8		7	35766.3	
			223.5			224.1			224.6
16	7	35553.0		7	35546.7		7	35541.7	
			221.9			221.6			223.2
15	9	35331.1		10	35325.1		10	35318.5	
			225.6			224.6			225.1
14	8	35105.5		8	35100.5		8	35093.4	
			223.6			225.5			224.3
13	10	34881.9		10	34875.0		10	34869.1	
						224.1			225.2
12			2×225.5	8	34650.9		8	34643.9	
						225			222.9
11	8	34430.9		7	34426		8	34421.0	
			223.9			223			
10	9	34207.0		9	34202.6				
						221.7			
9			2×222.9	9	33980.9				
						224.5			
8	10	33761.2		9	33756.4				
			222.8			221.7			
7	5	33538.4		5	33534.7				9×223.6
						223.6			
6			3×223.2	10	33311.1				
						222.0			
5				9	33089.1				
4	9	32868.8				2×224.6			
			222.3						
3	8	32646.5		8	32640				
2			2×224.4			2×224	7	32408.3	
1	2	32197.7		5	32192.0				
	Mean differences		223.7			223.6			223.9

	Series 38			Series 37			Series 36		
m	I	N	D	I	N	D	I	N	D
23									
22									
21									
20									
19							5	36194.8	(2768.80)
									221.6
18	7	35979.8	(2779.34)				8	35973.2	
			222.7						222.3

TABLE III—*Continued*

	Series 38			Series 37			Series 36		
m	I	N	D	I	N	D	I	N	D
17	8	35757.1					9	35750.9	
			223.1						225.0
16	8	35534.0		8	35529.4	(2814.57)	8	35525.9	
			221.3			221.7			224.9
15	10	35312.7		10	35307.7		10	35301.0	
			224.9			224			221.5
14	8	35088.0		8	35084		9	35079.5	
						224.0			223.8
13			2×223.9	9	34860.6		9	34855.7	
						224.6			
12	9	34640.2		9	34635.4				2×222.9
						221.6			
11			2×222.2	9	34413.8		8	34409.9	
						221.7			
10	9	34195.7		9	34192.1				2×223.5
			221.4			223.9			
9	9	33974.3		10	33968.2		9	33962.9	
			222.8			221.7			221.5
8	9	33751.5		10	33746.5		9	33741.4	
									222.6
7			2×222.3			2×222.8	6	33518.8	
									224.6
6	9	33307.0		9	33300.8		9	33294.2	
			223.8			223.8			223.9
5	9	33083.2		9	33077.0		8	33070.3	
			222.4			225			
4	9	32860.8		9	32852				2×224.4
			224.8						
3	8	32636.0				2×225	8	32621.5	
									222.7
2			2×224.8	7	32402.3		6	32398.8	
						221.4			224.6
1	5	32186.5		4	32180.9		2	32174.2	
	Mean differences		223.1			223.2			223.4
	Series 35			Series 34			Series 33		
23									
22									
21									
20									
19				5	36187.4	(2763.39)			
						223			
18	7	35967.9	(2780.26)	8	35964		7	35960.2	(2780.85)
			223.2			225			
17	9	35744.7		8	35738.8				2×225
			225.1			224.9			
16	9	35519.6		9	35513.9		9	35510	
						221.5			222
15			2×222.6	9	35292.4		9	35287.6	
						222.5			221.9

TABLE III—*Continued*

	Series 35			Series 34			Series 33		
m	I	N	D	I	N	D	I	N	D
14	9	35074.4		9	35069.9		10	35065.7	
			221.6			222.6			
13	9	34852.8		9	34847.3				
			223.6			222.5			
12	10	34629.2		10	34624.8				3×222.7
			224.6						
11	10	34404.6					10	34397.7	
			222.6						223.7
10	9	34182.0					8	34174.0	
			224.1						
9	9	33957.9							2×222.6
8							8	33728.9	
7			3×223.5			10×223.4			2×222.8
6	9	33287.4					8	33283.4	
			221.8						223.7
5	8	33065.6					8	33059.7	
			224.4						
4	8	32841.2							2×224.6
			222.3						
3	7	32618.9					7	32610.6	
			223.7						
2	6	32395.2		6	32390				
						222			
1				1	32168.2				
	Mean differences		223.3			223.3			223.3

	Series 32			Series 31			Series 30		
m	I	N	D	I	N	D	I	N	D
23									
22									
21									
20									
19	4	36175.7	(2764.29)	6	36169.6	(2764.75)			
			223.0			223.7			
18	8	35952.7		6	35945.9				
			224.6			222.8			
17	7	35728.1		7	35723.1		6	35718.1	(2799.70)
			223			223.5			222.1
16	9	35505		9	35499.6		9	35496.0	
			222						223.9
15	9	35282.5				2×221.8	9	35272.1	
			221.1						221.0
14	10	35061.4		10	35056.1		9	35051.1	
			222.0			222.1			223.9
13	9	34839.4		9	34834.0		9	34827.2	
			221.2			221.5			224.0
12	10	34618.2		9	34612.5		9	34603.2	
						223.4			222
11			2×224.4	10	34389.1		9	34381	
						222.1			
10	8	34169.5		8	34166.9				2×222
			221.0			223.6			

TABLE III—*Continued*

	Series 32			Series 31			Series 30		
m	I	N	D	I	N	D	I	N	D
9	9	33948.5		9	33943.3		9	33936.8	
									221.8
8			2×222.6				8	33715.0	
									2×223
7	6	33503.2				3×223.4			
6			2×224.8	7	33273.1		7	33269	
						222.3			223
5	8	33053.5		7	33050.8		8	33045.8	
			223.5						
4	8	32830.0							2×224.4
			225						
3	6	32605				4×223.8	6	32597.1	
			221						
2	5	32383.6							
			221.6						
1	1	32162.0		3	32155.8				
	Mean differences 223.0					223.0			222.9

	Series 29			Series 28			Series 27		
m	I	N	D	I	N	D	I	N	D
23									
22									
21									
20									
19	7	36163.2	(2765.24)	6	36156.6	(2765.75)	5	36151.2	(2766.16)
			224.3			223.9			224.8
18	6	35938.9		6	35932.7		7	35926.4	
						222.9			
17			2×223.8	5	35709.8				2×223.0
						223.6			
16	9	35491.3		10	35486.2		10	35480.4	
			225.1			224.1			222.3
15	8	35266.2		8	35262.1		8	35258.1	
			222.5			222.6			223.4
14	9	35043.7		9	35039.5		9	35034.7	
			222.5			222.9			224.5
13	9	34821.2		8	34816.6		8	34810.2	
									221.0
12			2×222.5				8	34589.2	
									223.3
11	8	34376.3				3×222.9	8	34365.9	
			221.4						223.3
10	8	34154.9		7	34148.0		7	34142.6	
9						2×224.2			
8			3×222.2	5	33699.5				4×222.8
7	5	33488.2				2×221.7			
			223						
6	5	33265		4	33256.1		4	33251.3	
			224			221.7			222.3
5	7	33041.4		6	33034.4		6	33029.0	
			224.4						222.9

TABLE III—*Continued*

	Series 29			Series 28			Series 27		
m	I	N	D	I	N	D	I	N	D
4	6	32817.0					6	32806.1	
3			2×224.6			3×223.2			2×223.4
2	5	32367.8		4	32364.7		3	32359.2	
			221.7			222.6			223.1
1	3	32146.1		1	32142.1		1	32136.1	
	Mean differences		223.2			223.0			223.1

	Series 26			Series 25			Series 24		
m	I	N	D	I	N	D	I	N	D
23									
22									
21							8	36584.2	(2733.42)
20									2×224.4
19	5	36146.5	(2766.52)				4	36135.4	
									222.6
18				7	35919.4	(2784.01)	7	35912.8	
						225.0			225.3
17			3×223.1	5	35694.4		5	35687.5	
						224.1			222.1
16	10	35477.2		9	35470.3		9	35465.4	
									222.6
15			2×224.6			2×223.8	8	35242.8	
									223.9
14	9	35028.1		9	35022.8		9	35018.9	
			223			224.7			225
13	8	34805		8	34798.1		8	34794	
						222.5			224
12			2×221	7	34575.6		6	34570	
11	8	34362.7				2×223.0			2×222
			223.8						
10	7	34138.9		6	34129.6		6	34125.9	
			223.9			223.0			223
9	6	33915.0		5	33906.6		5	33903	
			223.0						223
8	5	33692.0					5	33680.4	
			225.0						221.8
7	4	33467.0				3×222.8	4	33458.6	
			222.2						
6	3	33244.8		3	33238.3				2×223.6
			221.8			221.2			
5	6	33023.0		6	33017.1		6	33011.5	
						224.4			222.7
4				6	32792.7		5	32788.8	
									221.6
3			3×222.3			2×221.6	4	32567.2	
2	[illegible]	32356.0		3	32349.5				2×225.0
			225.1						
1	1	32130.9					2	32117.1	
	Mean differences		223.1			223.1			223.4

TABLE III—*Continued*

	Series 23			Series 22			Series 21		
m	I	N	D	I	N	D	I	N	D
23									
22									
21									
20									
19									
18	7	35907.5	(2784.93)						
			224						
17	5	35683		5	35673.3	(2803.22)			
			224			222.2			
16	9	35459.4		9	35451.1		9	35446.9	(2821.12)
			221.5			222.8			223.4
15	8	35237.9		8	35228.3		7	35223.5	
			225.1			222			223
14	8	35012.8		8	35006		8	35001	
			223.3			223			225
13	8	34789.5		9	34783.2		8	34776.0	
			223.7			222.1			222.5
12	6	34565.8		5	34561.1		6	34553.5	
						220.9			
11			2×222.0	5	34340.2				2×221.9
						223			
10	5	34121.7		5	34117		4	34109.7	
			222.3			222			
9	5	33899.4		5	33895.1				2×222.8
						222.3			
8				5	33672.8		3	33664.2	
									222.4
7			3×223.2			2×224.4	2	33441.8	
									222.9
6	3	33229.8		3	33224.0		2	33218.9	
			225.1						
5	5	33004.7				2×223.5			2×224.3
			222.9						
4	4	32781.8		5	32777		4	32770.3	
			221.3						
3	4	32560.5							
2			2×224.7						
1	2	32111.2							
	Mean differences		223.3			222.8			223.0

	Series 20			Series 19			Series 18		
m	I	N	D	I	N	D	I	N	D
23									
22									
21				7	36554.3	(2735.66)			
20									
19	4	36113.1	(2769.08)			3×223.1			
			224.2						
18	6	35888.9		6	35884.9		7	35880.6	(2787.02)
			222.9			224.8			
17	5	35666.0		5	35660.1				
			224.2			221.5			

TABLE III—*Continued*

	Series 20			Series 19			Series 18		
m	I	N	D	I	N	D	I	N	D
16	9	35441.8		9	35438.6				3×224.3
			222.2			224.7			
15	7	35219.6		6	35213.9		7	35207.7	
						223.7			222.1
14				8	34990.2		8	34985.6	
13			3×223.3			2×222.1			
12	7	34549.7		7	34546.0				4×222.6
			223.1			223.6			
11	5	34326.6		5	34322.4				
10							4	34095.4	
9									2×222.3
8			5×222.7			5×223.1	1	33650.8	
7									
6	2	33213.0		2	33206.9				
5			2×222.7			2×223.0			7×224
4	5	32767.7		4	32760.8				
3									
2						3×223			
1				2	32091		1	32083	
	Mean differences		223.0			223.1			223.4

	Series 17			Series 16			Series 15		
m	I	N	D	I	N	D	I	N	D
23									
22									
21				7	36541.6	(2736.61)	6	36530.0	(2737.48)
						223.3			224.6
20	2	36324.8	(2752.94)	3	36318.3		4	36305	
19			2×223.9			2×223.4			2×223
18	6	35877.0		8	35871.6		7	35859.5	
			223.3			223.5			222.1
17	6	35653.7		6	35648.1		6	35637.4	
									223.6
16			2×224.6			2×224.6	6	35413.8	
									222.5
15	7	35204.4		7	35198.9		7	35191.3	
			223.3			223.2			221.7
14	8	34981.1		8	34975.7		7	34969.6	
			221.7			224.8			223.4
13	6	34759.4		5	34750.9		6	34746.2	
									222.4
12			2×222.5			2×224.2	7	34523.8	
11	4	34314		4	34302.4				2×224.3
			224.7						
10	4	34089.0					3	34075.2	
9						3×221.8			2×222.8
8			4×222.9	1	33637.1		1	33629.6	
7						2×223.8			2×223.1
6	2	33197.3		2	33189.6		2	33183.4	
									221
5			2×225			2×223.0	3	32962	

TABLE III—*Continued*

	Series 17			Series 16			Series 15		
m	I	N	D	I	N	D	I	N	D
4	3	32747		2	32743.6				2×222.5
3							3	32517.0	
2			3×223			3×223			2×223.5
1	1	32079		1	32074		1	32070	
	Mean differences 223.5					223.4			223.0

	Series 14			Series 13			Series 12		
m	I	N	D	I	N	D	I	N	D
23									
22									
21	6	36724.9	(2737.86)						
20				4	36207.4	(2755.02)			
19			3×223.0			2×224.4			
18	9	35855.7		9	35848.7				
			223.5			220.9			
17	6	35632.2		7	35627.8		7	35622.9	(2807.18)
			223.4			222.6			221.5
16	6	35408.8		7	35405.2		7	35401.4	
			223.1			224.4			
15	7	35185.7		7	35180.8				2×224.2
			221.0			221.5			
14	7	34964.7		7	34959.3		7	34953.1	
			222.6						
13	7	34742.1							
12			2×225			3×224.0			3×223.1
11	5	34292		6	34287.3		7	34283.9	
			222			221.8			
10	4	34070.3		4	34065.5				
			221			222			
9	3	33849		3	33844				3×222.6
						223			
8			2×224	1	33621.1		1	33616	
7	2	33402							2×223.1
			222						
6	2	33179.8				3×223	2	33169.5	
									223
5			2×222.0	2	32952		3	32947	
						222			224
4	2	32735.8		2	32729.9		2	32723	
3									
2									
1									
	Mean differences 222.9					223.0			223.1

	Series 11			Series 10			Series 9		
23									
22				8	36733.2	(2722.33)			
21	6	36514.9	(2738.61)						
20									
19			3×224.5			4×224.6			

TABLE III *Continued*

	Series 11			Series 10			Series 9		
m	I	N	D	I	N	D	I	N	D
18	9	35841.5		8	35834.8		8	35828.7	(2791.06)
			223.4			222.6			221.4
17	7	35618.1		6	35612.2		6	35607.3	
			222.6			221.1			221.7
16	7	35395.5		7	35391.1		7	35385.6	
			223.2			224.4			223.7
15	8	35172.3		8	35166.7		8	35161.9	
			223.8			224.6			
14	7	34948.5		7	34942.1				
			224.6						
13	5	34723.9							4×222.2
			223.1						
12	6	34500.8							
			223.8						
11	8	34277.0					8	34273.1	
			221.5						
10	5	34055.5							2×222.7
			221.6						
9	3	33833.9				10×222.6	4	33827.7	
									224.3
8							1	33603.4	
7			3×223.1						2×222.7
6	1	33164.5					1	33158.0	
5									2×223.8
4				2	32715.9		2	32710.2	
									224
3						2×225.1	3	32486	
2				½	32265.7				
1									
	Mean differences		223.4			223.4			222.9

	Series 8			Series 7			Series 6		
m	I	N	D	I	N	D	I	N	D
23							7	36933	(2707.6)
22									2×224
21	6	36497.4	(2739.92)	6	36490.2	(2740.40)	6	36485	
20									
19			3×224.7			3×223.7			3×223.3
18	7	35823.2		7	35818.9		7	35815.3	
						222.8			223.8
17			2×222.5	6	35596.1		6	35591.5	
						222.4			222.8
16	7	35378.2		6	35373.7		6	35368.7	
			221.4			221.1			222.6
15	8	35156.8		8	35152.6		7	35146.1	
			221.8						222.8
14	6	34935.0					6	34923.3	
13						4×223.4			
12			3×222.9						3×223.0
11	9	34266.4		8	34258.0		7	34254.2	
						223.0			223.3
10			2×223.0	10	34035.9		10	34030.9	
						223.6			223.0

TABLE III—*Continued*

	Series 8			Series 7			Series 6		
m	I	N	D	I	N	D	I	N	D
9	5	33820.4		7	33812.3		7	33807.9	
8									
7			3×223.4			4×223			4×223
6	2	33150.1							
			224						
5	2	32926		2	32921		2	32916	
			222			224			
4	2	32704.0		2	32697				2×223
3							3	32470.3	
									224
2							3	32246	
1									
	Mean differences 223.1					223.1			223.2

	Series 5			Series 4			Series 3		
m	I	N	D	I	N	D	I	N	D
23									
22									
21				6	36480.7	(2741.18)	5	36474.4	(2741.65)
20									
19						4×224			3×223.8
18	7	35807.9	(2702.68)				7	35802.9	
			221.6						224.8
17	7	35586.3		7	35583		8	35578.1	
						222			222.4
16			2×222.2	6	35361.2		6	35355.7	
						223.0			222.5
15	7	35142.0		7	35138.2		7	35133.2	
						220.0			
14				5	34916.0				2×222.0
13			4×222.8			2×223.4	3	34689.3	
									224.1
12				9	34469.1		8	34465.2	
						224.6			225.1
11	7	34250.6		8	34244.5		8	34240.1	
									222.2
10			2×223.2			2×222.2	10	34017.9	
									222.6
9	8	33804.1		8	33800.1		8	33795.3	
									221.9
8			2×224			2×224.1	8	33573.4	
7	5	33356		7	33351.9				2×224
			223						
6	2	33133					8	33125.4	
			222						225
5	2	32911					3	32900	
4			2×223						
3	3	32464.3							
2									
1									
	Mean differences 222.9					223.5			223.4

TABLE III—*Continued*

	Series 2			Series 1		
m	I	N	D	I	N	D
23						
22						
21	6	36470	(2742.0)			
20						
19			3×224			
18	7	35799.1				
			224.9			
17	7	35574.2				
			224.0			
16	6	35350.2				
			223.2			
15	8	35127.0				
			222			
14	7	34905				
13			2×222.5	4	34676.7	(2883.70)
12	8	34459.5				2×221.9
			223.4			
11	8	34236.1		9	34233.0	
			223.2			224
10	9	34012.9		9	34009	
						223
9			2×223.0	9	33785.6	
8	9	33566.8				2×223.6
			223			
7	9	33344		9	33338.3	
			223			221.2
6	8	33120.7		9	33117.1	
						223.1
5				4	32894.0	
4			3×223			2×223.9
3	3	32452		4	32446.2	
			225			
2	½	32227				
1						
	Mean differences		223.3			223.1

It is obvious from Table III that equality of the observed intensities is not rigidly held in the series; there is too great a probability of error between the observed and actual values to make strict conformity to such a condition needful.

TABLE IV

I	N	D	I	N	I	N	I	N	I	N
9	34000.0		7	36685.9	6	35644.4	9	34385	3	33450.8
		225.1								
10	33774.9		5	36656.4	10	35473.9	5	34336.5	4	33260.7
		2×223.0								
10	33328.8		5	36640.4	9	35455.0	9	34226.5	9	33112.8
		2×224.8								
10	32879.3		5	36643.1	6	35418.0	6	34134.8	7	33037.4
		2×222.8								
5	32433.7		8	36507	8	35382.8	5	34046	5	32999
			6	36312.0	9	35276.0	9	34024	3	32886
	Mean differ.	223.7	5	36206.0	8	35233.7	8	33930.2	8	32633
			5	36013	3	34680	5	33909.8	2	32573.1
			7	35985.3	9	34606.1	9	33791.8	3	32478
			7	35903.8	9	34597.1	5	33689		
			8	35864.9	9	34479.5	8	33577		

ERRORS

1. *In wave-lengths.*—The wave-lengths were determined by measurements made on the lines on two plates, six measurements of one plate and four of the other. The difference between the mean values of the wave-length of any one line obtained from the two plates is in many cases less than 0.1, in the majority of cases less than 0.15, although in some cases this difference amounts to 0.2 Å. The following are examples of the mean values of wave-lengths obtained from the two plates:

Plate 26	Plate 28	Plate 26	Plate 28
2811.85	2811.91	2916.83	2916.82
2812.27	2812.28	2917.39	2917.14
2812.69	2812.72	2918.20	2918.41
2813.16	2813.23	2918.87	2919.01
2813.52	2813.68	2919.66	2919.66
2814.23	2814.19	2920.09	2920.28
2814.55	2814.58	2920.89	2920.88
2814.79	2814.90	2921.46	2921.45
2815.37	2815.33	2922.24	2922.17
2815.77	2815.82	2922.87	2922.91

2. *In wave-numbers.*—Let

N = true wave-number,

λ = true wave-length,

ν = error in N,

e = error in λ.

Then

$$N-\nu=\frac{1\times 10^8}{\lambda+e},$$
$$=N-\frac{e\times 10^8}{\lambda^2}.$$

For the upper limit in the spectrum, $\lambda=3200$, hence $\nu=10e$; for the lower limit $\lambda=2700$, hence $\nu=13e$; thus an error of 0.1 Å in λ corresponds to an error of 1 or 1.3 in N. Some of the lines, whose wave-lengths have been given to the second place of decimals and their wave-numbers to the first place, may not be correct to 1 in those places, but they have been left to the nearest calculated value there when the probable error in the mean value of the wave-length was only about 0.03. This was done because giving the wave-numbers to the units place only would have frequently doubled the error in the first differences. The close agreement with one another of the mean values of the common differences of the arithmetical progressions seems to justify this course in many cases. For lines with larger probable errors, wave-lengths are given only to the first place of decimals and wave-numbers to the units place.

3. *In intensity.*—The want of accuracy in estimation by eye of the relative intensities of lines is well recognized. The difficulty in such estimation increases greatly with the number of lines to be observed, which here amounts to nearly 600. Further, errors must be present in the estimated intensity of absorption of those lines, which occur very near a broad metallic line strong enough to be transmitted through the gas. For example, the intensity of wave-number 33538.4 would most probably be represented by a higher number if full allowance were made for the increased intensity of the background due to the breadth of the metallic line 2980.79. Fig. 1 shows the position of some of the metallic lines transmitted; the absence of absorption lines at some of these places suggests that they are hidden by the metallic lines. The plates show several metal lines broadened by the capacity required to produce the continuous spectrum.

SIMILAR REGULARITY IN OTHER SPECTRA

1. *In emission spectra.*—Equal first differences in wave-numbers have been found by: Ames[1] in zinc and cadmium between the

[1] *Philosophical Magazine* (5), 30, 33–48, 1890.

frequencies of the first and second and between the second and third lines of most of the triplets; Kayser and Runge[1] in tin, lead, arsenic, antimony, and bismuth; Rydberg in copper[2] and in the red spectrum of argon;[3] Kayser[4] in the elements of the platinum group; Snyder[5] in rhodium, in the lines measured by Kayser; Olmsted[6] in the oblique series of barium, strontium, and calcium; Messerschmitt[7] in the heads of those bands of selenium which lie in the ultra-violet region. In these spectra, with the exception of the last two named, the constant first difference is different for different groups of lines of the same element.

In contrast with the above spectra, Professor Wood[8] has found in the resonance spectrum of sodium vapor series of lines having equidistant wave-lengths.

2. *In absorption spectra.*—Friedrichs[9] found in the absorption spectrum of Mn_2Cl_7 two kinds of bands, arranged in groups, those of one kind strong, the other weak, the groups alternating with each other. A constant first difference appeared between the wave-numbers of the edges of the first bands of the strong groups; also between those of the weak groups, as follows:

Strong Group		Weak Group	
N	ΔN	N	ΔN
1896		1919	
	76		76
1972		1995	
	76		76
2048		2071	
	76		74
2124		2145	
	77		
2201			
	77		
2278			

[1] "Ueber die Spectren der Elemente," 7 Abschrift, *Abhandl. Berl. Akad.*, 1894.

[2] *Astrophysical Journal*, 6, 239–243, 1897.

[3] *Ibid.*, 6, 338–348, 1897.

[4] "Ueber die Bogenspectren der Elemente der Platingruppe," *Abhandl. Berl. Akad.*, 1897.

[5] *Astrophysical Journal*, 14, 179–180, 1901.

[6] *Zeitschrift fur wissenschaftliche Photographie*, 4, 255–291, 293–333, 1906.

[7] *Ibid.*, 5, 249–278, 1907.

[8] *Astrophysical Journal*, 30, 339, 1909.

[9] *Zeitschrift für wissenschaftliche Photographie*, 3, 154–166, 1905.

Here the first differences are approximately the same for the two groups.

Another example of this structure in absorption spectra is in that of the vapor of paraxylene, found by Mies.[1] The approximately constant first differences are between the wave-numbers of the heads of bands, which are not resolved into lines; hence, as might be expected, there are comparatively large variations in the constants.

In an earlier work by Käbitz[2] on the spectrum of the vapor of CrO_2Cl_2 this simple structure can be found. He found five series of *Absorptionstreifen* (broad lines) in this spectrum; his values of the wave-lengths with their first and second differences are quoted below in Table V. The writer has calculated their wave-numbers and placed them in columns parallel with those containing the wave-lengths. The first four columns are quoted from Käbitz' paper; for the fifth and sixth columns the present writer is responsible. n denotes the number of the line in the spectrum.

It is obvious from the fifth and sixth columns of this table that the *first* differences of the wave-numbers of these series are approximately equal; further, that this constant difference, namely 135, is approximately the same for all the series. Hence this spectrum does not obey Deslandres' third law.

TABLE V

SERIES I. PRINCIPAL SERIES

n	λ	D_1	D_2	N	ΔN
3	5846.5			17104	
		46.3			137
8	5800.2		1.1	17241	
		45.2			135
13	5755.0		0.5	17376	
		44.7			136
18	5710.3		0.9	17512	
		43.8			136
23	5666.5		0.4	17648	
		43.4			136
28	5623.1		2.1	17784	
		41.3			131
33	5581.8			17915	

Mean $\Delta N = 135$

[1] *Zeitschrift für wissenschaftliche Photographie*, **7**, 357–368, 1909.
[2] *Ueber die Absorptionsspectra der Chlorsaüren*, Dissertation, Bonn, 1904.

TABLE V—*Continued*

SERIES II. SUBORDINATE SERIES I

n	λ	D_1	D_2	N	ΔN
4	5837.1			17132	
		45.3			134
9	5791.8		0.1	17266	
		45.2			136
14	5746.6		1.0	17402	
		44.2			135
19	5702.4		0.1	17537	
		44.1			136
24	5658.3		1.4	17673	
		42.6			134
29	5615.8		1.3	17807	
		41.3			132
34	5574.4			17939	

Mean $\Delta N = 135$

SERIES III. SUBORDINATE SERIES II

n	λ	D_1	D_2	N	ΔN
5	5827.5			17160	
		46.1			137
10	5781.4		1.2	17297	
		44.0			135
15	5736.5		0.6	17432	
		44.3			136
20	5692.2		0.5	17568	
		43.8			136
25	5648.4		2.4?	17704	
		41.4			131
30	5607.0		0.6	17835	
		40.8			127
35	5567.2			17962	

(Omitting the last) Mean $\Delta N = 135$

SERIES IV. SUBORDINATE SERIES III

n	λ	D_1	D_2	N	ΔN
1	5865.9			17048	
		46.6			136
6	5819.3		0.4	17184	
		46.2			138
11	5773.1		1.3'	17322	
		44.9			135
16	5728.2		0.5	17457	
		44.4			137
21	5683.8		0.9	17594	
		43.5			136
26	5640.3		0.6	17730	
		42.7			135
31	5597.6			17865	

Mean $\Delta N = 136$

SERIES V. SUBORDINATE SERIES IV

n	λ	D_1	D_2	N	ΔN
2	5855.3			17070	
		46.1			135
7	5809.2		0.8	17214	
		45.3			135
12	5763.9		0.9	17349	
		44.4			135
17	5719.5		0.7	17484	
		43.7			135
22	5675.8		0.9	17619	
		42.8			134
27	5633.0		0.8	17753	
		42.0			133
32	5591.0			17886	

Mean $\Delta N = 135$

CONCLUSION

The spectra of SO_2 and CrO_2Cl_2 exhibit a very simple and definite structure, which is built up of a number of series of lines characterized by equidistant frequencies, this equal distance having approximately the same value in every series; for SO_2 it is 223, for CrO_2Cl_2 it is 135. Thus the different members of the structure are simple in themselves,

being expressed by arithmetical progressions, whose common difference is the difference in frequency. These members are united into a regular structure by the simple tie that the mean frequency difference of every series is approximately the same.

Bryn Mawr College
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THE ABSORPTION SPECTRA OF CERTAIN SALTS IN AQUEOUS AND NON-AQUEOUS SOLUTIONS

By FRANK L. COOPER

Introduction.—There is perhaps no other investigation connected with molecular vibrations which is of greater interest than that which endeavors to trace the connection of the spectrum of an element and that of its compounds.

It was first considered that an element preserved its spectrum when entering into combination, so that, for instance, the oxide of a metal would show only the metallic lines except in so far as oxygen lines might be visible. This idea had to be given up, but the absorption spectra of liquids were considered at first to be evidence in favor of the assumption of permanence of the spectrum of an element when combined with others. A large amount of work has already been done on the absorption of light by solutions, but there still remains some doubt regarding the nature of the "absorber," and it was for this reason that the present investigation was undertaken. Before it was entirely completed, however, a monograph by Jones and Anderson entitled "The Absorption Spectra of Solutions"[1] appeared, in which the authors arrived at some of the conclusions which were inferred from the present investigation. Since the method of investigation in the two cases is not entirely the same, it was thought desirable to give the results of the present investigation as confirming the conclusions of Jones and Anderson.

Before taking up in detail the work that is here recorded, a brief discussion of some of the more important work in the same field seems desirable.

Historical review.—We owe the first systematic investigation of the absorption spectra of solutions to Gladstone,[2] who examined the absorption of solutions of salts, each constituent of which was colored. He came to the conclusion that generally, but not invariably, the following law holds good: "When an acid and a base combine, each

[1] *Carnegie Institution of Washington, Publication No. 110*, 1909.

[2] *Phil. Mag.*, **14**, 418, 1857.

of which has a different influence on the waves of light, a solution of the resulting salt will transmit only that light which is not absorbed by either, or, in other words, which is transmitted by both." Thus, for instance, chromic acid in solution cuts off the more refrangible half of the spectrum, transmitting only the blue near F, in dilute solutions, while the less refrangible part is transmitted perfectly. The characteristic absorption of chromic acid remains when the acid is combined with such metals as copper, nickel, uranium, potassium, and chromium, but the salts formed by combination with copper and nickel also show their own influence, when combined with chromic or other acids, by absorbing part of the red end of the spectrum. Potassium salts are colorless when combined with colorless acids and, therefore, potassium chromate shows the same spectrum as chromic acid. Chromate of chromium forms an exception to the rule, for, although the absorption peculiar to chromic acid exists, the absorption visible in ordinary chromium salts does not appear. Gladstone also examined the effect of chlorine, bromine, and iodine when combined with different metals. The bromides of gold, palladium, potassium, and platinum gave spectra which are identical with the spectrum of bromine water; the same applies to a concentrated solution of bromide of copper which, in addition, shows the red absorption characteristic of copper. Similar results were obtained with the chlorides and iodides.

In pointing out that it is generally, though not universally, true that a base or an acid retains its absorptive properties in different combinations, Gladstone calls attention to the exception of ferric ferrocyanide which, when dissolved in oxalic acid, transmits blue light, although the same light is generally absorbed both by ferrocyanides and by ferric salts.

Babo[1] observed that cobalt chloride in concentrated solution was colored blue by absolute alcohol at ordinary temperatures. At more elevated temperatures a few drops of alcohol were sufficient to produce a blue color. A solution of calcium chloride or magnesium chloride readily changes the color of cobalt chloride to blue, especially at the boiling-points of the solutions. A concentrated solution of

[1] *Ber. über der Verhandl. der Gesell. für Beford. der Naturm. zu Freiburg i. B.*, No. 17, 283, 1857; *Jahresber.*, p. 72, 1857.

zinc chloride, on the other hand, gave only red color with cobalt chloride, even when the mixture was warm. Babo concluded that the change of color was due to the formation of a double salt, and that whenever there was a transformation from a reddish to a blue color there was hydration of the cobalt salt.

Erhard[1] studied the absorption spectra of some salts in which chromium plays the rôle of a base. He found that as a general rule these salts absorbed the yellow end of the spectrum transmitting the blue; the exact position of the maximum of absorption, however, and the intensity of the absorption band varied considerably with different salts, and even for the same salt at different temperatures, and the results were complicated by the fact that heating the salts produced a permanent alteration in the absorption. The insoluble chloride of chromium showed a behavior differing from the other chromium salts. There was, however, a general resemblance in the absorption of different chromium salts but no identity.

Nitric acid and the nitrates of transparent bases such as potassium, sodium, and ammonia showed spectra according to Soret[2] which were not only qualitatively, but also quantitatively identical; that is to say, a given quantity of nitric acid in solution gives a characteristic absorption band of exactly the same width and intensity whether by itself or combined with a transparent base. It also showed a continuous absorption at the most refrangible side, beginning with each of the salts mentioned, at exactly the same position in the spectrum. The etherical nitrates,[3] however, gave different results.

In most of the cases hitherto discussed, the characteristic absorption of the substance under examination extended over a considerable range; the substance either absorbed a large part of the light or at least showed absorption bands which were broad and increased considerably in width with increase of concentration. When, however, absorption bands become narrower and more definite, so that they can be examined under high dispersion, their behavior under different circumstances becomes more interesting, for we can trace smaller differences and more minute changes.

[1] Inaugural Dissertation, Freiburg, without date.

[2] *Bibliothèque Universelle Arch. Sc. Ph.*, **61**, 322, 1878.

[3] *British Assoc. Rep.*, p. 55, 1880.

It was Bunsen[1] who first showed that such small changes do occur. While studying different salts of neodymium, he found that although all the salts showed spectra so nearly identical that, with the ordinary one-prism spectroscope, they could easily be mistaken for each other, higher dispersion revealed some very interesting and characteristic changes. His conclusions are best quoted in his own words:

> Very remarkable and noteworthy are the small alterations in position which occur in the minima of brightness in the didymium spectrum, depending upon the nature of the compound in which the metal occurs. These changes are too minute to be seen with the small, though seen with the large, instrument. I have as yet investigated them completely only in the case of three didymium salts, viz., the chloride, sulphate, and acetate. It is, however, more than probable that the same phenomena will also be found to occur with the absorption spectra of other crystals of didymium salts, and perhaps may be exhibited with the luminous spectra of the oxide and other compounds of didymium. The atomic weight of didymium chloride is 95.9, and that of the anhydrous acetate is 106.9. It will be noticed that all the groups of bands in the case of the salts under examination approach the red end of the spectrum in the order of their increasing atomic weight.
>
> These differences here noticed in the absorption spectra of different didymium compounds cannot, in our present complete state of ignorance of any general theory for the absorption of light in absorbing media, be connected with other phenomena. They remind one of the slight and gradual alterations in pitch which the notes from a vibrating elastic rod undergo when the rod is weighted, or of the change of tone which an organ pipe exhibits when the tube is lengthened.

Some interesting cases of the shifting of bands in different compounds of the same metal have been found by Russell,[2] who subjected the cobalt salts to a very careful and most instructive examination. He worked with a fused chloride, with its solution in concentrated hydrochloric acid, and with its solution in various alcohols. He showed that the spectrum in the concentrated acid was very similar to that of the fused salt, the bands being displaced a little toward the blue. The spectrum of cobalt chloride, when dissolved in the various alcohols and in glycerol, were practically the same, independent of the nature of the solvent.

Russell also worked with aqueous solutions of different concentrations, studying their absorption spectra and observing the effect of temperature-changes. He concluded that the color of the aqueous solutions was due to the presence of hydrates.

[1] *Phil. Mag.* (4), **32**, 177, 1866.

[2] *Proc. Roy. Soc.*, **37**, 258, 1881.

W. H. Hartley,[1] in his elaborate investigation of absorption spectra, has included a number of salts of cobalt. His experimental work consisted in observing and photographing the spectra of a large number of solutions of chlorides, bromides, iodides, nitrates, etc., of a fairly large number of metals, including cobalt. Some of the more interesting and important conclusions at which he arrived, stated in nearly his own words, are the following: When a definite crystalline hydrate is dissolved in a non-aqueous solvent, upon which it does not act chemically, the molecules of the salt remain unchanged in chemical composition.

In a series of anhydrous salts which do not form definite crystalline hydrates, the effect of rise in temperature up to 100° C. does not produce any alteration in their absorption spectra other than that which results with substances which undergo no chemical change with such rise in temperature. The change in question is usually an increase in the intensity of the absorption, or a slight widening of the absorption bands. Crystallized hydrated salts dissolved in a minimum amount of water at 20° C. undergo dissociation when the temperature is raised. The extent of the dissociation may proceed as far as complete dehydration of the compounds, so that more or less of the anhydrous salt may be formed in the solution.

The most stable compound that can exist in a saturated solution at 16° C. or 20° C., has not always the same composition as the molecule of the crystallized solid at the same temperature, since the solid may undergo a partial dissociation from its water of crystallization when the molecule enters into solution. When a saturated solution of a colored salt suffers a great change in color or any remarkable change in its absorption spectrum upon dilution, the dilution is always accompanied by a marked evolution of heat.

Hartley[2] closes his paper on "The Absorption Spectra of Metallic Nitrates" with the following significant paragraph, as quoted by Jones and Anderson:

The ultimate conclusion drawn from this work is that the operations of dissolving a salt and diluting the solution do not cause a separation of the compound

[1] *Trans. Roy. Dub. Soc.* (2), 7, 253–312, 1900; *Jour. Chem. Soc.*, 81, 571, 1902; 83, 221, 1903.

[2] *Jour. Chem. Soc.*, 83, 245, 1903.

into ions, but only a dissolution of such a character that the molecule is shown to consist of two parts, the movements of one being influenced by those of the other, so that the molecule of the salt is, in fact, not completely resolved into ions, but is in a condition of molecular tension. The application of external energy, such as light or electricity, may, however, readily cause a separation such as may be brought about by electrolysis, or by static electricity, and in some instances by photographic action.

Donnan and Bassett,[1] working with cobalt salts, came to the conclusion that the blue color of solutions of cobalt salts is due to the formation of complex anions containing cobalt.

Hartley[2] calls attention to certain inaccuracies in the above paper. It is not at all certain that, when hydrochloric acid is added to a solution of cobalt chloride, the blue color is due to the same cause as when an aqueous solution is heated. Indeed the absorption spectra in the two cases are quite different. Hartley points out further that the color of hot aqueous solutions was not supposed to be due to the anhydrous cobalt chloride, but to the dihydrate. Again, the spectra of a solution of cobalt chloride saturated at 20°, and taken at the temperatures 23°, 33°, 43°, 53°, 73°, and 93° C., are all quite different from the cobalt chloride to which hydrochloric acid had been added.

Of the more recent work, that of Jones and Uhler[3] is very important. They worked with aqueous solutions of copper and cobalt salts, to which some dehydrating agent was added; and also with non-aqueous solutions of these salts to which water was added.

They found that in all cases where the ion would be expected to be hydrated the most, the bands were the narrowest.

Jones and Strong,[4] working with salts of potassium and uranium, found that the absorption bands of uranyl nitrate in water are all farther toward the violet than for any other uranyl salt investigated, or for uranyl nitrate in other solvents.

THE PRESENT INVESTIGATION

In the present investigation, two entirely distinct methods have been used. In the first method the spectrum of the salt was examined

[1] *Jour. Chem. Soc.* (London), **81**, 939, 1902.

[2] *Ibid.*, **83**, 401, 1903.

[3] *Carnegie Institution of Washington, Publication No. 60*, 1907.

[4] *Proceedings Amer. Phil. Soc.*, **48**, No. 192, 1909.

visually with a spectroscope and a quartz spectrograph and also photographed by means of a large spectrograph, while the condition of the salt remained the same. By the other method, the changes in the absorption spectrum of the solution were studied with both a spectroscope and a quartz spectrograph, while a chemical change was taking place. So in the first method the condition of the substance was the same throughout the investigation, while in the second case the condition was slowly varying.

Apparatus.—Three spectroscopes were used in this work; the first, which we will designate as the small spectroscope, consisted of one 60° prism; the second had six 60° prisms through each of which the light was made to pass twice. The third spectroscope was of the direct-reading type and was obtained from Hilger of London. A description of this form of instrument has been given by Jones and Uhler.[1] The spectrograph consisted of a plane Rowland grating of about 14,000 lines to the inch. The light after passing through the slit was rendered parallel by a condensing lens, and after striking the grating it was brought to a focus at the camera by means of a second lens, each lens having a focal length of about two and a half feet. The whole apparatus was inclosed in a light-tight box and rested upon a steel frame in the bottom of the box.

Through the courtesy of Professor H. A. Bumstead of the Sloane Physical Laboratory, it was also possible, for part of the work, to use the smaller Rowland concave grating belonging to that laboratory. This grating has 14,000 lines to the inch and is of 11 feet radius of curvature.

Sources of light.—For wave-lengths between the extreme red and below about 0.326 μ the Nernst glower was used. These were 220-volt D.C. glowers carrying a current of 0.5 of an ampere. A variable resistance was placed in the circuit to maintain a constant current. For wave-lengths between the strong ultra-violet of the Nernst glower and the extreme ultra-violet, the arc with Norway iron electrodes was used. It was also used as the source of light when working with the concave grating.

Photographic material, etc.—The plates used were the "panchromatic" plates made by the Wratten & Wainright Co., of Croydon,

[1] *Carnegie Institution of Washington, Publication No. 60*, p. 170, 1907.

England. These plates were of very thin glass and were found to be very uniformly sensitive to light of all wave-lengths, from the very short wave to about λ 7400. Besides, as a check on the results, considerable work was done, making use of Seed's Gilt Edge plates No. 27, Cramer's Trichromatic, and also the Standard Orthonon plates. The developer used was a hydroquinone solution made up according to Jewell's[1] formula.

Solutions.—The solutions were made up in the following manner, which is the same method used by Jones and Uhler:[2] A chosen volume of mother-solution of a colored salt was measured out from a burette into a measuring flask of known capacity. The portion of the solution in the flask was then diluted by the addition of pure water until the volume of the resulting homogeneous liquid was exactly equal to the fixed capacity of the flask. The concentrations will always be expressed as multiples of normal. The term "normal" will be used to mean gram-molecular normal, i.e., a liter of solution which contains just as many grams of anhydrous salt as there are units in the number expressing the molecular weight of the salt is defined as normal.

Cells.—There were three cells used in this work, each of which was made with quartz-plate sides. They were constructed so that the length of the absorbing layer would be 1 cm, 2 cm, or 4 cm, according to the cell used.

RESULTS—FIRST METHOD

Solutions of cobalt salts.—Several salts of cobalt have been examined in this investigation. Plates have been obtained for aqueous solutions of cobalt chloride, bromide, iodide, sulphate, nitrate, nitrite, and acetate. These salts were also studied when dissolved in absolute ethyl alcohol, absolute methyl alcohol, and glycerine, also qualitatively when the salts were dissolved in various acids; some of the oxygen salts, however, were not absolutely anhydrous.

The concentrations of the solutions used in making the spectrograms varied from 2.0 normal to 0.1 normal, each of the solutions

[1] *Astrophysical Journal*, **11**, 240–243, 1900.

[2] *Carnegie Institution of Washington, Publication No. 60*, 1907.

being examined with each cell so that the lengths of the absorbing layer were 1 cm, 2 cm, and 4 cm.

The solutions were examined with both the Hilger spectroscope and the quartz spectrograph as well as with the grating spectrograph. Whenever the absorption spectra of two aqueous solutions, for example, were compared, it was for corresponding concentrations. In making the exposures with the spectrograph, the light of both the Nernst glower and the iron-arc was used, although most of the spectrograms were obtained with the Nernst lamp. In studying the ultra-violet region of the spectrum with the quartz spectrograph, the light from the iron-arc was used.

Cobalt chloride in water.—Commencing with the smallest and increasing the concentration, the absorption bands appeared in the following order: There were two regions of absorption in the ultra-violet, one in the extreme ultra-violet, and the other near the visible spectrum at about λ 3300. As the concentration increased, these two bands came together, appearing as one band and extended toward the red to about λ 4000.

The second band to appear was in the green, center at about λ 5200, although this band appeared about the same time as the band at λ 3300. With increasing concentration, this band widened, extending from λ 4730 to λ 5340. In very concentrated solutions, a faint band appeared at about λ 6090 and also one in the red extending from λ 6750 to λ 6900. With the smaller concentrations, these last two bands could not be seen without increasing the length of the absorbing layer.

Cobalt bromide in water.—In this case as with the cobalt chloride solutions, the absorption bands in the extreme ultra-violet appeared first, although the solutions of cobalt bromide were more transparent in this part of the spectrum. The band which, with the chloride solution, extends from about λ 3000 to λ 4000 in very concentrated solutions was not present in the spectra of the bromide solutions.

The second band to appear was the strong band in the green. It first appeared at λ 5200, that is, at the same position in the spectrum as was found for the corresponding band in the green with the chloride.

The red band could not be seen in the more dilute solutions, but as the concentration was increased a faint band appeared with the center at about λ 7000.

Cobalt iodide in water.—Owing to the ease with which the iodide of cobalt dissociates, its absorption spectrum was more difficult to study. Several spectrograms have been obtained, however, which show the general nature of the spectrum, although they are not definite like those obtained for the chloride and bromide solutions.

The absorption spectra of these solutions were also examined carefully with the small spectroscope, the Hilger direct-reading spectroscope, and with the quartz spectrograph. It was impossible to determine the limit of transmission in the ultra-violet, but the band in the green was present, although its exact position was not determined.

There was also a very slight absorption in the red, but this band was farther toward the red end of the spectrum than with either the chloride or bromide. It should be said, however, that its position was determined from the ethyl alcohol and not from the water solution, and therefore would not be the position of the band in the aqueous solution, could it have been determined.

Cobalt sulphate in water.—The spectrum of the sulphate solutions differed from those of cobalt chloride, bromide, and iodide, in that they were entirely transparent in the red for all concentrations, as far as λ 7400. The dilute solutions were also quite transparent in the ultra-violet, but with increasing concentration the absorption in this region increased. As there were no sharp absorption bands in the ultra-violet, it was impossible to determine the exact limits of transmission in this region.

In the green there was strong absorption and the band was more intense. The center of the band was in about the same position in the spectrum as was found with the cobalt chloride and bromide solutions.

Cobalt nitrate in water.—The spectrograms, as well as the visual observations, showed two distinct regions of absorption, one in the green and the other in the ultra-violet. The center of the band in the green appeared to have moved toward the violet end of the spectrum, but this can be explained, in part at least, by the fact that the band did not widen symmetrically as the concentration of the solution increased. This explanation was confirmed by the visual observations with the Hilger spectroscope.

The absorption in the ultra-violet was probably due to two bands, although they were not resolved.

All the solutions transmitted the red as far as λ 7400, in this respect also being similar to the spectra of the solutions of cobalt sulphate.

Cobalt nitrite in water.—The work with cobalt nitrite was taken up especially to ascertain whether the spectra of the solutions of two salts, differing in composition only in the amount of oxygen contained, were similar. For this reason all the work was limited to the visible part of the spectrum.

By referring to Fig. 1 it will be seen that the spectra in the two cases are not the same. The spectrum of the nitrate shows only a band in the green, while the spectrum of the nitrite solution of the same concentration and length of absorbing layer, shows also a strong band in the blue and violet. It will be noticed, however, that the green band is in exactly the same position and is of the same width in the two spectra, showing that it must be due to the same kind of "absorber."

Cobalt acetate in water.—As was found with the solutions of cobalt nitrate and cobalt sulphate, there were also two regions of strong absorption with the solutions of cobalt acetate, one in the ultra-violet and another in the green. With the more dilute solutions, the limit of transmission was in the extreme ultra-violet, but the absorption band widened, extending farther toward the red end of the spectrum, as the concentration of the solution was increased.

The center of the band in the green appeared to change position slightly as the concentration of the solution changed, but this can be explained by the fact that the band did not widen symmetrically.

The solutions easily transmitted the yellow, orange, and the red as far as λ 7400.

Cobalt chloride in absolute ethyl alcohol.—The concentration of the solutions used varied from 0.10 to 0.01 normal, and the lengths of the absorbing layer were 1 cm, 2 cm, and 4 cm.

The sources of light used in making the spectrograms were the Nernst glower and the iron-arc. For the work with the quartz spectrograph the iron-arc was used as a source of light.

The solutions of cobalt chloride in ethyl alcohol showed a region of absorption in the extreme ultra-violet and also two other bands

which were nearer the visible spectrum, about λ 3100 and λ 3500 respectively. The last two bands could be seen only with the more concentrated solutions.

The green band could not be seen except in the solutions of greatest concentration and then only faintly. It was evident, however, that the center of the band was farther toward the red end of the spectrum than in the aqueous solutions, about λ 5270, as determind by the Hilger spectroscope.

The first bands to appear, even with the most dilute solutions, were the bands in the red, while with the water solutions these bands were the last to appear, and then only in very concentrated solutions. With the ethyl alcohol solutions, five bands could be seen, although two were very faint; a narrow band with its center at λ 5920, a band extending from λ 6020 to λ 6073, a stronger band with its edges at λ 6160 and λ 6235. There also appeared to be a faint band about λ 6420 and two stronger bands, one extending from λ 6520 to λ 6605 and the other from λ 6809 to λ 7000. As the concentration increased, the absorption bands in the red gradually widened and appeared as one broad band.

It is significant that, with the ethyl alcohol solutions, the absorption bands in the red appeared first, even in very dilute solutions, and the band in the green could not be seen except when the solution has become concentrated, while just the opposite was true in the case of aqueous solutions of cobalt chloride. Here the band in the green was strong even in very dilute solutions, but there was complete transmission in the red except in the most concentrated solutions.

Cobalt bromide in absolute ethyl alcohol.—The concentration of the solutions used was the same as in the case of cobalt chloride.

The absorption spectra of the solution of cobalt bromide in absolute ethyl alcohol showed strong absorption in the extreme ultraviolet, but the band extended farther toward the visible spectrum than in the case of the solutions of cobalt chloride in ethyl alcohol. The limit of transmission in the case of the cobalt bromide solutions was about λ 3000. There was no trace of any other absorption bands nearer the visible spectrum corresponding to those at about λ 3100 and λ 3500, which were found with cobalt chloride.

In the green there was no absorption except a trace with the longest

layers of the most concentrated solutions. The position of this band was approximately the same as with the chloride, although it was too faint to be determined accurately. The bands in the red were similar to those of the chloride, although they appeared to be shifted slightly toward longer wave-lengths.

In order to be sure that we were dealing with an entirely different set of bands from those obtained with the chloride, the spectrum of the chloride was superimposed upon that of the bromide. The light was first allowed to pass through two like cells each containing a solution of cobalt chloride of a definite concentration, and a spectrogram made. The same thing was repeated using cobalt bromide solution of the same concentration in both cells. Then a third spectrogram was made, one cell being filled with the cobalt chloride and the other with the solution of cobalt bromide. The absorption band, in the latter case, was the result of the superposition of the two entirely distinct bands, showing that the "absorber" in case of the cobalt chloride solution must be entirely distinct from that which produced the red absorption bands in the spectrum of cobalt bromide.

Cobalt iodide in absolute ethyl alcohol.—The concentration of the solutions was the same as used in the case of cobalt chloride and cobalt bromide, the length of absorbing layer and the sources of light also being the same.

The same difficulty was experienced with the ethyl alcohol solutions of cobalt iodide as with the water solutions. The spectrograms, however, showed an apparent displacement of the bands toward longer wave-lengths, but the bands were not sharp so it was impossible to determine the exact limits of transmission.

Cobalt nitrate in absolute ethyl alcohol.—The concentration of the solutions and all other conditions were the same as those used with the other cobalt salts. Since every condition was the same, their spectra could be compared, but it should be stated that the cobalt nitrate was not absolutely anhydrous. No attempt was made to determine the exact position of the absorption bands in the solution of cobalt nitrate, the object being simply to ascertain whether there were any bands in the red region of the spectrum. There was strong absorption in the ultra-violet, and also in the green, but no trace whatever of any absorption in the red could be found as far as λ 7400, either from

the spectrograms or from visual observations. The band in the green could not be seen except with concentrated solutions. Since there were no absorption bands in the red, the color of the solutions of cobalt nitrate in absolute ethyl alcohol, and also in water, was always red. The same thing was true of the solutions of cobalt sulphate, acetate, and nitrite, all of which had no absorption bands in the red.

Cobalt chloride in absolute methyl alcohol.—The concentration of the solutions used was varied from 0.30 to 0.01 normal, and the lengths of absorbing layer were 1 cm, 2 cm, and 4 cm. The sources of light used were the Nernst filament and the iron-arc. There was a region of absorption in the extreme ultra-violet in about the same position in the spectrum as in the case of the solution of cobalt chloride in ethyl alcohol. The two bands at λ 3100 and λ 3550 were also present in about the same position, although the exact position of these bands in the ultra-violet was not determined. There was strong absorption in the green. This band was more intense than with the ethyl alcohol solution and was displaced slightly toward the violet, the center of the band being at λ 5245. It was not as strong, however, as the corresponding band with the aqueous solutions of the same salt. The bands in the red were more sharply defined than in the ethyl alcohol solutions. There was a narrow band at λ 5915 which appeared only in the more concentrated solutions. There was, also, a band having its center at λ 6045, which was seen in the spectrum of the concentrated solutions. The third band was about 70 Ångström units in width, the center being at λ 6125. At λ 6418 there was another narrow band; besides, there were two others, one extending between the limits λ 6525 and λ 6650, and the other having its center at λ 6700.

Cobalt bromide in absolute methyl alcohol.—The absorption in the extreme ultra-violet was stronger than with the chloride dissolved in methyl alcohol. There were no bands, however, corresponding to the bands at λ 3100 and λ 3550 in the spectra of the chloride solutions.

The bands in the red were seen only in the spectra of very concentrated solutions.

Cobalt salts in other solvents.—Besides the work above recorded a qualitative study of cobalt salts dissolved in different acids was also

made. For this work the small spectroscope, the large spectroscope with six 60° prisms, and the quartz spectrograph were used.

No attempt was made to determine the exact position or to resolve the bands, but only to show that certain bands were characteristic of certain salts, although the condition of the salt itself might have been changed when dissolved in the acids. The results are recorded in the following table:

BANDS

Solution	Red	Orange	Green	Violet	Ultra-Violet
$CoCl_2$ in HCl	very intense	intense	faint	faint	faint
CoC_2O_4 in HCl	intense		faint	faint	faint
$CoCO_3$ in HCl	intense	intense	faint	faint	faint
$Co(NO_3)_2$ in HCl	intense	faint	faint	very intense	faint
$CoSO_4$ in HCl	intense	intense	faint	faint	faint
$CoCl_2$ in $HC_2H_3O_2$	intense	faint	intense		intense
$CoCO_3$ in $HC_2H_3O_2$			intense		intense
$Co(NO_3)_2$ in $HC_2H_3O_2$	I.R.		intense	intense	intense
$CoSO_4$ in $HC_2H_3O_2$	I.R.		faint		intense
CoC_2O_4 in H_2SO_4			faint		intense
$CoCO_3$ in H_2SO_4			faint		intense

From the above table it will be seen that in every case where chlorine is present with the cobalt there is always strong absorption in the red, and in no case is there any absorption in the red if chlorine is not present.

In the case of cobalt nitrate and cobalt sulphate dissolved in acetic acid, a band appeared to approach from the infra-red and absorb part of the red. It is indicated in the table above by I.R. This absorption, however, could be observed only in the case of very concentrated solutions.

RESULTS—SECOND METHOD

Cobalt salt solutions.—For this part of the work both the small and the large spectroscope with six prisms were used. The method was as follows:

Light from a Nernst lamp was focused upon the slit of the spectroscope by means of a condensing lens, the light having passed through a cell 1 cm long containing absolute ethyl alcohol. Anhydrous cobalt chloride was then added to the alcohol until the bands in the red region of the spectrum became just distinctly visible. To

this solution of cobalt chloride in ethyl alcohol was then added a very small quantity of cobalt nitrate containing part of its water of crystallization. The spectrum was then examined carefully, and in a short time all the absorption bands in the red began to disappear slowly, while the band in the green gradually became stronger. This continued until all traces of any absorption in the red had vanished, although the band in the green grew more intense. A few drops of sulphuric acid were then added to the solution, and immediately the bands in the red reappeared.

It is reasonable to suppose that in the first case, that is, in the solution of anhydrous cobalt chloride in absolute ethyl alcohol, the "absorber" was the undissociated molecule of cobalt chloride, and that after the cobalt nitrate containing some water had been added to the solution, there was hydration and also dissociation of the cobalt chloride molecule sufficient to cause the absorption bands in the red to disappear. Since the chloride solution was made to such a concentration that the absorption bands in the red just appeared visible, the small amount of water present in the cobalt nitrate would be sufficient to cause this change in the spectrum.

From the odor and appearance of the solution it was suspected that oxidation of the chloride was slowly taking place. That such was probably the case was shown by the following experiment: The cell was cleaned and refilled with absolute ethyl alcohol. Then anhydrous cobalt chloride was added just sufficient to cause the absorption bands in the red to appear distinctly. A little cobalt nitrate containing some water of crystallization was added, as before, and the absorption bands in the red again began to disappear while the band in the green gradually became more intense. This continued until there was no trace of any absorption in the red region of the spectrum. The band in the green, however, continued to widen until the green, blue, and violet were absorbed, although the absorption in the violet was partly due to the bands in the ultra-violet region of the spectrum extending into the visible spectrum.

The solution was then allowed to stand for about fifteen hours and the spectrum examined at frequent intervals. After about five hours the spectrum appeared to be gradually changing, but this time it was neither the spectrum of cobalt chloride nor that of cobalt nitrate in

absolute ethyl alcohol. The band in the green gradually separated, and a strong narrow band appeared in the yellow with its center about λ 5850, and another band in the red at about λ 6020. The limit of transmission in the red was about λ 6070. There was also strong absorption in the violet and ultra-violet region as before. This spectrum is similar to that found by Russell for cobalt oxide.

The fact that this spectrum was not the same as either that given by the cobalt chloride or cobalt nitrate solutions is what would be expected if the "absorber" which produced the bands in the red was the undissociated molecule of the cobalt compound. Moreover, the absorption band in the green should always be present if the "absorber" which produced it was the cobalt atom or kation. That this was true was confirmed both by the spectrograms and by visual observations of the spectrum for every solution.

DISCUSSION OF THE RESULTS WITH COBALT SALTS

For the sake of comparing the spectra of the solutions of different salts of cobalt we have in Fig. 1 plotted the position of the bands in the spectrum of each salt studied. In some cases the edges of the bands are not sharply defined as indicated in Fig. 1, but the position of the center of each band was as represented. The limit of transmission in the ultra-violet, however, was not accurately determined.

The exact position of the absorption bands with the iodide solutions could not be definitely determined, although the spectrograms indicated a displacement of the bands toward the longer wave-lengths. The ultra-violet region of the spectrum of cobalt nitrite was not examined, as these solutions were studied only to ascertain whether the spectrum of a solution of cobalt nitrite differed from that of a solution of cobalt nitrate of the same concentration.

If we compare the spectra of the different salts in aqueous solutions, we find that the band in the green was always present, and in about the same position, especially in the dilute solutions. Moreover, the slight change in the position can easily be explained by the unsymmetrical widening of this band. This indicates that the band in the green must be produced by an "absorber" which is common to all of the salts, that is, that it must be due either to the cobalt atom or to the cobalt kation.

The view of Jones and Anderson that the "absorber" is the atom itself, and not the cobalt kation, is confirmed by the present work.

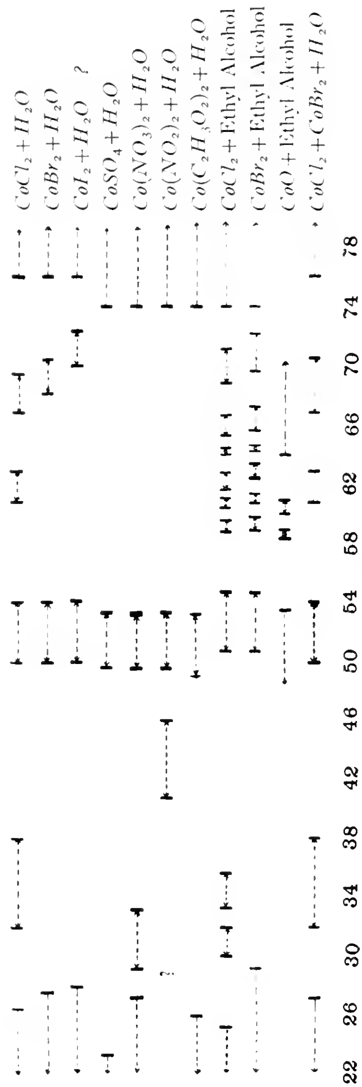

Fig. 1

If it were due to the cobalt kation, then, as stated by Jones and Anderson, the solutions which are strongly dissociated ought to show the

bands of much greater intensity than solutions which are less dissociated. This, however, was not the case, for the bromide and chloride, which are most dissociated, showed a much less intense absorption band in the green than did the acetate, sulphate, and nitrate, for the same concentration and length of absorbing layer.

The positions of the absorption bands in the red region of the spectrum were not the same for any two salts in aqueous solutions; but, on the other hand, some solutions had absorption bands in the red which are characteristic of that solution itself, while others, such as the nitrate, sulphate, and acetate, transmitted perfectly all the light as far as λ 7400. This shows that the "absorber" which produced the absorption in the red must be different for each solution, as we have already shown that the bands in the spectrum of a solution of cobalt chloride, for instance, were an entirely different set of bands from those in the spectrum of cobalt bromide. Moreover, the "absorber" which produced the absorption bands in the red must be different from that to which the band in the green is to be ascribed.

If we compare the spectrum of the aqueous solution of a salt with the spectrum of the same salt dissolved in ethyl or methyl alcohol we find that the position of the absorption bands was not the same. Moreover, the bands in the red region of the spectrum of an ethyl alcohol solution of cobalt chloride, for instance, always appeared first, and were always present even in very dilute solutions; the band in the green appearing only after the solution has become more concentrated. On the other hand, the band in the green region of the spectrum of an aqueous solution of cobalt chloride always appeared first, while the bands in the red could not be seen until the solution had become very concentrated. This is what we would expect if the absorption is produced by different kinds of "absorbers."

In the absolute ethyl alcohol solution there is very little dissociation, so we would obtain the spectrum of the undissociated molecule of cobalt chloride even in dilute solutions; while in the case of the more dilute water solutions there would be nearly complete dissociation and, therefore, the bands due to the molecule would not appear until the solution had become concentrated. Moreover, if the band in the green is due to the cobalt atom and not to the cobalt kation, then we would expect the intensity of this band to be roughly propor-

tional to the concentrations, as it was in the case of the solutions studied.

The most reasonable explanation of the fact that the absorption bands in the red region of the spectrum of an absolute ethyl alcohol solution of cobalt bromide, for instance, are not in the same position in the spectrum as the corresponding bands in an aqueous solution of the same salt, is that there is some "simple" hydrate of the molecule formed and that the "absorber" which produces these bands in the red is these "simple" hydrated molecules.

The spectrograms as represented in Fig. 1 show very clearly that the oxygen present in the molecule of the salt plays a very important rôle in its absorption, for in the case of cobalt nitrate and cobalt nitrite their only difference in composition is in the amount of oxygen in the radical which is combined with the cobalt atom.

RESULTS WITH COPPER SALTS

For most of the work with the solution of copper salts, the Rowland concave grating was used. Only aqueous solutions of copper salts were examined. The concentration of the solutions used was varied from 5.0 to 0.1 normal and the lengths of the absorbing layer were 1 cm, 2 cm, and 4 cm. The source of light for all the work with the concave grating was the arc with Norway iron for electrodes.

Copper chloride in water.—There was strong absorption in the blue, violet, and ultra-violet and also a strong band in the red. The absorption of the short wave-lengths was probably due to more than one band, as was evident both from the spectrograms and the visual observations made with the quartz spectrograph when using very dilute solutions. With greater concentration of the absorbing solution, however, the limit of transmission was about λ 4800, giving the appearance of a single absorption band.

In the red the edge of the absorption band was not so sharply defined as in the case of those in the violet and ultra-violet. This band appeared to move up from the infra-red, that is, became wider as the concentration of the solution was increased, and in more concentrated solutions there was complete absorption as far as λ 5900.

Copper nitrate in water.—The ultra-violet absorption in the case of copper nitrate did not extend so far toward the longer wave-lengths

as in the case of copper chloride. The limit of transmission in the case of the most concentrated solutions of copper nitrate was about λ 3550, while there was absorption as far as λ 4800 in the case of copper chloride. With very dilute solutions the absorption in the blue and violet could not be seen. The band at λ 3550 was probably due to NO_3, while the absorption of the copper chloride in the blue and violet was evidently produced by the molecule of copper chloride, or to the molecule and whatever it was associated with, and not to the copper ions.

The absorption band in the red was in practically the same position as that in the case of copper chloride, showing that these bands were undoubtedly produced by the same kind of "absorber."

Copper acetate in water.—In the case of the concentrated solutions of copper acetate the limit of transmission in the shorter wave-

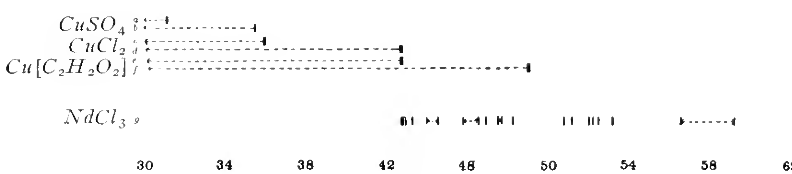

FIG. 2

lengths was farther toward the red than with either the copper chloride or nitrate, as is shown in Fig. 2. In Fig. 2, (*c*) and (*d*) represent the spectrum when the light was passed through a solution of copper chloride at different concentrations, while (*e*) and (*f*) give the spectrum when solutions of copper acetate of corresponding concentrations were used. It is evident that the absorption in this region of the spectrum must be ascribed to different "absorbers" in the case of the different salts.

The absorption band in the red was in about the same position as that of the chloride and nitrate.

Copper bromide in water.—For the work with copper bromide only very dilute solutions could be used, as there was too much general absorption with the solutions of greater concentration. The absorption bands with the bromide were similar to those of copper chloride, but they were displaced in position. With the more concentrated solutions the limit of transmission in the violet was λ 4525.

In the red the most concentrated solution transmitted everything as far as λ 6400, but when the solutions were less concentrated there was transmission as far as λ 7300.

Copper sulphate in water.—The solutions of copper sulphate showed less absorption in the violet and ultra-violet than did the other salts of copper, the limit of transmission being farther toward the ultra-violet. The red band, however, was in about the same position as in the case of the other salts of copper.

In Fig. 2, (*a*) and (*b*) represent the variation of the absorption as the concentration of the copper sulphate solution is increased. The concentration of these solutions was the same as that of the solutions of copper chloride and copper acetate, the absorption spectrum of which is also represented in Fig. 2.

Copper chlorate in water.—The work with copper chlorate was undertaken for the purpose of comparing its absorption with that of copper nitrate, the object being to have the composition of the salts the same except the chlorine and nitrogen.

With dilute solutions, the spectra of the solutions appeared about the same, but as the concentration of the solutions was increased, the color of the solutions differed more and more. This difference was less pronounced than in the case of cobalt nitrate and cobalt nitrite, where the only difference in composition of the salts was in the amount of oxygen that the salt contained. It is evident, however, that the chlorine and nitrogen play a rôle in the absorption of the salts in solution.

Results with copper salts.—By comparing the absorption spectra of the various solutions of copper salts we find that some of the bands are common, while there are others which are characteristic of each spectrum. This is what would be expected if there were two kinds of "absorbers," the one producing the red band being the copper itself, while the molecule would be responsible for the characteristic bands.

Moreover, since the color of the solutions of copper chlorate and copper nitrate was not the same for the same concentration of the solution, it is evident that the chlorine and nitrogen must play a rôle in the absorption.

Ferric chloride in water.—For the work with ferric chloride the

second method of investigation was employed, in which the spectrum was examined visually by means of the quartz spectrograph and the prism spectroscopes.

The method of carrying out this part of the work was similar to that used in the case of cobalt chloride: light from an iron arc and the Nernst filament were focused respectively upon the slit of the quartz spectrograph and the prism spectroscopes, by means of quartz condensing lenses, the light having passed through a cell 1 cm thick containing pure water. The ferric chloride was then slowly added to the water and the absorption bands examined. There appeared to be two regions of absorption, one in the extreme ultra-violet and another farther toward the red, extending from the ultra-violet into the violet and blue. These two bands appeared as one except when the violet band first became visible. The latter band, however, did not appear until the solution was quite concentrated.

The same experiment was repeated by adding the ferric chloride to dilute nitric acid. When this was done free chlorine was given off and the band in the ultra-violet and violet could not be seen. This is what would have happened if the absorption in the ultra-violet and violet had been produced by the combined iron and chlorine or by any "simple" hydrate of this salt.

Neodymium chloride in water.—The work with neodymium salts was not completed until after the paper by Jones and Anderson appeared.

The absorption spectrum of neodymium salts was examined both with the grating spectroscope and with the quartz spectrograph.

The spectrograms showed several strong absorption bands in the visible region of the spectrum, and also several others in the ultra-violet could be seen by means of the quartz spectrograph. There were also several bands in the red which were not shown on the spectrograms. In Fig. 2, (*g*) represents the absorption bands in the visible region of the spectrum of neodymium chloride, beginning with the broad band in the yellow and extending as far as the sharp narrow band at λ 4275. The band in the yellow extending from λ 5660 to λ 5930 was broken up into five narrow bands, but they were too faint to be well reproduced. Below is given a list of the bands which were measured.

λ	Character
4275	Strong and well defined
4290	Faint
4330	Broad, hazy
4405–4458	Broad, hazy
4590–4655	Edges hazy
4695	Quite sharply defined
4750–4768	Edges sharp
4820	Hazy
5090	Quite sharp
5120	Hazy
5205	Sharp
5221	Sharp
5252	Hazy
5320	Hazy
5660–5922	Sharp on violet side

The wave-length of the absorption bands in the ultra-violet region of the spectrum was not determined, as the spectra were examined with the quartz spectrograph and the bands were too faint to be determined. There were also several in the red which were not measured.

Neodymium bromide in water.—The absorption spectra of neodymium bromide was very similar to that of neodymium chloride, although the solutions were a much deeper red than those of the chloride. For corresponding concentrations, however, the bands of the bromide solutions were much more intense than for the neodymium chloride. The bands were apparently slightly displaced, but this slight displacement cannot be reproduced in the spectrograms.

Neodymium sulphate in water.—The spectra of the solutions of neodymium sulphate differ considerably from those of the solutions of neodymium chloride. This difference was not apparent, however, unless the spectra were examined carefully, and it was much more easily seen in the case of the more concentrated solutions. The bands in the spectra of the sulphate solutions were broader and more hazy than those of the solutions of neodymium chloride.

Neodymium salts in other solvents.—The spectra of neodymium chloride and bromide were also examined when these salts were dissolved in other solvents than water. Both of these salts were dissolved in each of the solutions of absolute ethyl alcohol, glycerine, and acetic acid, and the absorption spectra studied. This work,

however, has not been entirely completed. In each case the spectrum was found to differ from the corresponding spectrum when the salt was dissolved in some other solvent. Not only did the spectra of different salts differ from each other, but some of the bands which were intense when the salt was dissolved in one solvent were narrow and faint when this salt was dissolved in some other solvent.

SUMMARY

The conclusions which have been reached from this work may be stated briefly as follows:

1. The absorption of solutions of the salts which have been studied is due to different kinds of "absorbers," one of these, at least, being the molecule of the salt and another the atom.

2. The absorption spectra of these salts in solution depends to a great extent upon the solvent, which probably forms some "simple" hydrate.

3. Each constituent of the molecule of these salts plays some rôle in their absorption.

SHEFFIELD SCIENTIFIC SCHOOL
YALE UNIVERSITY
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A SIMPLE CRITERION FOR THE DETECTION OF ANOMALIES IN THE ORBITS OF SPECTROSCOPIC BINARIES

By D. F. COMSTOCK

It has often been remarked that discoveries are oftenest made through close examination of observed exceptions to well-known laws or wide generalizations. Anomalies in the field of stellar motion are as important as elsewhere, but when the moving stars constitute a spectroscopic binary it is not always very easy to find out whether the observed velocity-curve is explainable by simple motion of two bodies under the law of gravity or whether some disturbing cause must be sought.

In looking for a particular type of apparent anomaly which was to be expected on a theory of light-velocity to be discussed elsewhere, the author had occasion to study the problem of two bodies, with a view to obtaining, if possible, a simple relation which was characteristic merely of motion under gravity and independent of the constants of any particular orbit.

A relatively simple relation was found which is best stated in terms of the curve which is the integral of the common radial velocity-curve and which I will call the "distance-curve." The distance-curve is easily obtained by plotting as abscissae the times and as ordinates the *area* of the velocity-curve between zero velocity and the time-ordinate chosen. Thus if in Fig. 1 v is the velocity-curve, then d is an approximate representation of the distance-curve. If the axis OP is so chosen that the area of the v-curve above it is just equal to the area below, then the ordinates of the d-curve give the distance of the star at any instant from the nearest point of the orbit, this point being of course considered as moving with the center of gravity of the system.

The relation to be proved is then between the width of the distance-curve, such as MN, at any point and its height LM at the same point.

It may be written thus:

$$\frac{w}{W}=\frac{1}{\pi}\left\{\cos^{-1}\left(2\frac{h}{H}-1\right)-2\left(2\frac{h_1}{H}-1\right)\sqrt{\frac{h}{H}\left(1-\frac{h}{H}\right)}\right\} \qquad (1)$$

where

h = height of distance-curve at any point;

w = width of distance-curve at same point;

h_1 = height of distance-curve at the abscissa corresponding to the maximum point of velocity-curve (KS, Fig. 1);

H = maximum height of distance-curve;

W = total width (i.e., length of base) of distance-curve.

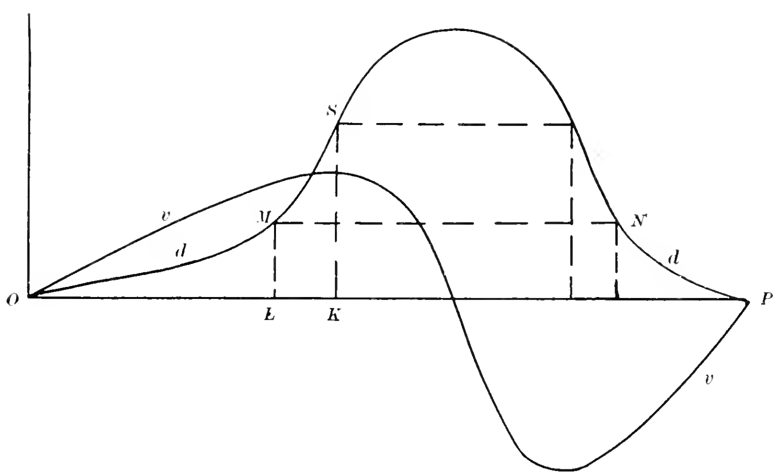


FIG. 1

PROOF OF RELATION

In order to prove equation (1) it will be convenient to state six theorems which are quite simple in themselves and on which this equation may be based.

Theorem I.—This is a geometrical theorem and states that if (Fig. 2) in any circle are drawn two parallel chords CB and MN, two tangents HRL and AVY parallel to these chords, and the diameter RV perpendicular to the tangent, and cutting the chords

in P and Z; then if F is any point on the chord CB and the lines MF and FN are drawn, the following relation holds:

$$\frac{\text{Area } MFNBRCM}{\text{Total area of circle}}=$$

$$\frac{1}{\pi}\left\{\cos^{-1}\left(2\frac{VZ}{D}-1\right)-2\left(2\frac{VP}{D}-1\right)\sqrt{\frac{VZ}{D}\left(1-\frac{VZ}{D}\right)}\right\} \tag{2}$$

where D is VR, the diameter of the circle.

This theorem can readily be proved by subtracting the area of $\triangle MFN$ from segment MRN.

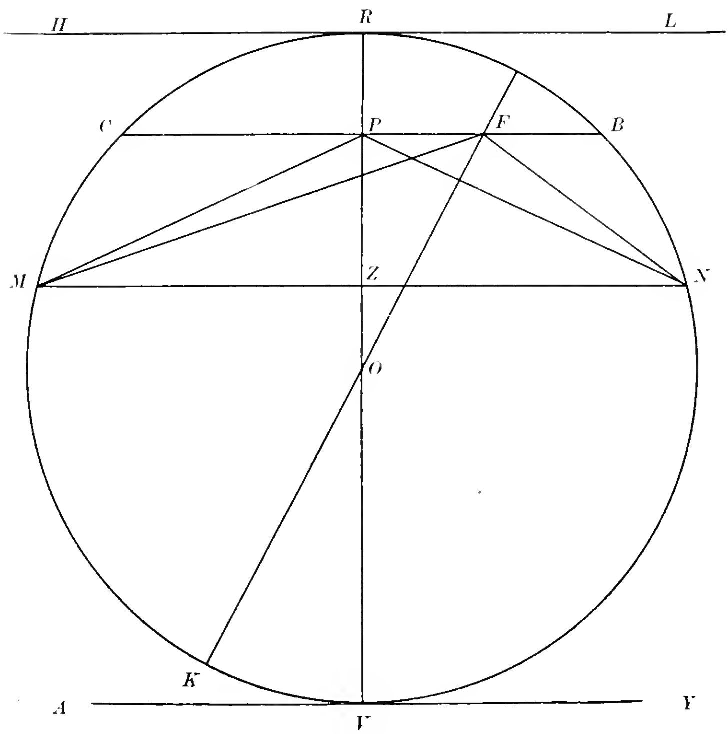


FIG. 2

Theorem II.—If four planes, each perpendicular to the line of sight, be imagined drawn in connection with the real orbit (Fig. 3), two tangent to the orbit, one through the focus, and one at any distance (z) from the nearest tangent plane, then the orbit, together

with the intersection of these planes with the plane of the orbit, may be completely represented by the projection of a certain circle similar to that of Fig. 2, with the various lines as there drawn.

This is readily seen to be true if the point F and the direction of the lines HL, CB, etc., are properly chosen on the circle of Fig. 2. For clearness Fig. 3 is lettered similarly to Fig. 2.

Theorem III.—The relation of Theorem I holds also for the ellipse of Fig. 3, if the corresponding letters are substituted. That is,

$$\frac{\text{Area } M'F'N'B'R'C'M'}{\text{Total area of ellipse}}=$$

$$\frac{1}{\pi}\left\{\cos^{-1}\left(2\frac{V'Z'}{D'}-1\right)-2\left(2\frac{V'P'}{D'}-1\right)\sqrt{\frac{V'Z'}{D'}\left(1-\frac{V'Z'}{D'}\right)}\right\} \tag{3}$$

This follows from the fact that, since each area and line of Fig. 3 is a projection of the corresponding line or area of Fig. 2, we can write

$$\frac{\text{Area } M'F'N'B'R'C'M'}{\text{Total area of ellipse}}=\frac{\text{Area } MFNBRCM}{\text{Total area of circle}} \tag{4}$$

$$\frac{V'Z'}{D'}=\frac{VZ}{D} \tag{5}$$

$$\frac{V'P'}{D'}=\frac{VP}{D} \tag{6}$$

Theorem IV.—The ratio $\frac{V'Z'}{D}$ in the *right*-hand side of the relation of Theorem III may be found from an examination of the experimentally determined distance-curve of Fig. 1. In fact

$$\frac{V'Z'}{D'}=\frac{h}{H} \tag{7}$$

where h and H are the quantities in equation (1).

To prove this it is merely necessary to remember that the ordinates of the distance-curve (Fig. 1) give the distance of the body at any instant from the nearest tangent plane to the orbit. From this it is evident that h and H are simply projections of $V'Z'$ and D' on the line of sight.

Theorem V.—The ratio $\frac{V'P'}{D'}$ on the right of the relation in Theorem III may also be found from an examination of the experimentally determined distance-curve of Fig. 1. In fact

$$\frac{V'P'}{D'} = \frac{h_1}{H} \tag{8}$$

where h_1 and H are the quantities in equation (1).

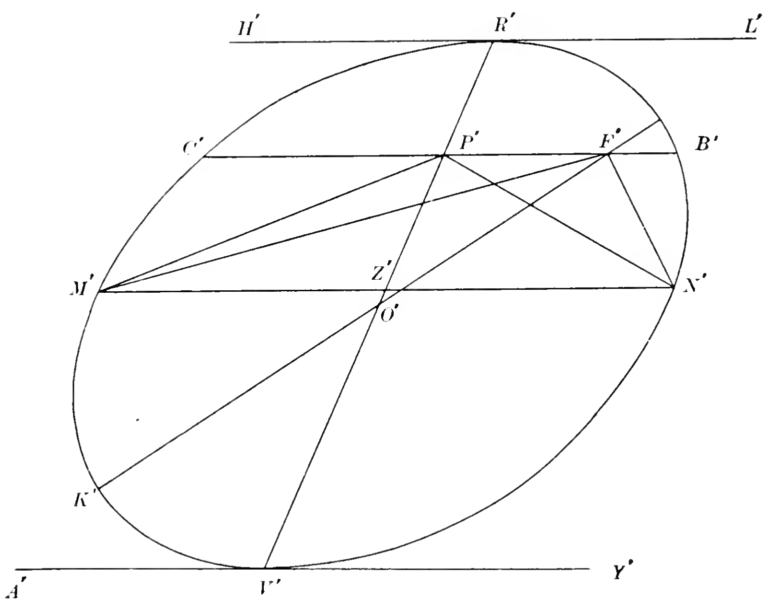


FIG. 3

This follows from the fact that h_1 was defined as the ordinate of the distance-curve which had the same abscissa as the maximum point of the radial velocity-curve. It follows, however, from the simplest expression of Newton's Laws as applied to central forces that the radial velocity is a maximum when the body is in a plane through the focus drawn perpendicular to the line of sight. Thus the height h_1 of the distance-curve represents the distance of the focus of the orbit from the nearest tangent plane.

Theorem VI.—The ratio which forms the *left*-hand side of the relation of Theorem III may be found from an examination of the experimentally determined distance-curve of Fig. 1. In fact

$$\frac{\text{Area } M'F'N'B'R'C'M'}{\text{Total area of ellipse}}=\frac{w}{W}. \tag{9}$$

To prove this it is merely necessary to remember that from Kepler's Law the ratio

$$\frac{\text{Area } M'F'N'B'R'C'M'}{\text{Total area of ellipse}} \tag{10}$$

is equal to the time that it takes the body to go from M' to N' divided by the total time to go once around the orbit. By construction M' and N' are at equal distances from the nearest tangent plane and hence the positions M' and N' are represented by equal ordinates on the distance-curve. Hence the *width* of the distance-curve corresponding to the $M'N'$ ordinate, divided by the total width (the base) of the curve, is the time it takes the body to go from M' to N' divided by the periodic time and hence is equal to the above ratio (10).

It is evident that from Theorems IV, V, and VI the relation of equation (1) follows directly. We thus have

$$\frac{w}{W}=\frac{1}{\pi}\left\{\cos^{-1}\left(2\frac{h}{H}-1\right)-2\left(2\frac{h_1}{H}-1\right)\sqrt{\frac{h}{H}\left(1-\frac{h}{H}\right)}\right\} \tag{11}$$

SUMMARY AND CONCLUSION

The criterion for the detection of anomalies in the orbit of spectroscopic binaries which has been set up in this paper, may be outlined in words as follows:

If a plane be drawn through any simple double star orbit perpendicular to the line of sight, then the time during which one of the stars is on the far side of this plane is a simple function of the position of the plane along the line of sight, and of the position of the focus along the line of sight. All three of these quantities can be determined with great ease from the true radial-velocity-curve of one star.

If the time be expressed in terms of the periodic time, and the distance along the line of sight, measured from the nearest point of

the orbit, be expressed in terms of the total length of the projection of the orbit on the line of sight, then the equation contains *no constant of the orbit except the distance of the focus along the line of sight expressed in the same way.*

This criterion is uniquely suited for the detection of any apparent anomaly in the motion of binary stars due to a possible dependence of the velocity of light on the velocity of the source. It was developed solely for this reason and the author is now using it with the purpose of detecting such an effect, does it exist. It was thought, however, that, since the criterion is so general and the relation itself so simple, compared with the complexity of the integral function connecting velocity and time, it might be found useful in other problems than the one for which it was developed.

Massachusetts Institute of Technology
March 4, 1910

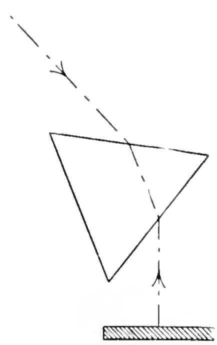
Minor Contributions and Notes

AN IMPROVED METHOD OF USING QUARTZ PRISMS

In all quartz spectrographs in which the optical train is composed of crystal quartz for work in the ultra-violet it has been found necessary to make the dispersing prism of two halves, one of right and the other of left-handed quartz, to eliminate the doubling of the spectrum lines due to the two oppositely polarized circular vibrations.

This trouble may be overcome in another way, which is at the same time simpler, cheaper, and makes a gain in defining power.

A solid 60° quartz prism was constructed by Hilger of London with the axis of the prism running parallel to its base. This prism is mounted in an instrument of the Littrow type with an object-glass of about 90 inches focus. Behind the prism is mounted a metal reflector so that the light passes through the prism on to the mirror and is reflected back through the prism and along its original path (see diagram). If now the image of the slit be examined, it will be found that the doubling effect is entirely eliminated.



This prism was replaced by a Cornu prism of the same size, but no difference could be detected except that the solid prism gave better definition, due probably to the fact that four surfaces had to be worked on the Cornu prism instead of two on the solid one.

I must say, however, that it is extremely important that the optic axis of the prism should run parallel to the base or it will be found necessary to turn the prism slightly out of minimum deviation to get the single image. I have not heard of a solid quartz prism being used in this way before but it is quite possible that it has been. If so, I should be glad to hear of it.

Of course, a 30° prism can be used in a similar way.

F. Stanley

The Laboratory, Adam Hilger, Ltd.
London
January 21, 1910

DETERMINATION OF ABSOLUTE WAVE-LENGTHS WITH OBJECTIVE-PRISMS[1]

Nearly all of the photographs of stellar spectra, made by the aid of the Henry Draper Memorial, have been taken with objective-prisms. The principal objection to this method of photographing the spectra of stars has been that it does not provide the means of determining their velocities in the line of sight. Various methods of remedying this difficulty have been tried here, and were described in 1891, in *H.A.*, **26**, chap. xxi. Among these methods are absorption by didymium and hyponitric acid, variation in length of a known portion of the spectrum, and the use of an auxiliary prism. The first of these methods appeared to be the simplest and best, but the absorption bands were too wide and hazy for precise measurement. The second method was then shown to be impracticable, but has since been recommended by Orbinskij, and is commonly ascribed to him. Another method, turning the prism 180°, was proposed in *H.C. 110*, and is now generally regarded as the best method yet proposed.

After discussing this problem with Professor Robert W. Wood of Baltimore, at my request, he prepared the following statement:

My attention was drawn recently by Professor E. C. Pickering to the great need of a ray-filter giving one or more sharp and narrow absorption lines, for the determination of stellar velocities with the objective-prism.

It occurred to me that the 4272 band of neodymium might answer the purpose. In a solution of the double nitrate of neodymium and ammonium (now procurable in large quantities as a by-product in the manufacture of Welsbach mantles) I found that the band was much too wide for use in the determination of stellar velocities. The solution of the pure chloride, however, was quite different, for the band was found to have contracted to a width of less than three Ångström units

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Editorial Note.—The pressure upon the space in this *Journal* during recent volumes made necessary the adoption of a general rule favoring priority of space to articles which would not be published elsewhere in English. This unfortunately has made it impossible for the Editors to bring before the readers of this *Journal* many valuable circulars and bulletins upon astrophysical topics, issued from various observatories, such as could formerly be reprinted in these pages. The managing editor of course reserves to himself some elasticity in the application of such a general rule, and he is glad to find space for this circular by Professor Pickering, which records such a decided step of progress in the more precise quantitative utilization of plates taken with the objective-prism. E. B. F.

[1] *Harvard College Observatory Circular 154.*

PLATE X

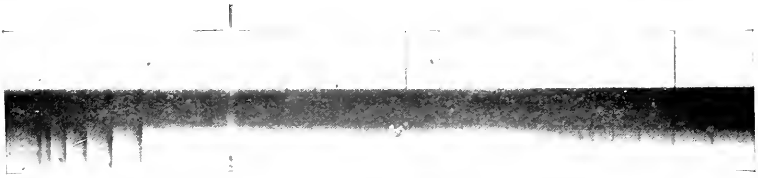


FIG. 1.—Spectra with Grating (R. W. Wood)

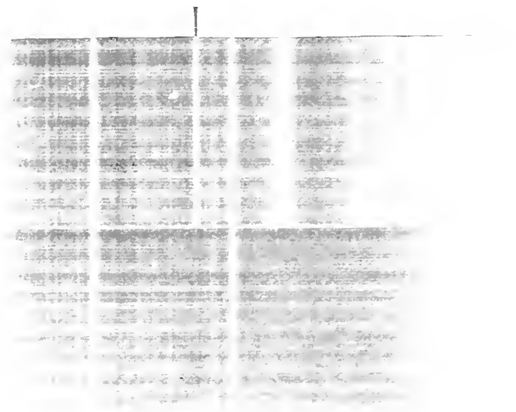


FIG. 2. *α Canis Minoris*

FIG. 3. *φ¹ Orionis*

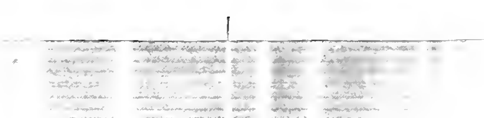


FIG. 4. *β Aurigae*

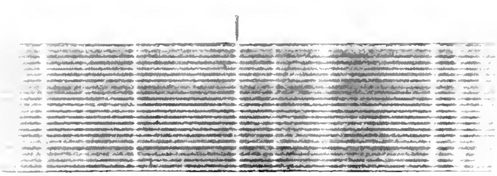
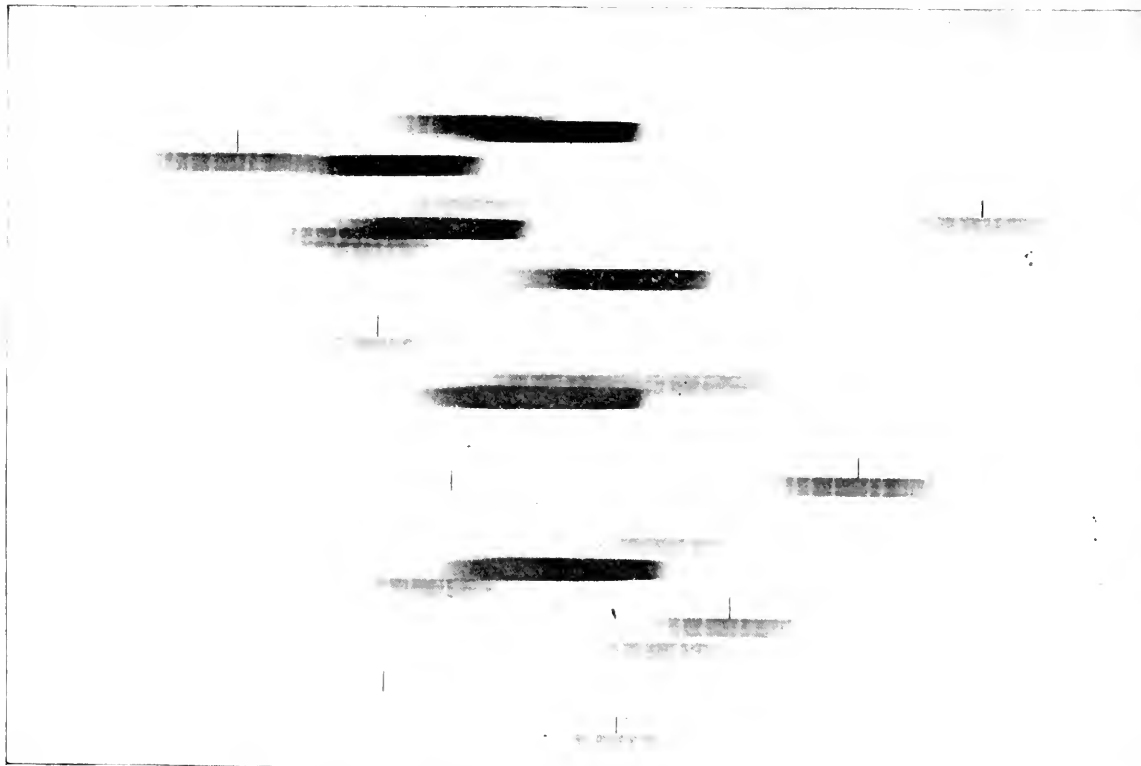


FIG. 5. *β Orionis*

STELLAR SPECTRA WITH ABSORPTION LINE OF NEODYMIUM CHLORIDE

PLATE XI



Spectra of the *Pleiades*, with Neodymium Comparison Line

and its center could easily be determined to within 0.15 Å.U. The absorption spectrum was photographed with a concave grating of 7 feet radius, together with the iron spectrum, and it was found that wave-length determinations referred to the neodymium line could be determined certainly to within 0.10 Å.U. This would enable stellar velocities to be determined to within less than 10 kilometers per second. The strength of the solution must be regulated according to the thickness of the cell. For a cell 4 mm in thickness I diluted a saturated solution with about five parts of distilled water. It is very easy to determine the proper strength by trial. It may be possible to still further contract the band by the addition of some other substance. It has been found that lanthanum increases the sharpness of some of the didymium bands, also phosphoric acid. Experiments in this direction are in progress. The photograph of the absorption line with a rough scale of wave-lengths and the iron spectrum is reproduced in Fig. 1 (Plate X). The absorption line is almost in coincidence with the close double line of iron at wave-length 4271.5.

Professor Wood has also kindly filled a ray-filter with the solution named above, and a number of stellar spectra have been photographed through it.[1] Plate XI represents a photograph of the *Pleiades* taken with the 8-inch Draper telescope, on February 14, 1910, exposure 54 m., enlarged 2.72 times. The other spectra were photographed with the 11-inch Draper telescope, and were enlarged 1.56 times. The lengths of exposure are those of the portion of the photograph shown in the plate. On Plate X Fig. 2 represents the spectrum of α *Canis Minoris* taken with and without the cell interposed, on February 18, 1910, each exposure being 21 m. Fig. 3 represents the spectrum of *H.R.*, 1876, ϕ^1 *Orionis*, taken February 18 with an exposure of 55 m. Fig. 4 represents the spectrum of *H.R.*, 2088, β *Aurigae*, taken February 19, 1910, with an exposure of 17 m. This star is a spectroscopic binary and the stellar lines have become double, as may be seen in line K in the original negative. The neodymium line of course remains single. Fig. 5 represents the spectrum of *H.R.*, 1713, β *Orionis*, taken February 19, 1910, with an exposure of 8 m.

It will be seen from Plate XI that a large number of spectra may be photographed on a single plate, with sufficient distinctness to enable the lines to be measured with accuracy. This is also shown in *H.A.*, **26**, Plate VII. Several absorption bands are shown in Fig. 3, but only one, λ 4272, is sufficiently well defined to be satisfactorily meas-

[1] When placed just in front of the photographic plate.—EDS.

ured. It occurs in a very convenient place, in the brightest part of the spectrum, between the lines $H\delta$ and $H\gamma$. It is marked by a black line on several of the spectra. The lines in the spectrum of *β Orionis*, Fig. 5, are well defined, and can be measured with accuracy. It will be seen that the neodymium line appears to be at least as sharp as the hydrogen lines in this star, or as the calcium line, K.

The determination of the approximate motion of stars in the line of sight in this way becomes very simple. It is only necessary to measure the distance of the neodymium line, λ 4272, from $H\delta$, λ 4101, and $H\gamma$, λ 4340, and perhaps from some of the other lines, and then to derive their wave-lengths, by Hartmann's formula, assuming the neodymium line to be constant. The actual determination with sufficient accuracy to render this method of value, however, is a very different matter. While it is not probable that it can compete with the slit-spectroscope in determining the motions of the brighter stars, it may be worth while to compare the advantages of each, and thus see how far the new method may be useful in supplementing the old. This cannot be decided until this method receives the same careful study which has been made of the slit-spectroscope.

The principal objections to the slit-spectroscope are, first, the great loss of light. It is maintained that less than 1 per cent of the entire light falls upon the sensitive plate. Accordingly, only the brightest stars can be measured with accuracy, and even for these it is necessary to use the largest telescopes yet constructed. A second objection is the great rigidity required, and the necessity of maintaining a perfectly uniform temperature. The simplicity of the method of the objective-prism renders it probable that the sources of error are few in number, and therefore readily corrected. The loss of light is small, so that very faint spectra can be photographed. The principal work so far done with the slit-spectroscope has been with stars of the fifth magnitude and brighter.

In the *Harvard Circulars* announcing peculiar spectra it will be noticed that many of the stars are not contained in either the northern or the southern *Durchmusterung*, and must therefore be fainter than the tenth magnitude. These spectra, however, have well-marked bands or bright lines, which are easily seen. From the classification of the spectra of stars in clusters given in *H.A.*, **26**, chap. xiv, it does

not seem unreasonable to expect that the spectra of stars of the first type as faint as the ninth magnitude can be photographed with sufficient detail to determine the approximate approach and recession. Plate VII of *H.A.*, **26**, is an example of a plate taken with an objective-prism. The distance between the lines $H\delta$ and $H\gamma$ is here about 1.6 mm. On this scale 1 Ångström unit would equal 1″2, corresponding to 75 km. An error of $\pm 0''1$ would therefore represent ± 6 km. Preliminary measures of the three lines in the spectra of five stars photographed with the 11-inch Draper telescope gave a probable error of 10, 7, and 11 km for the mean of five settings on the lines $\lambda\lambda$ 4101, 4272, and 4340, respectively. It will be noticed that the error in the case of the neodymium line is distinctly less than those of the hydrogen lines. The only lines that can be photographed in faint stars whose spectra are of the first type are those of hydrogen. As they are wider than the neodymium line, this limits the accuracy with which such stars can be measured. Spectra of the second or third type must be much brighter to show measurable lines, but when these are visible, greater accuracy may be expected. The agreement of the various estimates given above renders it probable that, with our present means, we can determine the motion of a ninth-magnitude star of the first type with an error of about 10 km, and of an eighth-magnitude star of the second type with somewhat greater accuracy. Much wider spectra are needed to classify spectra than to measure the position of the lines. Unfortunately, when objective-prisms are used, if long exposures are made, producing narrow spectra, the definition is poor.

The Henry Draper Memorial has furnished the Harvard Observatory with excellent means for photographing stellar spectra with objective-prisms. At Cambridge, prisms are available for covering the mirrors or objectives of the following instruments: 24-inch reflector, interval $H\delta$ to $H\gamma$, 0.5 mm, scale $60'' = 1$ mm; 16-inch Metcalf telescope (doublet), prism under construction; 11-inch Draper telescope, two prisms giving intervals 5.5 and 11.5 mm, scale $54'' = 1$ mm; 8-inch Draper (doublet), two prisms giving intervals 0.0 to 1.7 mm, scale $162'' = 1$ mm. At Arequipa, 24-inch Bruce telescope (doublet), interval 1.7 mm, scale $60'' = 1$ mm; 13-inch Boyden telescope, two prisms, intervals 7.3 and 13.9 mm, scale

42″ = 1 mm; 8-inch Bache telescope, two prisms, intervals 1.7 and 0.6 mm, scale 178″ = 1 mm.

With this equipment, the Harvard Observatory is prepared to take such photographs as may be desired, and it invites the aid of astronomers experienced in measurements in the line of sight.

EDWARD C. PICKERING

DETERMINATION OF STELLAR VELOCITIES WITH THE OBJECTIVE-PRISM

My attention was drawn by Professor E. C. Pickering to the great need of a ray-filter giving sharp and very narrow absorption bands, for the determination of line-of-sight velocities with the objective-prism. Having used solutions of salts of the rare earths, neodymium and praseodymium, for the isolation of spectral lines in my work upon the fluorescence of sodium vapor, it occurred to me that the fine absorption lines of these substances might meet the requirements. The excellent results which Professor Pickering has already obtained with a small cell, which I filled with a solution of neodymium chloride, makes it seem well worth while to improve the method in every possible way. I found that the absorption band at λ 4273, by proper adjustment of the concentration, should enable us to determine velocities to within about 10 km per second, provided that the definition given by the prism was sufficiently good to admit of measurement to this degree of accuracy.

By the addition of erbium chloride we can get another good reference line at λ 382, and if we use an isochromatic plate, there is a double band (neodymium) at λ 5220, the bright central band of which is not over five or six Angströms in width. With vapors, however, I feel sure that even better results can be obtained. I have examined a number already: perchloride of manganese gives groups of lines in the green resembling the *b* group in the solar spectrum, with low dispersion. There should be no trouble with the absorption part of the problem, for we can without doubt get numerous lines as fine as the prism and the atmospheric conditions permit of our resolving.

With the temperature of the prism accurately controlled, and "good seeing" conditions, I feel sure that the accuracy of the measurements can be greatly increased. It may be possible to find some salt of

neodymium which is superior to the chloride. The appearance of the band is quite different in the different salts. In the double salt with ammonium, in which form the salt comes as a by-product in the manufacture of Welsbach mantles, the band is very broad and diffuse. In the nitrate it is double, the two components being of very unequal intensity, and not quite as sharp as in the chloride. Other salts are in process of preparation.

Anderson's work has shown that water is probably the best solvent. In alcohol the λ 4273 line disappears entirely and two new lines, much wider and fainter, appear a little on the red side of it.

I have not yet tried solvents which become solid, but shall do so shortly. If it is possible to get it into solution in styrol, we may be able to make a solid screen, for by keeping styrol at a temperature of 100° for a day or two it is converted into a hard, glassy substance (metastyrol) which has the same refractive index as flint glass, and is insoluble in water, acids, and most organic solvents.

I used this substance twelve years ago for the preparation of solid masses which would show the Christiansen colors, mixing powdered glass with styrol and then converting it into metastyrol (see *Physical Optics*, p. 92).

R. W. Wood

Baltimore
April 4, 1910

NOTE ON THE ACCURACY OF RADIAL VELOCITY DETERMINATIONS

There has been of late some tendency, particularly on the part of writers who have not had opportunity to make or measure spectrograms, to exaggerate the accuracy of determinations of radial velocities with the best modern spectrographs. The statement is sometimes seen that the precision has now reached the tenth of the kilometer.

It seems to me only proper that a word of caution should be expressed at this time in order to avoid misunderstandings on the part of astronomers and physicists who have not had direct experience in this particular line of work.

A conservative point of view might succinctly be stated by raising the question whether we today know the radial velocity of any star in the heavens to the nearest kilometer.

Measurements of the same plate (obtained with good, high-dispersion spectrographs) by two different but equally experienced observers may yield results, based upon a dozen well-chosen lines, which differ in the mean by a whole kilometer, or even more. Where the process involves different instruments, observers, and sources of comparison spectrum, as well as different methods of measurement and reduction, the discrepancies may be of the order of two or more kilometers. It is highly desirable that all such elements should be as different as may be feasible, in order that from a comparative study of the results for the same star at different institutions, some idea of the systematic errors may be gained and a closer approximation to the true radial velocity be obtained. It was from this point of view that I suggested several years ago that certain standard stars should be observed with some regularity at different observatories. A large number of such spectrograms have been made here and at several other institutions. When these are all fully reduced and published, valuable data will be at hand for a study of present-day precision in line-of-sight work.

Without wishing to push too far the somewhat radical query above expressed, let us for a moment examine the case of *Arcturus*, a bright star involving only a short exposure with powerful spectrographs, whereby changes of temperature and of flexure are largely avoided. Its lines are sharp and the spectrum does not differ enough from that of the sun to involve many differences of blending of the lines. Results have also been published for this star by numerous observers, and it was very successfully employed by Professor Küstner for a determination of the solar parallax from the inferred radial velocity of the earth in its orbit. It was also included in the similar determination, more elaborately made at the Cape Observatory and recently published.[1]

The table shows some of the recent values, readily accessible at this time.

Unfortunately no values have yet been published for this star by the Lick Observatory, nor have results with the high-dispersion spectrograph recently installed at Paris by M. Hamy been announced

An examination of this list will probably convince the reader that

[1] *Annals of the Cape Observatory*, **10**, Part 3, 1909.

for this exceptionally well observed star the radial velocity lies between -4.5 and -5.0 km, particularly in view of the large weight to be assigned to the last determination, made at the Cape. One might question the use of the thousandth of a kilometer in expressing the value for each plate, as has been done throughout the longer series at the Cape, inasmuch as their values for this star from plates taken consecutively on the same night in some instances differ by 1.5 km.

RADIAL VELOCITY OF *Arcturus*

Epoch	Observers	Number of Plates	Velocity	Extreme Range
			km	km
1902.3	Frost and Adams	8	−4.3	2.1
1902.4	Newall	6	−5.8	2.7
1902.4	Lord	7	−3.7	3.2
1903.1	Frost and Adams	5	−4.8	1.3
1903.3	Newall	19	−6.6	4.5
1903.4	Belopolsky	9	−6.1	3.3
1904.4	Belopolsky	18	−5.4	2.6
1904.8	Küstner	18	−4.8	1.2
1905.3	Plaskett	11	−4.6	2.3
1905.6	Slipher	5	−4.7	1.5
1907.00	Halm*, Cape	46	−4.7	4.9
1907.15	Halm†, Cape	55	−4.9	2.9

* Measured with ordinary machine.

† Measured with Hartmann spectrocomparator.

In thus commenting on these results at the Cape, I desire to express my high opinion of them, and would cite them as representative of high-grade spectrographic work: they illustrate my point that such differences do occur with the best modern installations. Granting, then, that a single plate of a star having excellent lines may yield results uncertain by one or two kilometers, it is evident that with low-dispersion, one-prism instruments, generally having a dispersion of one-third or one-fourth that just under discussion, three or four times that uncertainty may exist in regard to a single plate. Now, one-prism instruments are, moreover, much more liable to flexure than the three-prism form, whence additional uncertainty arises. In many spectra of the first type, which are often better investigated with low dispersion, the lines are so broad and ill-defined that large additional errors may be introduced, some of a systematic nature. It is a fact that it is sometimes necessary, in forming a mean, to combine values for separate lines which differ by as much as 30 km. Accordingly

we may hardly expect from such stars a value reliable within less than 5 km from a single plate, and the uncertainty may sometimes be twice this.

I have thus called attention to the unreliability of determinations of radial velocity in order to bring out more clearly the danger of overestimating the present possibilities of determining radial velocities with the objective-prism, in spite of the excellent progress recorded in Professor Pickering's circular elsewhere in this number (p. 372).

The simplicity and efficiency of the instrument have been recognized since the days of Fraunhofer, and the campaign conducted with it, using dry plates, under Professor Pickering's direction has yielded results of inestimable value, necessarily largely qualitative. As he states, however, the development of the instrument has not received the minute attention given to the slit-spectrograph, and this is doubtless because of the lack of reference points in the spectrum to serve as the basis for rigorous measurements. As further study is given to the production of ray-filters yielding absorption lines where needed throughout the spectrum, to the maintenance of the prism or prisms at a constant temperature during the night's work, and to the prevention of flexures of the mechanical and optical parts, it is, therefore, highly probable that a great gain in accuracy will be made in the near future.

Fully sharing Professor Pickering's views as to the difference between the statement of the method and the actual determinations by it, as clearly put forward by him (p. 374), I regret that I must be pessimistic as to his estimate (p. 375) that it is "probable that, with our present means, we can determine the motion of a ninth-magnitude star of the first type with an error of about 10 km, and of an eighth-magnitude star of the second type with somewhat greater accuracy."

Most spectrographers would reject without measurement any plate on which by some mischance only a single comparison line could be utilized. Even if three good comparison lines were available, at the middle and extremities of the spectrum, so that a satisfactory Hartmann formula could be calculated for the plate, it would still be quite beyond the usual practice to make the displacement of a star line depend upon a comparison line distant more than a few tenth-

meters, a linear distance ordinarily of much less than a millimeter. Without this safeguard of comparison lines, the effects of optical distortions, lack of uniformity of the film and glass surface, etc., would greatly increase the range of error of a plate.

Further, in the slit-spectrograph every effort is made to have the star's rays traverse the optical system centrally, which must diminish errors. One of the great advantages of the objective-prism is that it secures many spectra on one plate. This involves much distortion for stars away from the center of the plate, so that the scale-values, (Ångström units per millimeter) would vary decidedly over the plate, and depart capriciously from the normal values for the central rays passing the prism at minimum deviation.

The small scale given by most of the objective-prism telescopes listed as available at Harvard—and these are among the largest in use anywhere—also operates to diminish the accuracy of the determinations. None of the one-prism combinations gives a linear separation between $H\delta$ and $H\gamma$ equal, for instance, to that of the Bruce spectrograph of the Yerkes Observatory as arranged for one prism (10.5 mm), and only the two-prism arrangement of the 11-inch Draper telescope at Cambridge (11.5 mm) and the 13-inch Boyden telescope (13.9 mm from $H\delta$ to $H\gamma$) exceed its scale.

For such reasons as these it seems to the writer very doubtful that values "*with our present means*" more accurate than about 25 km per second are likely to be secured.

The question is, however, largely academic: it can be tested practically by measurements on the Harvard spectra provided with the neodymium line which have elsewhere been measured with slit-spectrographs.

With the highest appreciation of the splendid advantages of the objective-prism for radial velocity work when it has been sufficiently developed and when new filters are provided yielding many comparison lines well distributed throughout the spectrum, it seems to the writer that we should not allow ourselves to overestimate the precision at present attained, if this shall in any way diminish efforts and experiments to further improve this highly promising method.

EDWIN B. FROST

YERKES OBSERVATORY

Reviews

Atlas Stellarum Variabilium. Series VI, supplementing Series I, II, and III. By J. G. Hagen. Berlin: Felix L. Dames, 1908. Pp. *circa* 70 and 63 charts. M. 92.

Series VI of this well-known atlas contains sixty-three charts of the fields surrounding sixty-five telescopic variables (*Z* and *RU Tauri* being on the same chart, also *Y* and *RS Pegasi*), with the corresponding catalogue sheets. The former series have proved so valuable that praise is superfluous, but it may be said that the present series is a distinct improvement over its predecessors, as the catalogue sheets contain a column of photometric magnitudes from the Harvard measures.

This series has been reviewed in various journals, but not from the standpoint of the faint stars in the vicinity of the variable. The reviewer has been engaged for the past two years, at the request of Father Hagen, in bringing out photographic prints on a scale of 10″ to the millimeter, of all of the *Atlas* fields in which the variable becomes as faint at minimum as magnitude 13. The prints are about one-half degree square, and are from negatives taken with the 2-foot Yerkes reflector, showing stars to about magnitude 16.

As these prints will be used for identifying the variables when near minimum, it was necessary to compare carefully the print with the chart, and check the identification of the variable on each by the configuration of the surrounding stars. The variables were very faint on many of the photographs, hence it is no reflection on the charts to say that it was not always possible to identify the variable with certainty on the photograph, as there were frequently several faint stars near the place, any one of which might be the variable. It became necessary, therefore, to find the limits of accuracy of the charts and catalogue, in order to limit the area within which the variable might be situated. For this reason particular attention was paid to the stars in a square 5′ each way from the variable, and any differences between the chart and the photograph, which might change the appearance of the group, were noted and measured.

Of the 63 fields in the series, 54 were photographed, and differences large enough to be noticeable were found on 28. As might be expected from the method of charting, the differences were nearly all in declination, the declination being read on a glass scale in the ocular of the Georgetown 12-inch refractor. It was found that differences as small as 0′3 would not be noticed unless the configuration were especially favorable,

the actual range among the 24 differences of declination being from 0′.3 to 1′.6, the average being 0′.7. This leads to the conclusion that in cases of doubtful identity, any star within 1′ of the calculated place may be the variable, but if the distance be greater than 1′, it is probably not the variable.

The list of differences thus found has been communicated to the author, who has announced his intention of publishing a list of corrections to the *Atlas*. It may be well, however, to mention here that Hagen's identification of the variable 7619 *RR Aquarii* differs from that given by Professor Abetti, the discoverer, who kindly marked a print sent to him, giving Hagen's No. 35 as the variable, 30″ north preceding the star marked by Hagen. Abetti's observations of this star are given in the *Arcetri* publications, No. 12, p. 21, 1900.

It should also be mentioned that the variable 2376 *S Lyncis* has a companion, of just about its minimum brightness, distant 12″.5 in position angle 155°. This is not mentioned in Hagen's work, and might easily be mistaken for the variable in its faint stages.

The only prominent star missing from the charts is one of magnitude about 10.8 near 5405 *RT Librae*, co-ordinates $-0^m 32^s$, and $-1'.0$. This star is not visible on the Harvard "Chart of the Entire Sky," Nos. 41 and 42, the limit of the plates being about ninth magnitude, near the corner of the plates.

Rather unexpectedly, the comparison of chart and photograph has been seldom affected by red stars of consequently faint photographic magnitude. The greatest difficulty encountered in this work has been that of finding the catalogue number of a given chart star, since the catalogue is arranged in order of brightness, instead of right ascension.

The photographic prints of 140 Hagen fields are now available, being for sale by the University of Chicago Press at cost, namely, 20 cents per sheet.

J. A. PARKHURST

YERKES OBSERVATORY
March 1910

The Moon in Modern Astronomy. By PHILIP FAUTH. Translated from the German by JOSEPH McCABE, with an introduction by J. ELLARD GORE. New York: D. Van Nostrand Co., 1909. 8vo, pp. 160, with 66 illustrations. Price $2.00 net.

This is an American reprint, from the original plates of the London edition issued a year previously, which was reviewed in this *Journal*, **29**, 91, January 1909. Nothing needs to be added to that review.

Zur Geschichte der astronomischen Messwerkzeuge von Purbach bis Reichenbach, 1450 bis 1830. By JOH. A. REPSOLD. Leipzig: W. Engelmann, 1908. Pp. viii+132, with 171 illustrations. M. 16; bound, M. 18.50.

This volume contains a most interesting collection of plates illustrating the gradual development of those mechanical contrivances by means of which astronomers have observed the heavenly bodies during the last four centuries. Accompanying the plates is an explanatory text characterized by the two very desirable qualities of brevity and completeness. The author has consulted original sources whenever possible; and it is almost unnecessary to add that the plates and press work are of the high quality to be expected in an astronomical publication coming from the house of Engelmann.

One might perhaps suppose that a work of this kind would be of interest to those only who possess the curiosity of the antiquarian—who desire to compare the supposed excellence of our present age with the feeble attempts made by men of old. But a careful examination of the book has inclined the present reviewer to the opinion that active practical astronomers and makers of modern instruments might not improbably derive ideas of value from a study of the plates. And certainly the whole body of astronomical teachers will find an invaluable source of lecture material. Descriptive teaching is given life by sketching the manner in which the earlier investigators solved their problems. Too keen an insistence on elaborate descriptions of the very latest advances is apt to obliterate both the relatively greater importance of older discoveries and the high inherent interest of astronomy as a disciplinary study. We know vastly more about the sun-spots than Galileo knew; but our added knowledge might be summarized easily within the limits of a single printed page. It is good to see the old masters receiving a little effective advertising in Repsold's book.

Many teachers will perhaps be surprised to learn that Galileo invented a clock-escapement: his apparatus was not merely a simple swinging weight suspended by a string. The book contains a good picture of the complete apparatus. There are also several valuable plates reproduced from the *Machina Coelestis*, showing Hevelius observing with his great sextant of 1659. In these pictures a lady appears, taking as complete a share in the observing as Hevelius himself. In more recent work, such as that of Fraunhofer and Reichenbach, the book is of course very complete; in fact, we recommend it strongly to the attention of all astronomical teachers.

H. J.

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THE PRESSURE OF LIGHT ON GASES

AN EXPERIMENTAL STUDY FOR THE THEORY OF COMETS' TAILS

By PETER LEBEDEW

The peculiar forms developed in the tails of comets in the neighborhood of perihelion led Kepler,[1] almost three hundred years ago, to the thought that the sun's rays exert pressure upon the matter vaporized in comets' heads and repel it from the sun.

Additional weight was later given to this idea by Fitzgerald,[2] when he sought to base such an effect of the rays on Maxwell's force of pressure. In order to be able to compute the magnitude of the forces occurring, Fitzgerald first proceeded on the assumption that the separate gaseous molecules are absolutely black spheres, and that these spheres behave in respect to the incident light-waves in the same manner as would black spheres of very much larger dimensions. In the case of very small spheres, the phenomena of diffraction become significant, as was proved by Schwarzschild,[3] who rigorously computed the pressure of light on small perfectly reflecting spheres. Debye[4] solved this problem in a general way for small bodies of any desired constitution, and thus it is possible to subject

[1] J. Kepler, *De Cometis*. Augustae Vindelicorum 1610 Opera Omnia. Ed., Dr. Ch. Frisch. **7**, 110, Frankfort, 1868.

[2] G. Fitzgerald, *Proc. Roy. Dublin Soc.*, **3**, 344, 1883.

[3] K. Schwarzschild, *Sitzberichte der Munchener Akademie der Wissenschaften, Math. Klass.*, **31**, 293, 1901.

[4] P. Debye, *Annalen der Physik* (4), **30**, 57, 1909.

to an accurate quantitative treatment the investigations suggested by Arrhenius,[1] which deal with the pressure of light on cosmical dust. It is not permissible for us to apply to the material of comets' tails, which, from their spectroscopic behavior, we must consider as consisting of separate fluorescing gaseous molecules, the computations which are valid for small spheres, as I pointed out long ago,[2] and in view of my proof that the separate molecules must be treated as resonators with selective absorption. Experiments which I made[3] with acoustic waves permit the observation of the continuous effect of these waves on movable acoustic resonators as sharply defined phenomena; the computations[4] which I made for electromagnetic waves permit us to infer an analogous effect of light-rays on separate molecules of gas. Debye[5] treated thoroughly the light pressure on a schematic molecule (a vibrating bipolar body) which is exposed to the solar rays in the same way as the gaseous molecules of a comet's tail; he then computed the numerical values of the repulsive forces thus arising.

Although the computations thus made, and the analogy with the acoustic resonators, scarcely leave any doubt as to the correctness of the idea proposed by Kepler, nevertheless it seemed to me to be in the interests of a theory of comets' tails based upon physical experience to see if direct experiments in the laboratory would give an unimpeachable proof of the repulsive effect of light on gases. Inasmuch as we are unable to deal with single molecules in such experiments, we are compelled to investigate the effect of light on a mass of gas which is compounded from the separate effects of the individual molecules. The resulting effect can readily be computed in this case, as was indicated by Fitzgerald,[6] who proceeded on the simple assumption that those rays only would exert Maxwell's pressure which were absorbed by the gaseous mass, and which, therefore, behave with respect to the gaseous mass like a black body. Then,

[1] S. Arrhenius, *Physikalische Zeitschrift*, **2**, 81–97, 1901. See also *Lehrbuch der kosmischen Physik*, and *Das Werden der Welten*, Leipzig, 1908.

[2] P. Lebedew, *Wied. Ann.*, **44**, 292, 1892.

[3] *Ibid.*, **62**, 168, 1897.

[4] *Op. cit.*, p. 170.

[5] *Op. cit.*, p. 97.

[6] *Op. cit.*, p. 345.

in case of a beam of parallel rays, the repulsive force p in the direction of the ray will be

$$p=\frac{aE}{V},$$

where a is the coefficient of absorption of the energy E incident per second of time, and V is the velocity of light.

I. *Method.*—If a beam of rays of white light passes through a selectively absorbing mass of gas, then the mechanical forces which are to be expected must reveal themselves by the fact that the gas thus penetrated begins to displace itself in the direction of the motion of the light. Inasmuch as the coefficients of absorption of gases are in general very small, the repulsive forces developed, even under the most favorable conditions of the experiment, hardly amount to the hundredth of the pressure which the same beam of rays would exert upon a solid black wall. In order to be able to observe these small forces, the experiment had to be so arranged that the gas could freely move in the direction of the beam and act upon a sensitive valve which could not be directly affected by the beam of rays. Fig. 1 represents the apparatus constructed for this purpose: the gas is placed in a parallelopipedal cavity G, having windows, F_1 and F_2, of fluorite, and is so traversed by the bundle of rays L_1 L_2 that no rays fall upon the walls. If the beam of rays L_1 L_2 exerts a force of translation upon the mass of gas, then must develop at the windows, F_1, F_2, differences of pressure in the gas which may become equalized by the dark space at the sides. This space at the sides is (almost) closed by an easily movable valve P; the valve P being hung on one arm of a torsion balance, the difference of pressure thus arising can be measured by the displacement of the valve P. After we have measured the diameter of the valve P, the directive force of the quartz-fiber Q, the length of the lever-arm of the torsion balance T, and the distance of the scale from the mirror of the reading telescope, we may readily compute in absolute measure the difference of pressure which corresponds to the deviation of the pressure apparatus by one scale-division. In this apparatus one scale-division $=1.4\times 10^{-6}$ dyne per sq. cm. A pencil of a Nernst lamp N (Fig. 2) served as a source of light: its rays were thrown on a rectangular diaphragm D, 2×3 mm, by the concave mirror S; then fell upon an

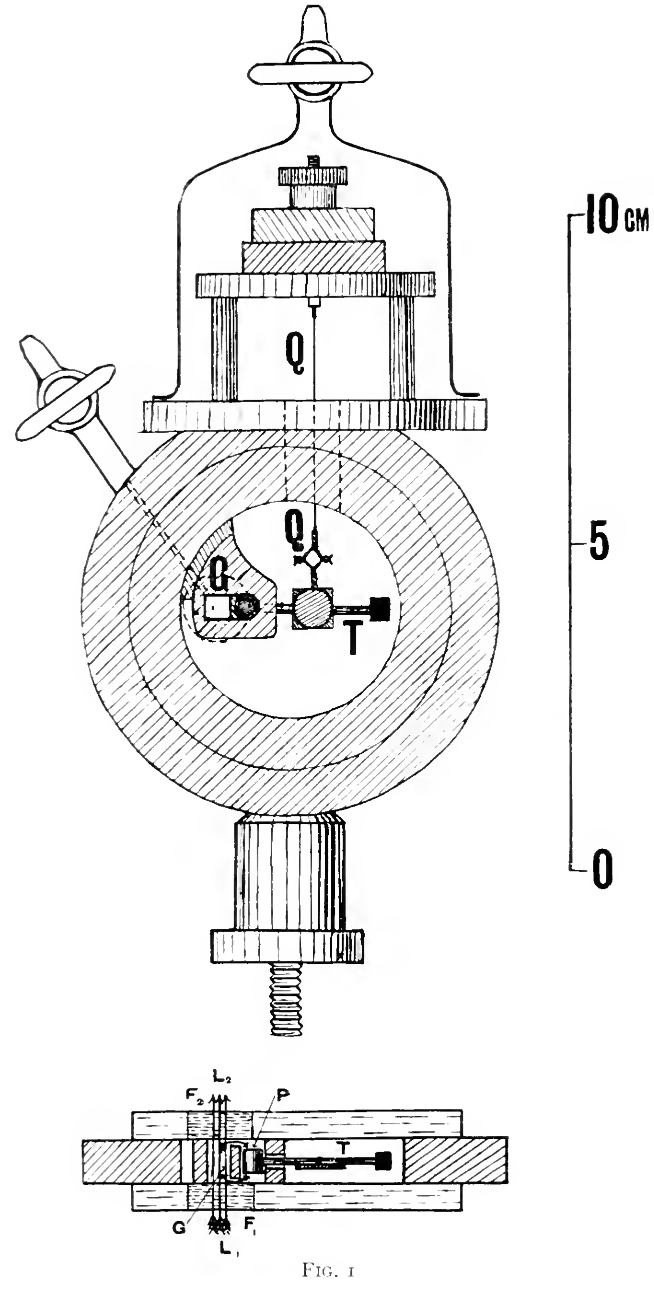

FIG. 1

inclined plane mirror P_1, and were united in a real image of the diaphragm by the concave mirror S_1 in the gaseous space G (Fig. 1) of the above-described pressure apparatus. The plane mirror P_1 can be replaced by P_2 without jar, by a pneumatic attachment, and the rays are sent by S_2 through the gaseous space of the pressure

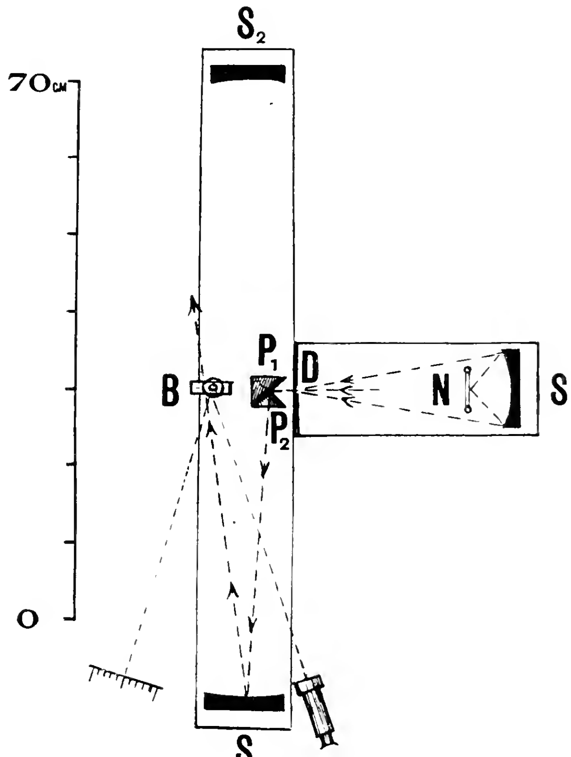

FIG. 2

apparatus, in the opposite direction. Thus the change in direction of the rays doubles the deflection of the pressure apparatus due to the pressure of light on the gas, while the direct effects, which are due to the radiation of the warmed gas on the valve of the pressure apparatus, as a result of the unavoidably small differences of symmetry in the apparatus (which are independent of the direction of the effective rays) disappear.

The coefficient of absorption a of the gas to be investigated was determined by the aid of two thermo-elements T_1 and T_2, which were attached close to the fluorite windows; the ratios of their electromotive forces were determined by the galvanometer when the space for gas was filled, first with air, and then with the gas under investigation, and thus the coefficient of absorption was derived in a simple manner.

The energy of the beam was measured with a calorimeter by allowing the rays to fall for five minutes upon a block of copper K

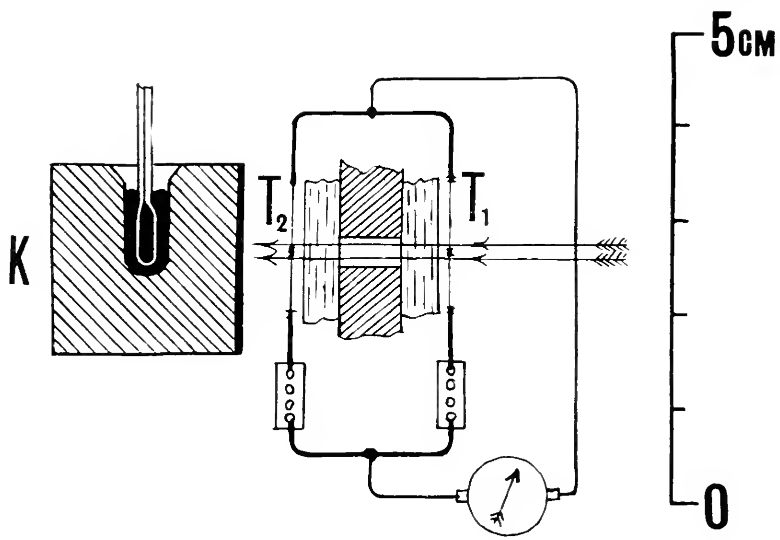

Fig. 3

(Fig. 3) previously given a black coating of platinum on its front surface, and having a known water-equivalent; the rise in its temperature was then measured. I cannot here go into the details of this very difficult experiment, but refer to its more extended description in a paper to appear in the *Annalen der Physik*. I will here only briefly mention that obstacles of two kinds hinder the quantitative experimental testing of the relation proposed by Fitzgerald:

a) The beam of rays can exert an appreciable translatory effect only upon gases which absorb selectively and which, therefore, are warmed by the radiation; change in their density gives rise to con-

vection currents, and thereby can displace the valve of the pressure apparatus. These disturbing effects of the warming can be determined, however, by exhaustive investigations, and are not injurious when the apparatus is correctly set up. As these disturbing forces are decidedly smaller in mixtures of hydrogen than in pure gases, the definitive experiments were made solely with hydrogen mixtures.

b) The simple relationships which Fitzgerald gave for a parallel beam cannot be realized experimentally, since in this case the energy of the beam cannot be made large enough. In a convergent beam the gaseous mass is not penetrated uniformly, differences of pressure arise in its interior, and the accurate computation of the effect of these differences of pressure on the valve apparatus cannot be made; we are, therefore, limited to estimates of the disturbing effects, and thus the computation of the absolute values of the forces of pressure to be measured from a and E are rendered decidedly more difficult and uncertain.

The result is that the relationships given by Fitzgerald can be tested quantitatively only to within about ± 30 per cent. It seemed to me necessary that I should content myself with this precision because the question as to the existence of the translatory effect of light on gases could be definitely decided, and because, on the other hand, the attainment of a greater precision was hindered by very great experimental difficulties.

II. *Results.*—The results of the definitive measurements are summarized in the following table, in which N denotes the current number of the observation, β the measured deflection in scale-divisions of the pressure apparatus, a the measured coefficients of absorption, and T_0 the measured rise in temperature of the calorimeter in five minutes, by which the incident energy E of the beam is measured. The column P_m contains the absolute amounts of the directly measured pressures of the light on the gas, in millionths of a dyne per sq. cm, as determined from the measured deflections β of the pressure apparatus, from its linear measurements, and from the torsion of the quartz-fiber.

In column P_c are given the pressures, also in millionths of a dyne per sq. cm, computed according to Fitzgerald from a and E. The ratio $P_m : P_c$ should be constant and differ only slightly from unity.

The table includes twenty series of observations made with four different Nernst pencils as sources of light. This explains the different values of intensity of the ratio T_0 and of the absorption coefficients a for the same mixtures of gases.

N		β	a	T_0	P_m	P_c	$P_m : P$
3......	0.5 Methane + 0.5 H_2	0.65	0.0065	0.48	0.91	0.76	1.20
6......	" " " "	0.60	0.0057	0.46	0.84	0.66	1.27
20......	" " " "	0.70	0.0071	0.55	0.98	0.98	1.00
1......	0.5 Propane + 0.5 H_2	2.05	0.0200	0.42	2.86	2.10	1.36
2......	" " " "	1.75	0.0175	0.43	2.45	1.89	1.30
11......	0.5 Butane + 0.5 H_2	2.10	0.0179	0.48	2.95	2.15	1.37
12......	" " " "	2.00	0.0172	0.48	2.80	2.06	1.35
13......	" " " "	3.10	0.0189	0.64	4.34	3.03	1.42
15......	0.1 Butane + 0.9 H_2	0.55	0.0063	0.55	0.77	0.87	0.88
17......	" " " "	0.70	0.0072	0.54	0.98	0.97	1.01
19......	" " " "	0.65	0.0067	0.55	0.91	0.93	0.98
4......	0.5 Aethylene + 0.5 H_2	0.60	0.0068	0.43	0.84	0.73	1.14
9......	" " " "	0.75	0.0075	0.50	1.05	0.94	1.12
16......	" " " "	0.80	0.0075	0.55	1.12	1.04	1.08
5......	0.5 Acetylene + 0.5 H_2	0.85	0.0080	0.50	1.19	1.00	1.19
10......	" " " "	0.85	0.0068	0.49	1.19	0.83	1.43
18......	" " " "	0.70	0.0063	0.53	0.98	0.77	1.27
7......	0.5 Carbonic Acid + 0.5 H_2	0.55	0.0055	0.50	0.77	0.69	1.11
8......	" " " " "	0.55	0.0061	0.48	0.77	0.73	1.05
14......	" " " " "	0.70	0.0072	0.51	0.98	0.92	1.06

The table shows that for each mixture of gases, the series of observations agree on the average within 10 per cent, corresponding to the possible errors of observation of the separate measures. For different mixtures of gases, in which the coefficients of absorption vary as 1:3 (methane and butane), and the density as 1:4 (butane), the ratios P_m : P_c exhibit differences which lie outside the errors of observation and indicate slight instrumental errors in the adjustment which could scarcely be overcome in such exceptionally difficult experiments.

The results obtained may be summarized in the following manner:

1. The existence of the translatory force exerted by light upon gases is experimentally established.

2. These forces are directly proportional to the quantity of energy incident and to the absorption coefficients of the masses of gas.

3. The relationship proposed by Fitzgerald is to be regarded as quantitatively proved within the limit of errors possible in these experiments and computations.

These experiments refer to masses of gas under atmospheric pressure, and the numerical values found cannot be directly applied to the excessively rare gases of comets' tails. They give, however, an experimental basis for the further exhaustive development of the physical theories of comets' tails first propounded by Kepler.

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February 1910

THE INTRINSIC BRIGHTNESS OF THE STARLIT SKY

By CHARLES FABRY

Important researches have been made in recent years on the number and distribution in the sky of stars of different magnitudes. To these studies are allied some of the most important problems of cosmogony and physics, such as the absorption of light and the distribution of stars in space. The statistical data which serve as a basis for all these calculations are so difficult to obtain that it is not without interest to find a direct verification of them. Such a verification may be sought by the measurement of the total light given by the starlit sky, or, better, of the mean intrinsic brightness of the sky in different regions of the celestial sphere. The importance of such determinations has been indicated by Newcomb,[1] who concluded that these numerical data "must be considered among the most important fundamental constants of astrophysics."

Two attempts, only, to measure have been made thus far—both visually and by elementary means, by Newcomb,[2] and by Burns.[3] The results obtained by these two observers can be summarized as follows:

1. From one region to another of the sky, no very large differences in the intrinsic brightness are found. Newcomb does not find any difference between the regions whose galactic latitude is greater than 25°; in the most brilliant parts of the Milky Way the brightness would be two or three times that of the non-galactic sky. This last result is also given by Burns.

This slight increase of intrinsic brightness in the Milky Way is, as Newcomb remarks, altogether unexpected. From the statistical data on the distribution of stars, Newcomb should expect to find in the Milky Way a brightness ten times greater than that at the galactic pole, and at 30° galactic latitude a brightness twice that at the pole. Instead of the numbers 10 and 2, the measures give 2 and 1.

[1] *Astrophysical Journal*, **14**, 297, 1901.

[2] "A Rude Attempt to Determine the Total Light of All the Stars," *ibid.*, **14**, 297, 1901.

[3] "The Total Light of All the Stars," *ibid.*, **16**, 166, 1903.

2. A square degree of non-galactic sky would be equivalent, according to Newcomb, to 1.15 stars of the fifth magnitude; according to Burns, to 2 stars of the fifth magnitude. These two numerical results are not as concordant as one might wish; it is necessary to remark, however, that Newcomb evaluates the probable error of his result at 25:100, and that that of Burns is the mean of numbers which vary in the ratio of 1:2.

The authors of statistical studies do not appear to have given much consideration to these measures of the intrinsic brightness of the sky.

Method.—Visual measures are difficult on account of the faintness of the intrinsic brightness to be measured. Photographic measures, on the contrary, are very easy, without excessive exposure-times, because each point of the plate can receive a very wide cone of rays, whereas the retina cannot.

The very simple apparatus which I have used is represented in Fig. 1. The objective A (which will be called the telescope objective)

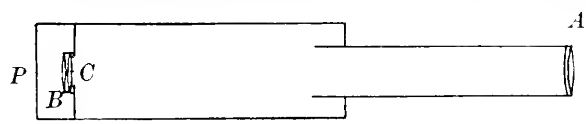

FIG. 1

has a focal length $F=48$ cm, and an aperture $D=5$ cm. In its focal plane is a diaphragm C, pierced by a circular aperture the diameter of which will be called d. Immediately behind this aperture is an optical system B, of very short focus and large aperture, having, consequently, qualities analogous to those of an objective of a microscope, and which will therefore be called the microscope objective. It is not necessary that this objective should have good optical qualities; it consists, in my apparatus, of two simple lenses, each of 20 dioptries, forming a system of focal distance $f=3.15$ cm, and of 3.5 cm usable diameter. This optical system projects on the photographic plate P the image of the telescope objective A. The whole forms a sort of photographic telescope, mounted on a simple, jointed support, supplied with leveling screws. A coudé finder, attached to the apparatus, makes it possible to point it exactly toward any desired region of the sky and, in particular, to center the image of a given star exactly on the opening of the diaphragm C.

Whatever the aperture of the diaphragm, and whatever the distribution of the stars in the region observed, there is formed on the photographic plate an image, uniformly illuminated, of the telescope objective *A*, the image being a circle 3 mm in diameter. It is formed by the light of all that portion of the sky of which the image is in the aperture *C*. The measure of the intrinsic brightness of a region of the sky will include two successive exposures, one on a comparison star, and one on the region of the sky chosen:

1. Let the aperture *C* be of very small diameter, and center the image of the comparison star on that aperture. The image obtained is then produced solely by the light of the star.

2. Next direct the telescope toward the region being studied, and give to the diaphragm a large aperture. The image is then formed by the light of all that portion of the sky which is projected through the aperture, the angular extent of which can easily be calculated. After a few trials, the diaphragm can be opened so that the photographic impression will be the same as in the first instance, the time of exposure being the same. An extremely simple calculation then gives the luminous intensity of one square degree relative to that of the comparison star.

For comparison star I have chosen *Polaris*, as being convenient on account of its almost complete immobility in the sky, and as having an intensity suitable for easy measures with my apparatus. In measures on the non-galactic sky, I was led to use a diaphragm of diameter $d=3$ cm; with Lumiere "Sigma" plates this gave a satisfactory impression in 10 minutes.

Instead of varying only the diameter of the diaphragm, it is more convenient, and not less precise, to proceed in the following manner: By a preliminary trial, determine once for all the diameter which it is necessary to give to the aperture of the diaphragm so that the exposure on *Polaris* and that on the region of the sky chosen should give, in the same time, about the same impression. The measures relative to this region of the sky will then be made with the aperture thus found but with varying times of the exposure. For example, having found that an aperture of 3 cm is satisfactory, make an exposure of 10 minutes on the region being studied with the diaphragm of 3 cm; then make exposures of 5, 10, and 15 minutes on *Polaris* (with a very

small aperture, so as to exclude all light except that of the star). On the developed plate, measure the opacity of the different exposures, drawing, for the exposures made on *Polaris*, the curve of opacity (or better, of its logarithm) as a function of the exposure-time; it is then easy to calculate the exposure-time for which *Polaris* would have given the same impression as the exposure on the sky. The time thus determined differs very little from 10 minutes; it is legitimate to apply the law of reciprocity for times differing so little.

Remarks.—The absorption by the lenses can be disregarded, for the light always passes through the same apparatus. It is true that, in the exposure on the comparison star, the light passes through the central part only of the microscope objective *B*, while, in the exposure on the sky, a part of the light passes through the edge; but the thickness of this objective is so little that the error which results ought to be negligible.

There enters into the calculation only the angular diameter of the circle of which the image is projected on the diaphragm; it is sufficient to know the focal distance *F* of the telescope objective *A*, and the diameter *d* of the diaphragm. The focal distance of the microscope objective does not enter. It is not necessary that this objective should have very good optical qualities; however, the one which I have used, made of two spectacle lenses, was perhaps a little too far from perfection.

The measure gives the mean intrinsic brightness of the portion of the sky whose image is in the aperture of the diaphragm; that is, in my apparatus, a circle of $3\overset{\circ}{.}5$ diameter. It is evident that if there is a bright star in this circle, the value found will not have any relation to the mean brightness of the sky. This is, however, easily avoided. Furthermore, it is easy to give to the apparatus slight irregular displacements around a mean position during the exposure, and thus obtain the mean intrinsic brightness in a more extended region.

Results.—I have made measures only in two regions of the sky.

A. The region near the celestial pole, near the star ϵ *Ursae Minoris* (but not including this star), in galactic latitude 30°. I find that the photographic intensity of one square degree = 0.103 of that of *Polaris*.

B. The region between β and γ *Cygni*, one of the most brilliant

of the Milky Way: photographic intensity of one square degree = 0.212 of that of *Polaris*.

According to King,[1] the photographic magnitude of *Polaris* ought to be considered as 2.62, if it is desired to make photographic and visual magnitudes coincide for the white stars (spectroscopic class A).

Using this value, we find: (1) Region of *Ursa Minor:* photographic intensity of one square degree = 0.92 star of 5th photographic magnitude; (2) Region of *Cygnus:* photographic intensity of one square degree = 1.90 stars of 5th photographic magnitude.

Comparison with the results of Newcomb and Burns.—From the point of view of relative intensities of different parts of the sky, my result is entirely in accord with the visual measures.

In absolute value, the values which I find for the photographic magnitude are smaller than those obtained visually. This naturally follows from the manner in which the scales of magnitudes have been chosen: the number 2.62 adopted for the photographic magnitude of *Polaris* is assigned so that the white stars (type A) should have the same photographic and visual magnitude; as it is certain that the mean color of the stars is more red than that of stars of type A, the photographic intensity ought to be less than the visual intensity. If we should adopt a photographic scale such that *Polaris* should have a photographic magnitude equal to its visual magnitude (2.12), we should find: one square degree in the region of *Ursa Minor* = 1.46 stars of magnitude 5, a result between those of Newcomb and Burns.

We may say then that, within the limits where agreement was possible, my results are in accord with those of the visual measures.

Comparison with the statistical data.—1. E. C. Pickering[2] has found that in the Milky Way the stars of each magnitude are twice as numerous as in the non-galactic sky. The intrinsic brightness of the sky would, therefore, be twice as great in the Milky Way as elsewhere, a conclusion agreeing with the result of the direct measures.

On the other hand, Pickering gives a table of the number of stars of each magnitude in the whole sky. We may deduce from it the total (visual) intensity of all of the stars of the sky. We thus find:

[1] *Harvard Annals*, **59**, No. 4.

[2] *Ibid.*, **48**, No. 5.

Visual intensity of all the stars = 306 stars of zero magnitude, and, consequently,

Mean intensity of one square degree = 0.74 star of magnitude 5.

As this number represents the visual intensity, and as the galactic region enters into the mean, it is probable that it is at least twice too small.

It is true that the table of Pickering does not include stars below magnitude 13.5; but, according to his opinion, the fainter stars would add very little to the total intensity.

2. The remarkable investigations of Kapteyn[1] lead him to results very different from those of Pickering; he gives a much greater importance to the very faint stars, and finds a distribution much more variable as a function of the galactic latitude. Here are some of his results:

Galactic Latitude	Visual Intensity by Square Degree
0°	7.42 star of magnitude 5
30°	1.26
90°	0.48

The number for latitude 30° ought not to be far from the truth; the result for latitude 0° is probably more than two times too great, and that for the galactic pole probably more than two times too small. It appears that Kapteyn has considerably overestimated the influence of the Milky Way.

For the celestial sphere as a whole, Kapteyn finds a visual intensity equal to that of 2384 stars of magnitude 1, which gives:

Mean visual intensity of one square degree = 2.3 stars of magnitude 5, a result which is probably not very far from the exact value.

Upon the whole, therefore, the data actually obtained on the number of stars are very far from being in accord with the result of measures of intrinsic brightness. Is the disagreement a result only of the inaccuracy of the measures and of the statistical data? That is not certain. If it were proved that the total intensity of the sky exceeds considerably the sum of the intensities of the observable stars, one could advance two hypotheses: either that there exists an immense number of stars too faint to be observed with our instru-

[1] *Publications of the Astronomical Laboratory at Groningen*, No. 18, 1908.

ments, or that there exists throughout the sky a sort of continuous nebulosity giving a uniform brightness. The statistical results do not appear certain enough, up to this time, to warrant the adoption of either of these hypotheses. If it should become necessary, it would be interesting, and perhaps not absolutely impossible, to obtain the spectrum of the total light of the sky.

Remarks bearing on further investigations.—The method described above gives, without much trouble, precise measures, but my results are very incomplete. It would be necessary to extend them over a great many more regions of the sky. I have no intention of doing it: in astronomy I am only an amateur; the measures can be made only very far from cities, and the results given above are only those of work done in vacations. A few evenings of observations in an observatory would give much more complete results. I wish simply to indicate what it would be possible to do.

Let F be the focal distance, and D be the aperture of the telescope objective A; f the focal distance of the microscope objective, and d the diameter of the diaphragm which is used in the exposure on the sky.

It is necessary to make two exposures: one, on the region of the sky to be studied, with the diaphragm d; the other, on the comparison star, with a very small diaphragm. This second exposure might be omitted, if one wished only to compare the different portions of the sky.

In the exposure on the sky with the aperture d, the time of exposure necessary to obtain a satisfactory photographic impression depends only on the ratio d/f; this time is independent of the telescope objective A (if one neglects the absorption), and increases as $(f/d)^2$. For the non-galactic sky, with Lumiere "Sigma" plates, the time of exposure would be about $(f/d)^2 \times 10$ minutes.

I have chosen practically $\frac{d}{f} = 1$. The use of a smaller aperture would necessitate longer exposures, which would make the observation more difficult and increase the chances of atmospheric variations. Conversely, it is possible to combine optical systems having a ratio d/f greater than 1 (this is the case in objectives of microscopes), and then make very short exposures, but the marginal rays would strike

the photographic plate very obliquely, which would introduce an error in the comparison of the sky with a star. This difficulty would not exist in the comparison of different regions of the sky (employing diaphragms in the form of sectors), and then comparisons could be obtained with exposures of a few minutes.

The luminous spot, the image of the telescope objective, which is projected on the photographic plate, is a circle whose diameter δ is given very nearly by the equation $\delta = D\frac{f}{F}$. This image should not be too small, and this condition, with a given telescope objective, would fix a minimum for the focal distance f of the microscope objective. In my apparatus, $\delta = 3$ mm, which is more than ample for measures of opacity. It is probable that an image of 1 mm would be sufficiently large.

The measure gives the mean intrinsic brightness of a portion of the sky bounded by a circle whose angular diameter is $\frac{d}{F}$ radians. It is possible, at will, to measure a very small portion of the sky, or a very large portion. The comparison star will need to be chosen accordingly. Its photographic magnitude will be about

$$m = 5 \log \frac{F}{d} - 3.4 \, .$$

I give below two examples, covering two extreme cases.

1. If it is desired to measure the mean intrinsic brightness on a circle of large radius, take a telescope objective of short focus, and a diaphragm of very large aperture. As the telescope objective can in this case have a small aperture, it will give a sharp enough field. Take, for example:

Telescope objective, $F = 13$ cm, $D = 0.4$ cm;
Microscope objective, $f = 3$ cm, $d = 3$ cm.

The comparison star will be of magnitude 0, the exposure will be 10 minutes, and the image on the photographic plate will have a diameter of 1 mm. The measure will give the mean intrinsic brightness of the portion of the sky inclosed in a circle 13° in diameter.

2. The mean intrinsic brightness can be measured in a circle having a diameter of a few minutes of arc, or even one minute, by

taking for the telescope a large objective of long focus, and as microscope objective a system of very short focus. For example:

Telescope objective, $F=20$ m, $D=1$ m;
Microscope objective, $f=10$ mm, $d=6$ mm.

The photographic image will be 0.5 mm in diameter; the comparison star will be of magnitude 14, and the exposure-time about 30 minutes. The measure will be of a circle 1′ in diameter. On such a small surface it will be easy to compare the total luminous intensity with that of stars really observable, to compare, to some extent, what one sees with what one does not see. A photographic impression might be obtained which was produced solely by invisible stars, concealing with screens the visible stars.

Finally, it would be easy to secure an idea of the color of the light of the sky as a whole, by making exposures with ordinary plates and with orthochromatic plates through suitable screens. The use of microscope objectives of large angular aperture would make moderate exposure-times possible, without introducing any cause of error in this special application.

Application of the method to stellar photometry.—The method which I have described in this paper might perhaps be useful for the photographic comparison of stars. In projecting, as I do, an image of the telescope objective on the photographic plate, a circle of uniform illumination is secured, which would not appear to be always the case with extra-focal star-images. It would not be necessary to have a microscope objective of large angular aperture, but it would be convenient to have one whose focal distance could be varied gradually. This could easily be arranged by making it of two parts whose distance could be varied.

University of Marseilles
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ADDENDUM

After the present article had gone to the printer, two papers on the same subject came to my attention.

In 1903, Sydney D. Townley[1] employed, to measure the intrinsic brightness of the sky, a photographic method very different from

[1] *Publications of the Astronomical Society of the Pacific*, **15**, 13, 1903.

that described above. Without interposing any lens, he compared the photographic impression produced by *Vega* with that of a convenient portion of the sky. The exposure-time necessary was one hour. He found that a square degree of non-galactic sky is equivalent to a star of 4.5 magnitude; and that the brightness of the galactic sky is 1.9 times greater.

Very recently, Yntema[1] has made a great many visual measures. The most important conclusion from his observations is that a great part of the light of the sky is of terrestrial origin. This result seems to be peculiar to the conditions under which Yntema has observed. No other observer has found, from one date to another, such large variations as those observed by him. The brightness of a square degree in the region of the celestial pole would equal 0.19 that of a star of magnitude 1, a result about four times as large as those found by other observers.

The only conclusion to be drawn from these great discordances is that measures made in different epochs and in different places would be very useful. The method I have described would permit of their being made with little difficulty, and with apparatus very easy to transport.

[1] *Publications of the Astronomical Laboratory at Groningen*, No. 22. See also Gavin J. Burns, *The Observatory*, March and April 1910.

STUDIES ON THE EMISSION OF GASES

I. MEASUREMENTS ON THE INTENSITY AND ENERGY IN SPECTRA

By H. KONEN and W. JUNGJOHANN

We report here on a series of experiments and measurements made by us, partly in collaboration and partly separately, and in part with Mr. J. Kyll, in the attempt to contribute to the solution of the questions: What are the factors which determine the emission of gases? and, Is it possible to separate these factors without giving the hypothesis so wide a field as has commonly been the case?

We shall begin in this first paper by presenting the points of view which may be taken in regard to the measurement of energy in gaseous spectra. A later paper will treat of the properties of the different sources of light we have employed, and a third will state the results of photometric measurements. Further communications will follow these in close succession.

1. In contrast to the case of the emission of solid bodies, the investigation of the emission of gases has hitherto not led to unambiguous quantitative results of general validity, if we leave out of account the relation between the wave-lengths in series-spectra and similar spectra. The separate qualitative results on the variation of spectra are indeed exceedingly numerous and varied. We shall, further, not discuss in detail the numerous theories of gaseous emission. It may be asserted that no one of the theories or hypotheses is able to include the wealth of facts; still less to make quantitative predictions without the assistance of an excess of "guesses" (*Vermutungen*). The reason for this unfortunate condition is partly to be sought in the fact that the available data are comparatively few for quantitative investigations as to the variability of spectra and the comparison between the variable data and the variables. Pressure-shift, the Zeeman effect, dispersion, magnetic rotation, measurements of energy in emission and absorption with different variables are the principal topics for which numerical results are available. We shall occupy ourselves here with the last-named topic. The number of papers

dealing with it and with the phenomena underlying it is already very considerable. Kayser's *Handbuch der Spectroscopie* gives a survey extending to the year 1902. We have collected in the footnote the papers appearing since 1902, and extend the list by the papers already named by Kayser. We name in the first group (Nos. 1 to 30), exclusively those papers in which are treated either the relative changes of intensity in spectra or measurements of emission; in the second group (Nos. 30 to 36) those in which the applicability of Kirchhoff's law to gases is quantitatively treated, or in which an attempt is made to determine the temperature of gases, for the most part flames, on the basis of a comparison of the emission of gases with that of solid bodies. The third group (Nos. 36 to 41) contains the papers in which the absorption of luminous gases is investigated; some of the papers from the first group also belong in this group. An extended analysis and criticism of all these papers may be found in a paper by Mr. Kyll soon to appear.

2. A comparison of the results of the different investigations indicates only slight agreement. Precise investigations in this respect have been made only on the spectra of mercury and of hydrogen, but there are also numerous, though less exact, researches on the band spectra of nitrogen.

For the latter, most observers state that the intensity of the bands increases proportionally to the current-strength. For mercury, there are precise investigations by Küch, Retschinsky, and Pflüger, from which it follows that the emission, as well as the absorption, increases with the increasing number of watts used in the mercury lamp, and, within any series of lines, the more rapidly for the lines of short wave-length. The change of emission is different for each line within the separate triplets. The absorption, further, does not proceed parallel to the emission. All this points toward a shifting of the maximum of emission toward the blue end with increasing tempera ture, within each series of lines, which is similar to the shifting of the maximum of a black body. Furthermore, in the case of hydrogen and a few other gases, a similar shifting of the maximum toward the blue end was first proven by Langenbach, or a more rapid rise for the short wave-lengths; from which Kayser was in fact able to compute a plausible value of the temperature of emission based upon the rules

for black bodies. Finally, the determination of the temperature of flames by experiments on reversal with the aid of black bodies has led to temperatures which are in agreement with the results of other measurements. It would appear from all this as if the previous experiments indicate that a curve of emission could be drawn for each series that is similar to that of the black bodies.

A careful comparison of the different experiments shows, however, that only the measurements of mercury vapor have been executed with the necessary precision and embrace a sufficient range of current. The measurements with hydrogen, as well as numerous ones of nitrogen, were carried out with discharges from condensers, so that only mean values could be observed. But the measures on nitrogen with a direct current have led to contradictory results, very small currents having been used for the most part. Geiger had indeed employed stronger currents, but he took into account only the strength of the current and not the density of the current. The investigations by him and others are contradictory in so far as it was found that the total color of the discharge changes while the intensity of each region of the spectrum is proportional to the current-strength. It should be further mentioned that other observers found linear relations.

Finally, as is well known, opinions are divided as to whether temperature is after all to be regarded as the cause and efficient variable in gaseous emission. On the one side, the possibility of bringing gases to luminosity by mere elevation of temperature is totally denied. On the other side, the astrophysicists are constantly figuring with the temperature of the luminous gases. We must here pass over the arguments adduced by the two sides, as well as a detailed discussion of the different experiments. It is clearly evident from what has been said that no agreement prevails either as to the change of emission with the current-strength or temperature, or even with the relation of the latter to the observed changes. This holds good above all for the band spectra, which have been only imperfectly investigated in comparison with the line spectra.

It therefore seems to us useful at first to summarize the ideas fundamental for measurements of the sort described, and to do this with the use of the facts already printed in the literature. New facts are not given therewith; but our remarks are intended merely to

illuminate the importance of the conclusions drawn, and to avoid statements that are not clear. We shall here not give references to the literature at every point.

3. If the variable factors of a gaseous spectrum are to be measured, then the nature of the dependent and the independent variables must first be established. The definition of both of these is beset with great difficulties.

The intensity or the energy of a definite point in the spectrum is commonly designated as the variable quantities here in question. The former is measured in comparison with a constant source of light, and serves for comparing the relative changes within a spectrum. The latter is either measured directly, as by the bolometer, or is derived by comparison with the intensity in the spectrum of a body whose distribution of energy is known. In both cases, principally in the latter, there have to be applied numerous corrections which are in part uncertain. Preference should be given to the determination of the energy of one point in the spectrum when this can be executed.

But with the quantities defined in this manner nothing can be done without further consideration. In the first place, practical difficulties are to be considered. It is indeed possible, in the case of some spectra having few lines, to measure the separate lines with the bolometer or photometer. This procedure is, however, in general excluded in the case of spectra having numerous lines, or in band spectra. For instance, the positive bands of nitrogen are only wholly resolved in the higher orders of large Rowland gratings, and the same is true of most other bands. It may be that intensities of separate components can be measured in a photographic manner, but until that can be done, it is only possible to measure by the photometer or bolometer average values of the energy or intensity, the interpretation of which depends upon the variation of each separate component, and the magnitude of which depends upon numerous subordinate factors, such as the position of the center of gravity of the band, the width of the slit, etc. But that changes of the components or of the position of the center of gravity may occur is proven by the investigations on the minimum in the cyanogen band λ 3884, and on the variations of intensity within the separate series of the nitrogen

bands. This is a circumstance which, as it appears to us, has been considered by none of the numerous observers who have investigated the nitrogen spectrum.

The same thing holds good in less degree for the line spectra. Cases are known in which the relative intensity of the components of compound lines changes simultaneously with the intensity of the excitation of the vapor. Further, it is practically impossible to produce a homogeneous stratum of luminous vapor. Therefore phenomena of broadening and reversal are observed, and this particularly with intense excitation. Both of these falsify the measures, the first particularly the photometric measures, the latter affecting all kinds of measures. Since in the photometry of lines a certain minimum width of slit must necessarily be employed, the assumption must be made, in absolute measures, that the brightness is independent of the slit-width employed. In relative measures it must be assumed at least that the dependence of the brightness upon the slit-width does not vary with the excitation of the luminosity. But if a broadening of the lines occurs, the assumption thus made will not apply. An investigation on this point is thus at least necessary to determine how far this is a fact. We shall very presently discuss the effect of the phenomena of reversal.

4. To the practical difficulties named are added those of principle. The absorption must be taken into account in addition to the specific emission of gases. If this is very small, the intensity increases approximately proportional to the thickness of the stratum, without alteration of the relative intensity of the lines or bands. But if, on the contrary, it does not vanish, then the brightness rises toward a maximum with increasing thickness of stratum. At the same time the relative intensity of the separate lines may change because of a difference in the absorption. Added to this is the fact that the absorption, for itself, is again a function of the variables upon which the emission depends.

If we could directly assume the validity of Kirchhoff's law of gases, this difficulty would be overcome; but this is by no means the case; examples in which the validity of the law is probable are contradicted by others in which, in spite of the strongest excitation, it has not hitherto been possible to prove an absorption.

In any case, every measurement of the energy in a discontinuous spectrum must be supplemented by a measurement of the absorption, even when relative measures only are made, and the thickness of stratum remains constant. This last point cannot be rigorously realized, however; for instance, if a gas is rendered luminous by an electric current, it is impossible to avoid having less highly excited strata of gas at the ends of the apparatus used. Their emission and absorption is in general different from that at the portions where the gas is most strongly luminous. Their size also changes with the amount of the excitation, so that in very many cases self-reversals are obtained, which are recognizable only when high dispersion is employed. No general statement can be made as to the magnitude of the error thus introduced, but it must be determined in every individual case.

5. Furthermore, the case often occurs that the spectrum to be investigated is a mixture of different spectra, e.g., of a continuous spectrum, a band spectrum, and a line spectrum. This introduces a double difficulty: (1) the share of the change in energy found for a definite point of the spectrum must be determined for each of the components, which is not always possible. An example is presented by the spectrum of mercury in a quartz tube through which mercury vapor is distilled: the energy of the lines λ 4348 and λ 4349 can then be measured only by uncertain indirect methods. Still greater, however, is (2) the difficulty thereby introduced, because with the change in excitation of the gas not only does the intensity at every separate point in the spectrum change, but also the composition of the spectrum is altered. For instance, if we are dealing with the superposition of a band spectrum upon a line spectrum, then not only will the intensity of both vary, but also the ratio of the intensity of the two. This effect is not limited to different classes of spectra, but even occurs between lines of one and the same spectrum. It may be asserted with a high degree of certainty that the number of emitting particles in a gas at a given time is small in percentage, and similarly that the different emissions correspond to different conditions of the emitting particles. On this assumption, the effect named may be described by stating that the number of particles producing a definite line shares in determining the intensity of the emission. According as this

number increases or decreases under given conditions, an increase or decrease of the emission may be observed at a definite point in the spectrum, although perhaps the mean emission of each separate particle has changed in the opposite sense.

Therefore, if we wish to obtain values of the emission in order to make a comparison for a definite point in the spectrum, or also for two different wave-lengths, then we must either determine the quantity of the luminous gas or eliminate it. If we wish to institute a comparison between different wave-lengths, then we must be certain that the waves in question have the same centers of emission. Kayser was the first to point out the decided importance of this point, which has often been overlooked in recent publications. The decision whether the same centers of emission can be assumed for given wave-lengths is often very difficult. All the facts must be taken into account which can give any hint as to the nature of the luminous particles. As was remarked first by Kayser, we may regard the fact that they belong to the same series as a fair sign of the same origin for different lines, since the simultaneous appearance of the lines of a series, and their similar behavior under the influence of pressure, of a magnetic field, and of other physical factors, as well as the relation between their vibration numbers, makes it in the highest degree probable that the lines in question were produced by the same vibrating system.

But there are still other cases in which we may assume the same thing; for instance, the series within one band of a band spectrum, which may be included by a formula similar to those of the series; or absorption lines which furnish the same series of fluorescence lines. The possibility is also not excluded that we may regard the whole system of bands as of the same origin. Dispersion, or the separation of luminous masses of gas in space, or in electrical or magnetic fields, in short all the physical factors, must be taken into account which characterize a definite spectral line in order to decide whether or not we may assume the same origin for given spectral regions.

6. The suggestion is obvious that instead of the emission or the absorption of a gas, we may introduce, as the variable to be measured, the ratio of the two, after the analogy of Kirchhoff's law. This

proposal has already been made,[1] but whether it may be adopted, and whether the resulting function, as the ratio of emission and absorption for each group of lines, follows simple laws and is independent in respect to its form from the nature of the instantaneous centers of the emission, must certainly be determined by experience and is not a priori obvious. The chain of reasoning on which is based the derivation of the Kirchhoff law and of the radiation function of the black body fails to apply to gases, as is well known, unless certain hypotheses are made. In the first place it must be assumed that the luminous gases absorb the same kind of light that they emit. Although this has been measured for different examples and is proven to be qualitatively applicable for many lines of the flame, arc, and spark, by experiments in reversal, the general proof has not been given. The opposite is expressly asserted for single examples, as the band spectra of many flames; and we cannot a priori exclude the possibility that in gases similar phenomena may occur as for fluorescent bodies, for which hitherto it has been impossible to prove the existence of a maximum of absorption corresponding to the emission, and where emission and absorption appear to belong to two different states of the luminous particles. Decisive experiments on this point are still awaited.[2]

But even if we assume that in every case a gas absorbs the identical wave-lengths which it emits, it nevertheless remains certain that in many cases the absorption is exceedingly small and below the limits of measurement. Then, if D represents the thickness of the luminous stratum, E the emissive power of an infinitesimal stratum of thickness dx, and a the coefficient of absorption, the emission of the gas for a wave-length λ will be

$$\epsilon=\int_{O}^{D} e^{-ax}E dx$$

or practically equal to DE. We therefore determine E only. For an infinitesimal stratum, ϵ must become equal to the emission of

[1] See A. Pflüger, *Annalen der Physik* (4), **24**, 575, 1907; also the article "Theorie der Strahlung," by W. Wien, Bd. 3, Heft 2, *Encyklopädie der mathematischen Wissenschaften*, p. 348, Leipzig, 1909.

[2] Kayser, *Handbuch der Spectroscopie*, Bd. 4, p. 964, where one of us discusses the matter fully.

the black body. But it will not always be possible to realize in practice an infinitely thick stratum by magnification of the thickness of the stratum or by applying mirrors in the proper way.

This indicates that in most cases the relation of the emission to the absorption cannot be determined at all, and we are compelled to draw our conclusions from the behavior of the emission only. But we must still, with Pflüger,[1] regard the measurement of the ratio $E:A$ as one of the most important problems in the quantitative investigation of gaseous spectra.

7. Finally, we must give some consideration to the question as to how far a spectral line is characterized by the data as to its emission and absorption, first leaving out of consideration all the other physical data which are significant for a line, as the Humphreys-, Zeeman-, and Doppler-effect, etc., and considering only the influence of lack of homogeneity. Every line of a band or line spectrum is to be regarded as practically a portion of a continuous spectrum, provided it does not have satellites. It is not known whether the light contained in it is equivalent to a portion of the spectrum of a black body in the same spectral region in this respect, but we can in any case assume, with Wien,[2] that this is so. There are, nevertheless, the greatest differences in respect to lack of homogeneity between different spectral lines: in the same spectrum, e.g. the spark spectrum of a metal, there occur extremely diffuse lines alongside of very sharp lines. Therefore, if we make our measurements with instruments of small resolving power, as a bolometer, then the total energy contained in the line will also be measured at the same time; hence it can happen that a diffuse line gives the same energy as an intense, but very sharp, line. If we measure with the eye, then, as already mentioned, it depends on the instrumental conditions whether we obtain a quantity nearer to the total energy of the line or to that of a definite wave-length. Finally, if the measurements are made photographically, we obtain primarily from the degree of blackening the intensity referred to a definite wave-length. The attempt has recently been made to derive from this value the total energy of the line by multiplying by the breadth of the line. (See below.)

[1] *Annalen der Physik* (4), 24, 515, 1907.

[2] *Op. cit.*, p. 348.

It is in general customary to regard the total energy in a line as characteristic of it. This leads, however, to some consequences of little plausibility: it does not agree with direct observation to call a broad, diffuse, and often hardly visible strip in a spectrum equal to a bright, strong line. This view proves to be properly established in certain respects; it depends on what we measure as the intensity of a line. If we start from the view that energy is contained in the broadening of a line which becomes appreciable as an increase of the total brightness of the luminous gas, then we are embracing the total emission of the luminous system under investigation. We must then integrate over the entire width of the line in question. But to the value thus obtained we must add the contributions of energy from all the other lines which we ascribe to the same center of emission; only in this way can we actually obtain the total emission, which we may then compare with the total emission of other centers or of solid bodies. But in practice we are concerned with something else: a measure of the energy in the sense of the optical intensity is desired; or the line in question is compared with a continuous spectrum under equal resolution. Then the matter depends on whether or not the maximum of the line for a given apparatus exceeds the intensity at the same place of the comparison spectrum. The ideal procedure would, therefore, be to determine the intensity- or energy-curve within each line; but as this is generally not possible, the attempt will be made to determine a quantity proportional to the maximum of the line. That it is generally this quantity, and not the integral, which is involved, appears from the fact that the experiments of the reversal and the comparison with the black body for fixed wave-lengths depend upon it.

We therefore propose to differentiate between:

a) The total energy of a line, which is obtained by integration from the energy-curve of the line; this value can be employed immediately only in the case where the luminous system is emitting this one line only.

b) The total energy of a system of lines; this is equal to the sum of the total energies of all the lines connected in the system (e.g., the lines of a series).

c) The intensity of a line; this is equal to the maximum of the

energy-curve of the line. If the energy-curve of a line cannot be obtained, then it is necessary to give the data (c) along with the mean value of the total energy of a line.

Similar statements may be made as to absorption.

8. A thorough discussion of the variables which condition the distribution of energy in the spectrum of a gas is not possible without taking up the numerous hypotheses as to the nature of gaseous emission. We therefore content ourselves with mentioning a few of the principal points; and we here would by no means claim to bring forward anything new. The purpose of our remarks lies in making the most precise statement possible of the assumptions in measurements of intensity, with the avoidance of all unnecessary hypotheses.

The variables upon which a gaseous spectrum depends differ according to the source. In an electric furnace, such as used by King, temperature is the primary variable; in the electric arc there come into question current-strength, potential-gradient, chemical processes, surrounding atmosphere, electrical data for the circuit, character of the electrons, pressure, quantity of vapor, effect of other elements present as impurities, and perhaps still other factors, which vary from point to point in the arc. With the oscillating spark there are added to all these factors the time of the observation (order of the phase of partial discharge). In flames the most different kinds of chemical processes play a rôle, along with the temperature, the pressure, the quantity of material, and such factors. For discharges in Geissler tubes all of the items named for the arc and spark occur together, and to these in all cases are added the pressure of the gas under investigation, and the degree and character of its ionization.

Among the variables thus mentioned, there are doubtless many of a secondary nature, but it is not decided to what degree this occurs. It is assumed by many that with constant pressure the temperature is the controlling variable. This is not to be measured as the average temperature of the gas, but refers principally to the luminous particles, indifferently whether these are assumed to be charged or not. The mean internal energy is supposed to depend on the temperature of the centers of radiation, and upon these again depends the energy of the radiation. Whether the inner energy of the radiating parts

sustains a simple relation with the temperature in an ordinary sense remains uncertain. But in any case, it seems possible that this may be so, and, further, that there exists a simple relation between the internal energy and the radiated energy, or also between the so-called temperature determining the inner energy and the radiated energy; in form, this relation may agree with similar laws of a black body. Chemical processes and ionization are then regarded as also determined by the temperature. But the temperature of the luminous centers in question, which possibly constitute but a small fraction of the gas, is not directly measurable, since all measurements furnish only a mean value. Therefore, even if we adopt the above point of view, we may not employ the temperature of luminosity as a variable; the converse process only can be employed. We may measure the change of energy in a spectral system (e.g., a series) and from the change, on the basis of some definite hypothesis, such as the assumption that the same rules hold good as for a black body, compute a temperature, the validity of which may be tested in some other way. This procedure was first carried out in logical manner by Kayser.

9. A modification of the view given here is to regard the temperature as the variable in the ordinary sense, at least in many cases. The temperature of luminosity of a radiating gas would then be defined as that temperature at which a black body is in radiation equilibrium with the gas in question. If we overlook entirely the fact that the proof must still be produced that a radiation equilibrium of this sort is possible, the objection may be made that practically in many gases the temperature of the gas could not be measured, because it would not be possible to produce a black body of sufficiently high temperature. Nevertheless, noteworthy results have been obtained on this assumption, and we intend to return to this point again.

10. In many cases it is not possible to produce in gases heated in closed receptacles the emission of spectra readily evoked in other ways; further, the luminous gases are in many cases appreciably ionized; finally, it can be proven that in many instances the ions are luminous; hence it is natural to think that the essential element in the emission of gases is to be found in this ionization, whether the emission is attributed to the recombination of ions or electrons, or to perturbations which the ions suffer when they are free. From

this point of view the temperature plays a secondary rôle: it determines under some circumstances the degree of ionization, and perhaps also the amount of excitation of the ions. We must expect that the factors determining the ionization, namely, current-strength, potential-gradient, position in the circuit, and chemical processes, play the principal rôle and determine the emission of the gas. This view has in its favor the fact that certain phenomena in vacuum tubes, also of the relation between ionization and luminosity, as well as the dominance of the chemical and electrical methods in the production of gaseous emission, are explained in a simple manner. It further permits us to include all the phenomena observed in the electrical conductivity of gases, and it is very capable of adaptation. We have a large number of possibilities of combination, and may regard the positive ions, or the negative ions, or modifications of them or the processes occurring in the formation or recombination, and similar effects, as determining the emission. But great disadvantages are opposed to these advantages: first, it is in no wise proven that in all cases the luminous gas is ionized, or, conversely, that any ions present are the carriers of the emission. We should first name here numerous spectra of compounds for which this proof is conspicuously lacking.

Even if we adopt the standpoint thus described, the practical application of the assumption in most cases goes to pieces from the complication arising from the many kinds of ions. For instance, in the case of helium, we should have to assume that there are at least six different kinds of ions, or perhaps as many different processes of excitation; and for elements having many lines these would be still more numerous. But this excludes the ionization as the independent variable, so long as it is impossible to separate and investigate the different kinds of ions. The assertion may be maintained that it has not hitherto been possible for anyone to isolate in an unquestionable manner ions or atoms of the same gas with emission proven to be different. Numerous examples have indeed been given from which such a conclusion has been drawn, but there have always been valid objections to these. The ionization of a gas would be practically treated as the variable only if the total emission in a given space, e.g. in the positive column, could be referred back to a single kind of ions (or process of ionization). In this case different conse-

quences would follow, to which we shall revert. It is obvious that these last considerations find their particular application in gases through which a current is passing, but first of all a relationship would have to be found between the number of ions and the energy emitted. It has already been asserted on the basis of such a process of reasoning that the emission must be directly proportional to the number of ions, hence to the strength of the current. But the objection may be made to this that the intensity of the excitation will change simultaneously with the number, if we may indeed assume that the luminous particles are capable of different degrees of excitation, which assumption would include a sort of temperature effect. In that case, we should select as the variable, not the current-strength, but rather the work done, hence the potential-gradient times the current-strength. This proposal has already been made, but without being logically carried out.

11. To the variables thus named there must be added, as a subordinate variable, the pressure of the gas, which plays a large rôle, particularly in electrical methods. Its greatest effect is due to the change of the discharge, and may be explained from the standpoint of the theory of the electrical conductivity of gases, if we regard either the temperature or the ionization and related processes as variables; but many experiments have shown that there are attendant phenomena pointing to a direct influence of the pressure on the emission: we mention here particularly the phenomena of broadening, and we shall revert to this point. Experiments on the emission of gases are greatly hindered by the condition that the pressure should be kept constant. This applies to the mercury lamps which are otherwise so excellent. Although experience shows that the pressure of a gas has the greatest effect on its emission, we cannot say a priori that the total pressure of the gas is the independent variable; it is thinkable that only the partial pressure of the luminous constituents was effective.

12. The different assumptions we have mentioned by no means exhaust the variables determining the luminosity of gases. It appears quite thinkable that there are different classes of emission, or that in individual cases several of the causes named operate together; but we abstain from depicting the possibilities that would thus arise.

We can decide between them only when it has been established by numerous examples what is the actual relation of the previously defined dependent and independent variable quantities, and whether the variation of the former can in any wise be satisfactorily represented with any of the latter. In a later communication we shall first report upon experiments of this sort, which seem to indicate that the relations are complicated, by finding different variables for different spectra.

Münster
March 5, 1910

NOTE ON THE INTERPRETATION OF SPECTROHELIOGRAPH RESULTS AND OF LINE-SHIFTS, AND ON ANOMALOUS SCATTERING OF LIGHT

By W. H. JULIUS

The puzzling character of solar problems is well illustrated by the fact that the images obtained with the spectroheliograph give rise to widely different explanations, and that it seems impossible as yet to answer in a satisfactory way even the fundamental question: What is the principal cause of the very unequal distribution of different kinds of light over the sun's disk?

Hale and Ellerman, in a paper "On the Nature of the Hydrogen Flocculi and Their Structure at Different Levels in the Solar Atmosphere,"[1] reject the hypothesis advanced by W. J. S. Lockyer, that the dark hydrogen flocculi indicate regions where there is a deficiency of hydrogen. They also refute Deslandres' argument, according to which those dark flocculi are not mainly due to a particular distribution of the emissive or absorbing power of hydrogen, but to a simple instrumental cause, an inherent defect of the spectroheliograph. In their own opinion, the best way to account for the observed phenomena is the hypothesis that the dark hydrogen flocculi are produced by increased absorption (probably resulting from greater depth and decreased temperature of the hydrogen gas in these regions of the solar atmosphere), while the bright flocculi represent regions of increased radiation. Finally Hale and Ellerman state that the results obtained in the high-dispersion work with the hydrogen lines are also in accord with certain inferences which I deduced[2] from the hypothesis first advanced in 1904,[3] that the distribution of the light in photographs taken with the spectroheliograph is mainly

[1] Hale and Ellerman, *Proceedings of the Royal Society*, **83**, 177, January 1910.

[2] Julius, "Anomalous Refraction Phenomena Investigated with the Spectroheliograph," *Contributions from the Mount Wilson Solar Observatory*, No. 29; *Astrophysical Journal*, **28**, 360, 1908.

[3] Julius, "Spectroheliograph Results Explained by Anomalous Dispersion," *Proceedings of the Royal Academy of Amsterdam*, **7**, 140, 1904; *Astrophysical Journal*, **21**, 278, 1905.

caused by anomalous dispersion. They wish to defer a general discussion of the effects of anomalous refraction in the solar atmosphere until many more observations have been made; but a preliminary survey of the results already obtained induces them to believe that the principal phenomena of the dark flocculi may be explained more satisfactorily as absorption effects, and that the evidence can hardly be considered favorable to my theory.

In the present note I wish not to combat the absorption hypothesis proposed by Hale and Ellerman, but only to show that their objections to an explanation on the basis of anomalous dispersion are easily refuted, and that the results so far obtained are by no means less favorable to the theory which ascribes the flocculi in the main to anomalous dispersion, than to that which explains them as mere absorption effects.

The intensity and width of the hydrogen lines, especially of $H\alpha$, differ greatly in different regions of the sun. If the widening of these lines is caused by increased absorption only, there is no reason to expect them to be asymmetrical (except perhaps by local displacements in consequence of motion in the line of sight). If, on the other hand, we are chiefly dealing with dispersion bands,[1] enveloping the real absorption lines, so that the widening results from the fact that the strongly refracted waves bordering the central lines have their origin on the average in less luminous regions—then, at first sight, it seems as if a marked and variable asymmetry must be the general appearance. Indeed, when comparing waves at equal distances from the center of the line on the red and violet sides, we must find the rays curved in opposite directions by the same density-gradients of the solar atmosphere. Hale and Ellerman think it improbable that equal amounts of light would reach the observer in both cases, and therefore conclude that, if anomalous dispersion were the principal cause, the spectroheliograph should, as a rule, give very different images when set on the one or the other side of the same line.

They tried the effect of photographing the flocculi simultaneously with light from opposite sides of the $H\alpha$ line at equal distances from the center. In general the two images proved to be almost identical in their principal features, though small differences of detail were

[1] Julius, *Astrophysical Journal*, **21**, 271–291, 1905; **25**, 95–115, 1907.

often visible. In the case of what they call "eruptive phenomena" the images were very unlike, as the distortion of the $H\alpha$ line would lead one to expect. It stands to reason that, if such distortions are satisfactorily explained on the basis of the Doppler effect, a corresponding explanation can be given of the unlike parts of the images just mentioned.

These are the considerations adduced by Hale and Ellerman in support of their conclusion that the results hitherto obtained are unfavorable to the anomalous dispersion theory.

On closer examination, however, the consequences of anomalous dispersion turn out to be in harmony with the observed phenomena. It will indeed prove very probable that R-light and V-light[1] selected at equal distances from $H\alpha$ should, in spite of their opposite curvature, produce images which are in general almost identical in their principal features, and that differences of detail should chiefly appear in much-disturbed regions, where steep density-gradients occur.

In a paper on "Regular Consequences of Irregular Refraction in the Sun"[2] I attempted to obtain a general idea of the optical effect which local condensations and rarefactions in the solar atmosphere must produce, if only the incurvation of rays is taken into account.[3] The result was as follows.

Let us first consider light for which the refracting power of the solar atmosphere ($n-1=R\Delta$) has a certain *positive* value. Somewhere on the central part of the disk we imagine in the gaseous envelope a region of any shape, only satisfying the condition that, from the outline inward, the density of the gases either diminishes or

[1] By R-light and V-light will be denoted waves on the red and violet sides of absorption lines within the limits where anomalous dispersion is perceptible.

[2] Julius, *Proceedings of the Royal Academy of Amsterdam*, **12**, 266, 1909; *Memorie della Società degli Spettroscopisti italiani*, **38**, 173, 1909; *Physikalische Zeitschrift*, **11**, 56, 1910.

[3] It may be well here to remark, that in the paper referred to, as well as in former publications on anomalous dispersion, I never thought of denying the probable effects of selective radiation, absorption, scattering, radial motion, pressure, radio-activity, magnetism; but because in solar literature full attention is generally paid to most of these subjects, whereas refraction and anomalous dispersion are little noticed, I wished to consider the latter agencies separately, and to inquire which solar phenomena may be produced or influenced by them. The object in view was not a theory of the sun, but a study of the cosmical consequences of anomalous dispersion.

increases continuously, so that the region includes either a minimum or a maximum of density. In both cases the image will show a dark rim. If in these two cases the density-gradients, though opposite in sign, were equal in magnitude, the optical images presented by the rarefaction or the condensation would be almost identical in their principal features. This is due to the fact that the light transmitted by our region comes from a source extending nearly symmetrically round the line of sight. As soon as the latter condition is not fulfilled, if, for instance, some of the rays, before entering our region, had already suffered strong deviation in a neighboring very marked density-gradient, the symmetry of the apparent source of light would be disturbed, and then the aspect of the rarefaction might sensibly differ from that of the condensation of the same shape.

Let us now consider light for which the refracting power of the solar atmosphere is equal in absolute magnitude, but *negative*. Such waves behave in a rarefaction just as the other waves, first considered, would do in the condensation that would be obtained by reversing the gradients. The optical effect is generally the same in its principal features. Consequently, confining our attention to the central parts of the disk, and excluding the much-disturbed regions, we must expect to find only small difference between spectroheliographic images taken with R-light and V-light selected at the proper distances from an absorption line.

Hale and Ellerman admit that the small differences, frequently observed when comparing images given by opposite sides of $H\alpha$, are, perhaps, due to anomalous refraction; I see no reason why the same principle should be inactive in the production of the remaining, almost identical, parts of the images.

As we approach the limb, the conditions of refraction are, however, modified. When seen projected on the disk at a sufficient distance from the center, a region with a minimum and a region with a maximum of density will appear different. With R-light the rarefaction shows *dark* on the side *opposite* the center of the disk, and may be *brighter* than the surroundings on the side *facing* the center, whereas the condensation shows *dark* on the side *facing* the center, and may come out *bright* on the *opposite* side. With V-light these effects are the reverse, rarefaction and condensation optically changing

parts.[1] So we have reason to expect that between spectroheliograms taken with light from the red and violet sides of a line, some systematic differences of detail—increasing as we proceed from the center toward the limb, and relating to distribution of brightness rather than to structure—will be observed.

It will prove necessary, however, to check the latter expectation, because there is a physical law, not hitherto considered in our argument, which tends to efface the differences just mentioned, and to promote similarity of the corresponding R-light and V-light images all over the disk. I mean the fact, discovered by Rayleigh, that the light is *scattered* by the molecules of a transmitting medium.

Effects of scattering on the character of the total radiation transmitted by stellar atmospheres were first considered by Schuster in a most interesting article, "Radiation through a Foggy Atmosphere."[2] It would lie beyond the scope of the present note to discuss the general bearing of the remarkable results, there described, upon conclusions deduced from the anomalous dispersion theory. One point, however, which may prove very important with respect to the explanation of spectroheliograph results, requires our special notice, viz., that scattering is a *selective* process. This peculiarity was alluded to by Schuster on p. 17 of the paper cited, but not further considered there.

Indeed, if we accept Rayleigh's formula, the coefficient of scattering, called s in Schuster's paper, depends not only on the number N of scattering particles per unit volume, and on the wave-length λ of the light under consideration, but also on the index of refraction n of the medium:

$$s=\frac{32\pi^3(n-1)^2}{3N\lambda^4} . \qquad (1)$$

The terms "anomalous dispersion" and "anomalous refraction" were until now used indiscriminately. We shall in future distinguish between the two expressions. By anomalous dispersion we denote the general property of matter, that its refracting power $\pm(n-1)$ varies rapidly as we approach an absorption line. This property, of course, subsists even when the density of the medium is perfectly

[1] *Proc. Roy. Acad. Amsterdam*, **12**, 26?, 274–276, 1909; *Memorie d. Soc. d. Spettr. ital.*, **38**, 175, 180, 1909; *Physikalische Zeitschrift*, **11**, 58–59, 63–64, 1910.

[2] *Astrophysical Journal*, **21**, 1–22, 1905.

uniform, and the propagation of light rectilinear. Whenever the density is not uniform, it may cause very different deviations of neighboring waves. That effect of anomalous dispersion—which I exclusively studied in former papers on the subject[1]—will be called *anomalous refraction*. Another effect, dependent on the same property, and now considered for the first time, is *anomalous scattering*.

Equation (1) shows that the coefficient of scattering passes through a sharp maximum in the neighborhood of every value of λ which corresponds to an absorption line, because there the factor $(n-1)^2$ increases rapidly as we approach the line from either side. In the nearest vicinity of the absorption lines of a mixture of gases, Rayleigh's formula is perhaps not rigorously applicable, but we may use it as a first approximation.

Even absolutely monochromatic absorption would thus, in an extensive atmosphere, give rise to a line of a certain width. If a group of neighboring waves are absorbed, the width of the resulting dark line will always exceed that of the spectral region of real absorption. Every absorption line of a stellar atmosphere is, therefore, enveloped in what we may call a *dispersion band*, because it depends upon anomalous dispersion. In an atmosphere of perfectly uniform density the dispersion band would be caused by anomalous *scattering* only; but if irregular density-gradients occur, anomalous *refraction* adds to the effect in two ways: (1) by directing back toward the luminous surface some of the strongly refracted rays,[2] and (2) by lengthening the paths along which the beams are subject to loss of intensity by scattering.

These notions may gain clearness if we imagine ourselves to be placed somewhere in the solar atmosphere, looking outward. Then

[1] I am very much indebted to Professor Lorentz of Leiden, who, at my request, was kind enough to subject my preceding work on the consequences of anomalous dispersion to a thorough criticism. According to him the weak side of my conclusions was, that I had not duly noticed the diminution of the light by scattering. I intend to discuss this important point more fully on a later occasion. The resulting new aspect of the anomalous dispersion problem will render necessary certain modifications of the theory (e.g., regarding the explanation of prominences), and may thus perhaps serve to reconcile opposite opinions on this matter.

[2] This process was more fully treated in my paper on "Regular Consequences of Irregular Refraction in the Sun," in the chapter "On the Origin of the Fraunhofer Lines," *op. cit.*

a spectroscope, if directed on the "solar sky," would show us the Fraunhofer lines bright on a less luminous ground, not only on account of luminescence or of selective temperature-radiation, but also because the scattering is more intense in the vicinity of absorption lines than in blank parts of the spectrum. The energy which thus returns to the sun by the *scattering* process is wanting in the Fraunhofer spectrum as seen on earth. Besides, the irregular density-gradients of the solar atmosphere would give rise to "mirage" on a large scale, also of a selective character. Distorted images of parts of the brilliant solar surface would appear everywhere in the sky, different in shape and extension for kinds of light that are differently refracted. This is the portion which anomalous *refraction* contributes to the returning energy, and withdraws from the radiation leaving the sun.

Applying our ideas on the combined consequences of anomalous scattering and refraction to the interpretation of spectroheliograph esults, we must remember: (1) that anomalous scattering darkens the solar spectrum almost equally on both sides of a strong absorption line,[1] thus reducing the differences which photographs made with R-light and V-light at equal distances from the same line would have shown, if anomalous refraction were the only agent; (2) that the width of a Fraunhofer line would be a minimum at points of the sun's image corresponding to regions of uniform density and composition in the solar atmosphere, because there anomalous scattering would be the only cause of the dispersion band; (3) that the same line will be wider, and, in general, darker in the spectrum of regions where irregular gradients disturb the rectilinear propagation of the light. (In this way we explain the varying width of $H\gamma$ as shown in Fig. 2 of Plate I, *Proceedings of the Royal Society*, **83,** 189, 1910. If, therefore, the camera-slit of the spectroheliograph is set, for instance, between the center and the edge of $H\alpha$, but nearer to the edge, the dark flocculi indicate regions where density-gradients with large components perpendicular to the line of sight are in

[1] It will be mentioned farther on, that especially the weaker Fraunhofer lines are asymmetrical by anomalous dispersion. So long as spectroheliograms are made only with light from the domain of strong lines, we may, in interpreting them, neglect that systematic asymmetry.

evidence. Almost the same structure must be revealed if the camera-slit is set on $H\beta$ or $H\gamma$, provided the distance from the center of these lines be taken smaller than with $H\alpha$, in order to catch waves that are refracted to the same degree as those in the former case. This effect was predicted in my paper in the *Astrophysical Journal*, **28**, on p. 369, and afterward found confirmed by Hale and Ellerman.)[1] (4) that gradients of exceptional magnitude and extension may produce marked irregularities in the distribution of the light within the range of a dispersion band; (5) that the composition of the solar atmosphere very probably varies with the level, but that convection currents tend to efface local differences of composition and temperature.

If these statements are kept in mind, it will be found possible to explain, on the basis of anomalous dispersion, at least as many particulars of the spectroheliograms as were explained by Hale and Ellerman on the basis of their temperature and absorption hypothesis. We will not, on this ocasion, enter into a comparison of the advantages of both points of view, our present aim being only to prevent a premature criticism of either of them.

With a similar object in view we shall now consider another important solar phenomenon—systematic displacements of Fraunhofer lines—which also was explained according to two entirely different theories.

I showed elsewhere[2] that anomalous refraction by irregular density-gradients causes the Fraunhofer lines to be asymmetrical, the narrower ones generally to a higher degree than the wider ones, thus producing an apparent displacement of the lines toward the red. The displacements must increase when passing from the center of the disk to the limb. These effects depend upon the rule that the refracting power of the mixture of gases constituting the solar atmosphere is on the average greater on the red side of an absorption line than on the violet side. Anomalous scattering also being determined by the values of the refracting power on both sides of the absorption lines, it co-operates in producing those systematic displacements.

[1] The optical effect produced by the systematized density-gradients near solar vortices requires special treatment.

[2] Cf. the paper on "Regular Consequences," etc., referred to above.

From a recent remarkable investigation of the displacements of the spectrum lines at the sun's limb, by W. S. Adams,[1] it appears that out of a total of 470 lines only one or two are shifted unmistakably toward the violet; the other lines all show displacements to the red, ranging from 0.000 to 0.014 Ångström. The various characteristics of the list of these lines will have to be studied in detail from the point of view of anomalous dispersion. I must defer that inquiry to a later date, and now confine myself to a few remarks on prominent statements made in Adams' paper.

Adams concludes that pressure is the effective agent in producing the displacements observed. He evidently paid very little attention to the possibility of explaining these phenomena by anomalous dispersion, for although he refers to the explanation which I recently published in the *Memorie della Società degli Spettroscopisti italiani*, and rejects it, the clue of my argument escaped his notice. Indeed, he writes:

> According to his [Julius'] point of view the photospheric light is anomalously refracted in the vicinity of the absorption lines produced by the metallic vapors, and, since in general the density-gradient decreases outward, the widening will be upon the red side of the lines producing the observed displacements. The fact that the sodium lines D_1 and D_2 are not displaced, although they show the largest amount of anomalous dispersion of any which have been investigated for this effect, is rather strongly opposed to this view.

In the first place, I do not quite understand why the decrease of the density-*gradient* should be material to the case. This, however, may be a lapse; probably the author intended to say: "since in general the density decreases outward." But then the inference expressed in the sentence as a whole is erroneous. A little reflection will easily show that near the limb the regular radial density-gradient assists R-light and hinders V-light in curving from the photosphere toward the observer. The result would be an apparent displacement of the dark line to the *violet*, not to the red. The radial gradient, therefore, if it is of any importance in this matter, counteracts the effective agent which produces the observed shifts toward the red.

The principal point overlooked by Adams is that, according to my explanation, the effective agent in producing the phenomenon is

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 43; *Astrophysical Journal*, **31**, 30–61, 1910.

the general asymmetry of the dispersion bands enveloping the absorption lines. It does not depend upon the incurvation which rays undergo in the regular radial density-gradient of the solar atmosphere, but is caused by anomalous scattering, and refraction in irregular gradients, combined with the fact that the refracting power of the mixture of gases is on the average greater for R-light than for V-light.

If we keep this in mind, we shall have a useful basis for investigating the relationship between anomalous dispersion and the results of Adams' measurements. That a simple comparison of Geisler's observations on anomalous dispersion of metallic vapors in the arc with displacements at the limb—as given by Adams on p. 28 (*op. cit.*)—could not possibly serve the purpose of finding such a relationship, is evident; for the amount of that part of the displacement which is due to anomalous dispersion is determined by the degree of asymmetry of the Fraunhofer line under consideration; and this asymmetry is not a mere property of the corresponding element itself, revealable in laboratory experiments, but depends upon the concentration with which that element is represented in the solar atmosphere. No shade of proportionality between the results of those two investigations could be expected. So it is not at all opposed to our view that the winged lines of sodium and calcium are little, or not at all, displaced at the limb, although they show strong anomalous dispersion. On the contrary, that result might have been foreseen; for if the wide wings are really owing to that cause, the wave-length corresponding to the zero value of the refracting power of the mixture, which always lies on the violet side of a Fraunhofer line, must be at a rather great distance from the absorbed waves,[1] thus making the asymmetry of the dispersion band imperceptible. The central part of the line, the true absorption line, cannot be displaced by anomalous dispersion.

A peculiar feature of our explanation is that both very strong and very weak anomalous dispersion make the displacements small, whereas intermediate values give larger displacements. Indeed, with decreasing width of the dispersion band, its asymmetry increases; but the resulting apparent displacement can never surpass half the width of the line. (Whenever greater shifts are observed, pressure, or magnetism, or Doppler effect certainly come into play.)

[1] Cf. Fig. 8, *Physikalische Zeitschrift*, 11, 68, 1910.

The largest displacements observed by Adams occur with many lines of iron and nickel. From the point of view of our hypothesis this means that near these lines the amount of anomalous dispersion of the mixture is most suitable for producing the phenomenon, neither too great, nor too small. Considerably smaller are the displacements for titanium, vanadium, and scandium—perhaps because these elements are less in evidence in the mixture of gases. That those iron lines which are most strengthened at the limb show smaller displacements than the average iron lines, also perfectly fits our point of view, for their asymmetry must be less conspicuous on account of their greater width. That the lines of the elements of very high atomic weight, such as lanthanum and cerium, show very small displacements is easily accounted for if we assume their vapors to be extremely rare in the solar atmosphere. This explanation is certainly not less simple than the one proposed by Adams on pp. 17 and 18 of his paper,[1] where he has to find a way out of the discrepancy to which in that case the pressure hypothesis appears to lead.

Various other characteristics of Adams' interesting list of displacements (e.g., the special behavior of the enhanced lines as a class) will be discussed on a later occasion, together with his equally valuable observations of the spectrum of sun-spots.

UTRECHT
April 1910

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 43; *Astrophysical Journal*, **31**, 46–47, 1910.

CORRECTIONS TO RADIAL VELOCITIES OF CERTAIN STARS OF THE *ORION* TYPE

By EDWIN B. FROST

In our paper[1] entitled "Radial Velocities of Twenty Stars Having Spectra of the *Orion* Type," Mr. Adams and I called attention, on p. 15, to the inaccurate wave-lengths of the three silicon lines at $\lambda\lambda$ 4553, 4568, and 4575, which we were obliged to employ, depending upon the approximate values given by Exner and Haschek for the spark spectrum.

I subsequently redetermined[2] the wave-lengths of these lines from plates made by Mr. Julius A. Brown with our $10\frac{1}{2}$-foot concave grating, using the spark between silicon electrodes. I pointed out that the considerable departures we thus found from the values of Exner and Haschek would very appreciably affect the radial velocities of the *Orion*-type stars previously published from measures on Bruce spectrograms (p. 160). The accordance of the velocities from the separate silicon lines was not particularly increased by the use of the new wave-lengths, but the agreement of the measures of these lines for different grating plates indicated an accuracy within a few hundredths of an Ångström unit. Using the wave-lengths as 4552.636 4567.897, and 4574.791, the corrections to plates reduced with Exner and Haschek's values, expressed in velocities, are respectively 7.51 km, 3.48 km, and 7.14 km, to be applied positively. In the spectra of seven of the twenty stars, the silicon lines were not measured, namely, ϵ *Cassiopeiae*, ζ *Orionis*, η *Leonis*, γ *Corvi*, τ *Herculis*, ζ *Draconis*, ϵ *Delphini*. Of these, γ *Corvi* was found to be a spectroscopic binary by Campbell and Curtis. The following table gives the corrections to the radial velocities, and the corrected radial velocities for the eight of the remaining stars which have not thus far been proven to be spectroscopic binaries. For convenience I have added the number of plates used and the epoch. For the five remaining stars having variable radial velocities, the values for the separate plates are given. The variation of β *Orionis* was established by Plaskett, of γ and ϵ *Orionis* by the writer, of β *Canis Majoris* and η *Lyrae* by Albrecht.

[1] *Publications of the Yerkes Observatory*, **2**, 1903.

[2] *Astrophysical Journal*, **22**, 157, 1905.

Star	No. of Plates	Correction	Corrected Radial Velocity	Epoch
γ Pegasi	8	+1.6 km	+ 7.0 km	1902.06
ζ Cassiopeiae	4	+1.6	+ 4.5	1902.10
ζ Persei....	5	+1.9	+24.0	1901.95
κ Orionis...	7	+1.6	+18.7	1901.88
ε Canis Majoris	3	+1.9	+29.1	1902.61
ι Herculis	4	+1.1	−15.3	1901.92
67 Ophiuchi	3	+1.3	− 1.8	1902.47
102 Herculis ...	4	+1.9	− 8.8	1902.62

In the case of the following spectroscopic binaries the values are given only for plates where the use of the silicon lines makes corrections necessary.

Star	Plate	Date	G.M.T	Correction	Corrected Radial Velocity
β Orionis... ...	A 262	1901 Oct. 3	19ʰ 26ᵐ	+0.7 km	+23.5 km
	B 207	Oct. 18	20 7	+0.7	+20.7*
γ Orionis...	A 224	1901 Sept. 11	21 32	+1.6	+17.0
	A 258	Oct. 2	21 34	+0.4	+16.2
	B 221	Nov. 8	20 38	+1.5	+20.1*
	B 253	Nov. 27	21 55	+1.3	+22.5
	B 262	Dec. 31	15 38	+1.2	+19.2*
	B 299	1902 Mar. 13	15 53	+0.9	+16.7
	B 317	April 9	15 4	+0.7	+19.4*
ε Orionis......	A 208	1901 Sept. 4	22 10	+1.5	+29.0
	B 228	Nov. 13	19 50	+0.7	+27.0*
	B 298	1902 Mar. 13	15 11	+0.8	+26.9*
	B 316	April 9	14 21	+0.3	+27.4*
β Canis Majoris	A 287	1901 Oct. 31	21 35	+2.2	+34.5*
	A 293	Nov. 1	21 26	+1.9	+36.7
	B 215	Nov. 7	21 0	+1.7	+33.4*
η Lyrae	B 409	1902 Sept. 13	17 57	+0.6	−10.0
	B 422	Oct. 15	14 12	+0.4	− 5.1*
	B 427	Oct. 16	15 32	+0.7	− 7.8*

*Mean of the measures by Frost and by Adams.

For the sake of uniformity with the earlier publication, the velocities are given to the tenth of the kilometer, but I do not attach any weight to the decimal.

Finality in respect to radial velocities is hardly to be obtained: with improvements in the accuracy of the stellar wave-lengths, the values published are subject to alteration. In measures of stars of the *Orion* type much dependence is necessarily placed upon the double helium line at λ 4472. We have here always employed the blended value 4471.676 resulting from the assignment of weights

6 and 1 to the two components, according to the estimates by Runge and Paschen. Variation in relative intensity of these components in different stars, or an uncertainty in the adopted blend, tends to introduce a systematic error in the radial velocity. Of course this may be adjusted by making the residuals of all lines used for a star add up zero, as is done by some observers. There are seldom lines enough, however, to furnish a thorough balancing of errors. This line is merely cited as a further illustration[1] of inherent uncertainties which make decimals of a kilometer illusory for most stars.

The data establishing the variable velocity of γ *Orionis* have not yet been published, and may be given here. I was recently led to take and measure additional low-dispersion plates of this star from the evidence of double lines I found on a casual one-prism plate.

γ *Orionis* ($\alpha = 5^h\ 20^m$; $\delta = +6°\ 16'$; Mag. $= 1.9$)

Plate	Date	G.M.T.	Taken by	No. Lines	Velocity	Quality
IB2282	1910 Feb. 18	$13^h\ 6^m$	F.	8	+11 km	v. g.
2284	Feb. 21	12 50	L.	5	+ 8	weak
2285	Feb. 22	13 12	F.	8	+12	g.
2293	Feb. 28	14 22	F., L.	8	+20	v. g.
2295	Mar. 7	12 44	L.	8	+ 1	fair
2303	Mar. 14	13 2	L.	6	+ 6	too strong

F.=Frost; L.=Lee; v.=very; g.=good.

On plate 2282 a faint component to λ 4472 was measured, which yielded a velocity of -108 km. Faint components were suspected in other instances.

It is our experience that low dispersion is more favorable than high dispersion for the detection of faint components of diffuse lines, and one-prism plates of some of these stars originally observed with three prisms have shown duplicities where we should not have been justified previously in suspecting them.

A re-examination of our three-prism plates of γ *Orionis* does not disclose any certain doubling of the lines.

The above data suggest that the period of γ *Orionis* is not very short. The only other published observations of the radial velocity of γ *Orionis* known to me are the measures of three plates at Potsdam, as follows, the values being the means of measures by Vogel and by Scheiner:

1888 Dec. 7, +9 km; 1891 Feb. 1, +8 km; 1891 Feb. 4, +13 km.

YERKES OBSERVATORY
April 26, 1910

[1] *Astrophysical Journal*, 31, 377, 1910.

THE CORRESPONDENCE BETWEEN ZEEMAN EFFECT AND PRESSURE DISPLACEMENT FOR THE SPECTRA OF IRON, CHROMIUM, AND TITANIUM[1]

By ARTHUR S. KING

The following is an attempt to present such evidence as is available concerning the connection between two phenomena which seem on certain theoretical grounds probably to be related, but for which a quantitative comparison has been almost entirely lacking. The material is still insufficient to make more than a beginning of the study of the relation from the quantitative side. It is hoped, however, that the accumulation of more data in this laboratory and elsewhere may soon add to the experimental evidence on the subject.

The view that there is a direct connection between the Zeeman effect and the pressure displacement of spectrum lines has been strongly advocated by Humphreys in a series of papers[2] which have been summarized[3] by him, together with all other pressure investigations up to the year 1908. Humphreys' hypothesis, briefly stated, is that the part of the atom to which the light impulse is due is a ring of electrons, rotating with a period of the order of the light vibration. Each of the electron rings will then set up a magnetic field of its own. The luminous gas will be in a condition of minimum potential energy when the planes of the rings are parallel and the electrons rotating in the same direction. We must, however, in view of the Zeeman effect, consider that different rings may rotate in opposite directions, and assume merely that the regular condition is a rotation of the electrons in orbits approximately circular, with a tendency for the planes of these to become parallel. The effect of pressure in the surrounding medium will be to bring the rings closer together, thereby altering their mutual induction. If two rings rotating in the same direction

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 46.

[2] *Astrophysical Journal*, **23**, 233, 1906; **26**, 18, 297, 1907; **27**, 194, 1908.

[3] *Jahrbuch der Radioaktivität und Elektronik*, **5**, 324, 1908.

are made to approach, the current in each ring will decrease, which means a retardation of the rotating electrons and an increase of period in the corresponding light vibration, resulting in a shift of the spectrum lines toward the red.

If rings of opposite rotation are forced closer together, their motion will be accelerated, resulting in a shift of the spectrum lines to the violet. Assuming that both directions of rotation are present for electrons producing each spectrum line, the general result will be a widening of all lines as the pressure increases, with a prevailing shift of the maximum of each line toward the red. This last is due to the fact that the condensing action of the pressure on rings rotating in the same direction is assisted by the effort of these rings to get into the strongest part of their mutual field; while for oppositely rotating rings the approach is opposed by the magnetic action, so that on the whole the retardation of the period for a given line is greater than the acceleration, and the line, while being widened toward both red and violet, has its maximum intensity moved toward the red.

Another theory is worked out by Richardson[1] which opposes the connection of pressure displacement with Zeeman effect. Instead of basing his reasoning on magnetic perturbations, Richardson considers the electron as an oscillator which sets up an alternating electrostatic field in its neighborhood. This field would produce forced vibrations in the electrons belonging to neighboring atoms, an effect increased by pressure in the medium. The electric field produced by the forced vibrations would then react on that of the radiating electrons. The mathematical development gives a change of wave-length proportional to the pressure and toward the red. Worked out numerically with the available data, the electrostatic resonance theory requires values for the pressure displacement many times greater than those observed experimentally. A modified conception of the equilibrium conditions might account for this discrepancy.

Richardson objects to Humphreys' theory largely on the ground that the magnetic disturbances of period would be far too small to account for the observed displacements of lines unless the magnetic field for any atom is greater than that corresponding to saturated iron, which Richardson holds to be an upper limit. This is replied

[1] *Philosophical Magazine* (6), **14**, 557, 1907.

to by Humphreys in a later paper[1] in which he questions the right to base the possible magnetic intensity of iron atoms upon the properties of iron in large masses, since the permeability and saturation point depend upon many factors of composition and physical condition. Going farther, Humphreys considers an ideal electron ring and deduces an expression for the change of rotation frequency brought about by an external magnetic field H, such as that due to a neighboring electron ring. This is found to give an expression for the change of wave-length $\Delta\lambda$ in the ether vibrations of original wave-length λ which reduces to $\Delta\lambda / H\lambda^2 = C$, a constant, which is Preston's law for the Zeeman phenomenon, indicating that the ideal electron ring is very similar in structure to the actual radiating particle. If this similarity is admitted, Humphreys is justified in his next step, which is the substitution of known values in the expression for the change of wave-length of ether vibrations produced by a change in the period of the electron ring. This gives a field intensity for the rotating ring of 45×10^7, which is about ten thousand times that of the strongest fields used in spectroscopic work. The change in mutual induction by pressing together electron rings having fields of this magnitude may be expected to give shifts of spectrum lines of the order of those measured.

A third theory is that presented by Larmor,[2] who treats the electron as a Hertzian doublet in a field of electric force. This field would be altered by any change in the distribution of material particles in the medium such as would result from increased pressure. A molecule approaching a vibrating electron would decrease the rigidity of the ether at that point. A lowering of the ether strain would tend to increase the period of the electron, and it is shown that this might give displacements of the magnitude observed for spectrum lines. A note by Humphreys[3] points out that several consequences of Larmor's theory agree only to a limited degree with observed facts, while the requirement that the shift should decrease in magnitude with the wave-length is contrary to the regular behavior in spectra.

It would seem that the interacting magnetic atoms of Humphreys provide the most plausible theory among those given; but experi-

[1] *Astrophysical Journal*, **27**, 194, 1908.

[2] *Ibid.*, **26**, 120, 1907.

[3] *Ibid.*, **26**, 297, 1907.

mental data have been lacking to show the probability of a connection between the effects of pressure and magnetic field on spectrum lines. Humphreys considers[1] that, in general, lines of large Zeeman separation are strongly displaced by pressure, but admits that there is scanty material on which to base this conclusion. The refusal of banded spectra, notably that of carbon, to show either Zeeman effect or displacement has often been cited as probably resulting from a connection between the two phenomena, and interesting developments on this point have recently been presented. Dufour[2] obtained Zeeman separations for the component lines of the band spectra of the chlorides and fluorides of the alkaline earths, the magnitude of separation being about the same as for line spectra. A short time after, Rossi[3] selected three of these, the fluorides of calcium, strontium, and barium, and obtained distinct pressure-shifts for the bands, the shifts being of the same order as for line spectra. Comparing his results with those of Dufour, Rossi did not find any general relation between the magnitude of the two effects.

A detailed comparison of the two phenomena for line spectra is very desirable and the writer found it possible to make a beginning by comparing a considerable amount of Zeeman material collected in this laboratory for the spectra of iron, titanium, and chromium with the pressure displacements for the same spectra given by Humphreys[4] and Duffield.[5]

The material on the Zeeman side was obtained from an extended investigation of the iron and titanium spectra by the writer through the range from λ 3660 to λ 6700, in which all lines of fair intensity were photographed, the character of the separation studied, and the components measured. The titanium spectrum has been entirely rephotographed under higher dispersion and stronger field since the publication of the writer's former paper on this spectrum.[6] This material for iron and titanium is being prepared for detailed presenta-

[1] *Astrophysical Journal*, **26**, 29, 1907.

[2] *Comptes Rendus*, **146**, 118, 229, 1908.

[3] *Proceedings Royal Society*, **82**, 518, 1909.

[4] *Astrophysical Journal*, **26**, 18, 1907.

[5] *Philosophical Transactions*, A, **208**, 111, 1908.

[6] *Astrophysical Journal*, **30**, 1, 1909.

tion in the *Publications* of this observatory. The measurements for chromium are from plates taken partly by Mr. Babcock and partly by myself. This series is not yet complete, although almost all lines given in the pressure tables were available for comparison.

The pressure measurements by Humphreys, while they cover only a fraction of the lines available on the Zeeman plates, give a sufficient number of the stronger lines of iron, chromium, and titanium as far as λ 5600 to show what may be expected as to general agreement between the two phenomena. The measurements of Duffield cover only a small region of the iron spectrum, but his values have been used to supplement the list of Humphreys and his classification of lines has proved useful in showing the rate of increase of displacement with pressure.

EXPERIMENTAL METHOD

The complete description of apparatus and methods must be left for the detailed publication. It may be said here, however, that the photographs used for these measurements were taken in the third order of a plane Rowland grating mounted in a vertical Littrow spectrograph,[1] the objective having 13 feet (4 m) focal length. The scale of the photographs was about 1.35 Ångström units to the millimeter. The light-source was a transformer spark between pieces of the metal under investigation, these being held between the poles of a large Dubois electromagnet. The light was taken at right angles to the lines of magnetic force and a Nicol prism above the slit used to transmit either the light with vibrations parallel to the magnetic force-lines or that vibrating in a plane at right angles to the force-lines. Several plates were usually taken for the same region of the spectrum, in order that lines of different intensities might be obtained favorable for measurement. The spectrum of the spark without the magnetic field was always taken for comparison outside of the spectrum with the field by moving an occulting plate above the slit.

EXPLANATION OF TABLES I, II, AND III

The wave-lengths given in column one are those of Rowland for the corresponding solar lines. The measurements of Kayser and

[1] Hale, "The Pasadena Laboratory of the Mount Wilson Solar Observatory," *Contributions from the Mount Wilson Solar Observatory*, No. 27; *Astrophysical Journal*, **28**, 244, 1908.

Runge and of Hasselberg were used by Humphreys and Duffield, but there has been as a rule no difficulty in selecting the proper line from the Rowland table. Column two describes the character of the Zeeman separation. This description includes the components given with the Nicol prism in two positions 90° apart and gives the appearance when the spark is observed at right angles to the force-lines without a Nicol. An interrogation point is often used to indicate that the type of separation is somewhat uncertain. When a triplet is thus marked, it usually means that the middle component is slightly widened, so that with a much stronger field it would probably be resolved into two. A questionable quadruplet has the components of its outer pair widened so that such a line may be a sextuplet. If the outer components are so much widened that each is certainly made up of two and possibly more, the line is a doubtful sextuplet, and correspondingly for the other types.

The "Weight" of the measurement for the Zeeman components is given in column three, and indicates the quality of the line as good, fair, or poor, corresponding to weights of 3, 2, and 1, respectively. Lines of weight 3 have sharply defined components, with an error of measurement in the third decimal place. Lines whose components are widened and probably compound or poorly defined for any reason do not admit of such close measurement and are graded 2. Those lines whose components are very diffuse, weak, or disturbed by blends, so that the measurement gives little more than the order of magnitude, are weighted 1.

The letters *n* and *p* (corresponding to *s* and *p* in German publications) are used in the table to denote the components having vibrations in a plane normal to the force-lines and parallel to the force-lines respectively. Column four gives the measured separation in Ångström units of the *n* components, the mean being taken when there are two or more pairs. The values of $\Delta\lambda$ for chromium and titanium may be slightly altered when the final tables are published, as the material for these spectra has not been fully worked over. The separations here given are accurate enough to show the order of magnitude. The behavior of the *p* component, which is often split up, is not given here, as it can scarcely enter into a comparison with pressure displacement, the electron in the latter phenomenon being

assumed to have a circular orbit. The character of the *p* component, as well as all description of the widening of components, is left for the complete Zeeman tables.

The measurements of pressure displacements by Humphreys are given in column five. These are in Ångström units and for a pressure of 42 atmospheres, his other measurements, for 69 and 101 atmospheres, being for only a part of the lines. For the iron spectrum, the displacements of Duffield for 41 atmospheres are given in the next column. Occasionally a line was not obtained by these observers for the given pressures, in which case an approximate value was deduced from the measurement for some other pressure and this noted in the "Remarks" column.

The two columns preceding "Remarks" contain ratios of Zeeman separation to pressure displacement, the one numerical, the other of letters denoting the order of magnitude. In the numerical ratios, the values of Humphreys are used for the sake of uniformity, those of Duffield for an almost equal pressure being taken when a line was not measured by the former. The letters stand for small, medium, and large values of separation and displacement. The limits covered by these classes are as follows:

	Separation	Displacement
S	<0.300	<0.060
M	0.301–400	0.061–100
L	>0.401	>0.101

The reasons for this classification are given in the discussion.

TABLE I

ZEEMAN SEPARATION AND PRESSURE DISPLACEMENT FOR IRON

λ	Character of Separation	Wt.	Separation $H=16,000$	Displacement 42 Atm (Humphreys)	Displacement 41 Atm. (Duffield)	Ratio Sep. to Displ.	Classes Sep. and Displ.	Remarks
3659.663	triple	2	0.176	0.050		3.54	S:S	
3669.666	triple ?	2	0.176	0.050		3.54	S:S	
3670.240	triple ?	2	0.261	0.047		5.55	S:S	
3676.457	triple	3	0.236	0.050		4.72	S:S	
3677.764	triple	3	0.184	0.052		3.54	S:S	
3680.069	sextuple ?	3	0.308	0.062		4.97	M:M	
3683.229	triple	2	0.480	0.040		12.00	L:S	n comps. have inner fringes
3684.258	triple ?	3	0.170	0.053		3.21	S:S	
3687.610	triple	3	0.313	0.090		3.48	M:M	
3689.614	sextuple ?	2	0.373	0.084		4.44	M:M	
3695.104	triple	3	0.261	0.070		3.73	S:M	
3704.603	triple ?	3	0.319	0.046		6.93	M:S	
3705.708	sextuple ?	3	0.294	0.054		5.44	S:S	
3709.389	triple	3	0.312	0.095		3.28	M:M	
3716.054	quadruple ?	1	0.290	0.107		2.71	S:L	
3720.084	triple ?	2	0.268	0.047		5.70	S:S	
3722.729	sextuple	2	0.286	0.050		5.72	S:S	
3724.526	triple	3	0.256	0.054		4.74	S:S	
3727.778	triple	3	0.324	0.100		3.24	M:M	
3733.469	quintuple	3	0.325	0.050		6.50	M:S	3 n, 2 p comps.
3735.014	triple	2	0.310	0.092		3.37	M:M	
3737.281	triple	2	0.254	0.040		6.35	S:S	
3738.454	triple	3	0.207	0.078		2.65	S:M	
3743.508	octuple	3	0.343	0.100 ?		3.43	M:M	5 n, 3 p comps. $\Delta\lambda$ mean of 2 pairs. [0.155 for 69 atm.
3745.717	triple	2	0.228	0.050		4.56	S:S	
3746.058	unseparated			0.050			O:S	
3748.408	?	2	0.171	0.040		4.28	S:S	n comps. have oute fringes
3749.631	triple	2	0.298	0.085		3.51	S:M	

3758.375	triple	3	0.278	0.090		3.09	S:M	
3763.945	triple	3	0.218	0.095		2.29	S:M	
3765.689	triple	3	0.232	0.106		2.18	S:L	
3767.341	unseparated			0.118			O:L	
3788.046	octuple	2	0.347	0.090		3.86	M:M	5 n, 3 p comps. $\Delta\lambda$ mean of 2 pairs
3795.147	triple	2	0.334	0.093		3.59	M:M	
3798.655	triple	3	0.326	0.085		3.84	M:M	
3799.693	triple	3	0.326	0.075		4.35	M:M	
3805.486	triple ?	3	0.204	0.092		2.22	S:M	
3813.100	triple ?	2	0.236	0.058		4.07	S:S	
3815.987	triple	2	0.264	0.110		2.40	S:L	
3820.586	triple	2	0.282	0.125		2.26	S:L	
3824.591	triple	3	0.345	0.040		8.63	M:S	
3826.027	triple	2	0.274	0.090		3.04	S:M	
3827.980	triple	2	0.256	0.102		2.51	S:L	
3834.364	triple ?	2	0.248	0.110		2.25	S:L	
3840.580	?	2	0.179	0.098		1.83	S:M	n comps. have outer fringes
3841.195	triple	2	0.178	0.100		1.78	S:M	
3850.118	unseparated			0.082			O:M	
3856.524	triple	3	0.341	0.038		8.97	M:S	
3860.055	triple	3	0.347	0.042		8.26	M:S	
3865.674	quintuple	3	0.350	0.103		3.40	M:L	3 n, 2 p comps.
3872.639	sextuple ?	2	0.289	0.108		2.68	S:L	
3878.720	triple	3	0.346	0.044 ?		7.86	M:S	0.066 for 69 atm.
3886.434	triple	3	0.348	0.056		6.21	M:S	
3887.196	sextuple ?	2	0.342	0.073		4.68	M:M	
3888.671	octuple ?	1	0.231	0.089		2.60	S:M	Probably 6 n, 2 p comps.
3893.542	triple	3	0.269	0.072		3.74	S:M	
3895.803	triple	3	0.347	0.030		1.16	M:S	
3899.850	triple	3	0.358	0.036		9.94	M:S	
3903.090	sextuple ?	2	0.290	0.095		3.05	S:M	
3904.052	triple ?	2	0.233	0.056		4.16	S:S	
3906.628	triple	3	0.352	0.050		7.04	M:S	
3920.410	triple	3	0.363	0.033		1.10	M:S	
3923.054	triple	3	0.356	0.032		1.11	M:S	
3928.075	triple	3	0.344	0.038		9.05	M:S	
3930.450	triple	3	0.354	0.047		7.53	M:S	

TABLE I—*Continued*

λ	Character of Separation	Wt.	Separation $H=16,000$	Displacement		Ratio Sep. to Displ.	Classes Sep. and Displ.	Remarks
				42 Atm. (Humphreys)	41 Atm. (Duffield)			
3948.925	triple	3	0.234	0.050		4.68	S:S	
3950.102	triple	3	0.364	0.066		5.52	M:M	
3956.819	triple	2	0.311	0.036		8.64	M:S	
3969.413	triple	2	0.378	0.089		4.25	M:M	
3977.891	triple	3	0.448	0.042		10.67	L:S	
3981.917	sextuple?	2	0.240	0.060?		4.00	S:S	Given by Rowland as *Ti.* 0.150 for 101 atm.
3984.113	triple	3	0.216	0.085		2.54	S:M	
3986.321	triple?	2	0.196	0.061		3.21	S:M	
3997.547	triple	3	0.270	0.048		5.63	S:S	
3998.205	triple?	2	0.226	0.066		3.42	S:M	
4005.408	7 or 9 comp.	2	0.399	0.103		3.87	L:L	
4009.864	?	2	0.342	0.040		8.55	M:S	*n* comps. have outer fringes
4014.677	triple	2	0.259	0.050		5.18	S:S	
4017.308	triple?	2	0.364	0.062		5.87	M:M	
4022.018	triple	2	0.272	0.037		7.35	S:S	
4045.975	triple	2	0.320	0.103	0.082	3.11	M:L	
4063.759	triple	2	0.269	0.107	0.082	2.51	S:L	
4071.908	triple	2	0.170	0.092	0.086	1.85	S:M	
4107.649	triple	3	0.410	0.060		6.83	L:S	
4109.953	sextuple?	2	0.322	0.062		5.19	M:M	
4118.708	triple	2	0.265	0.085	0.099	3.12	S:M	
4127.767	triple	2	0.214		0.082	2.61	S:M	
4132.235	sextuple?	2	0.415	0.105	0.108	3.95	L:L	Probably 4 *n*, 3 *p* comp.
4134.840	triple?	2	0.291	0.055?	0.086	5.29	S:S	0.138 for 101 atm. (Humphreys)
4143.572	triple?	2	0.280		0.095		S:M	0.048 for 21 atm.
4144.038	septuple?	3	0.393	0.116	0.099	3.97	M:L	Probably 4 *n*, 3 *p* comps.
4154.667	triple	1	0.379		0.086	4.41	M:M	
4156.970	quadruple	2	0.367	0.064?	0.065	5.73	M:M	0.096 for 69 atm.
4175.806	triple	2	0.296		0.065	4.55	S:M	
4181.919	triple	3	0.339	0.070?		4.84	M:M	0.170 for 101 atm.

4185.058	triple?	2	0.412	0.040	0.047	10.30	L:S	
4187.204	triple?	3	0.372		0.100	1.06	M:L	
4187.943	triple?	3	0.402		0.431	0.93	L:L	
4191.505	9 comps.	2	0.243		0.310	0.78	S:L	Probably 6 *n*, 3 *p* comps.
4195.492	triple?	2	0.306		Large		M:L	0.250 at 16 atm.
4196.372	triple?	1	0.359		Large		M:L	0.260 at 16 atm.; faint in spark
4198.494	triple	3	0.383		Large		M:L	0.147 at 15 atm.
4199.267	triple	3	0.276	0.073	0.065	3.78	S:M	
4202.198	sextuple?	3	0.331	0.071	0.078	4.66	M:M	
4204.101	triple	2	0.368		0.060	6.13	M:S	
4210.494	triple	3	0.806		0.157	5.13	L:L	
4219.516	triple	3	0.284	0.074	0.078	3.84	S:M	
4222.382	triple	3	0.457		0.358	1.28	L:L	
4227.606	triple?	2	0.337		0.431	0.78	M:L	
4233.772	9 comps.?	2	0.236	0.240	0.370	0.98	S:L	Probably 6 *n*, 3 *p* comps.
4236.112	triple	3	0.446	0.274	0.405	1.63	L:L	
4245.422	triple	2	0.464	0.060		7.73	L:S	
4250.945	?	2	0.277	0.089	0.082	3.11	S:M	Numerous *n* comps. blurred
4260.640	triple	3	0.442	0.246	0.177	1.80	L:L	
4271.934	triple	2	0.341	0.083	0.069	4.11	M:M	
4282.566	septuple?	2	0.360	0.043	0.056	8.37	M:S	
4294.301	sextuple?	2	0.319	0.084	0.086	3.80	M:M	
4299.410	triple	3	0.415		0.313	1.33	L:L	
4308.081	triple	3	0.322	0.090	0.060	3.58	M:M	
4315.262	triple?	3	0.558	0.036	0.041	15.50	L:S	
4325.939	triple	2	0.266	0.097	0.56	2.74	S:M	Given by Duffield as 4325.19, which is a very [weak *Fe* line
4337.216	sextuple?	2	0.295	0.090	0.082	3.28	S:M	
4352.908	septuple?	2	0.438	0.052	0.056	8.42	L:S	
4367.749	triple	2	0.339	0.060		5.65	M:S	
4369.941	triple	3	0.287	0.055	0.060	5.22	S:S	
4376.107	triple	2	0.471	0.039	0.047	12.08	L:S	
4383.720	triple	3	0.332	0.125	0.060	2.66	M:L	
4404.927	triple	3	0.334	0.110	0.056	3.04	M:L	
4407.871	triple	2	0.631	0.180		3.51	L:L	
4408.582	triple?	2	0.448	0.160		3.05	L:L	
4415.293	triple	2	0.338	0.087	0.078	3.89	M:M	
4422.741	sextuple?	1	0.310	0.065	0.046	4.77	M:M	

TABLE I—*Continued*

λ	Character of Separation	Wt.	Separation $H=16,000$	Displacement: 42 Atm. (Humphreys)	Displacement: 41 Atm. (Duffield)	Ratio Sep. to Displ.	Classes Sep. and Displ.	Remarks
4427.482	triple	3	0.460	0.055	0.043	8.36	L:S	
4430.785	triple	2	0.761	0.190	0.159	4.01	L:L	
4442.510	quadruple	2	0.510	0.190	0.164	2.68	L:L	
4443.365	triple ?	1	0.193	0.060	0.060	3.22	S:S	
4447.892	quadruple ?	2	0.674	0.180	0.172	3.74	L:L	
4454.552	quadruple	2	0.445	0.080		5.56	L:M	
4459.301	quadruple ?	2	0.464	0.160	0.172	2.90	L:L	
4461.818	triple	3	0.464	0.060	0.039	7.73	L:S	
4466.727	triple	2	0.384	0.056	0.046	6.86	M:S	
4476.185	triple	2	0.323	0.072	0.042	4.49	M:M	
4494.738	triple ?	2	0.353	0.200	0.168	1.77	M:L	
4528.798	triple	2	0.410	0.172	0.172	2.38	L:L	
4531.327	triple ?	2	0.400	0.075	0.078	5.32	L:M	
4548.024	triple	2	0.366	0.097		3.77	M:M	
4592.840	triple ?	2	0.416	0.110		3.78	L:L	Weak in spark
4603.126	triple	2	0.577	0.093		6.20	L:M	
4647.617	triple	2	0.392	0.070		5.60	M:M	
4691.602	triple	2	0.358	0.070		5.11	M:M	
4710.471	triple ?	1	0.242	0.060			S:S	Blend with air line in spark
4736.963	triple	2	0.426	0.085		5.01	L:M	
4787.003	triple	2	0.409	0.076		5.38	L:M	
4789.849	triple	2	0.352	0.080		4.40	M:M	
4859.028	octuple	2	0.564	0.390		1.45	L:L	5 n, 3 p comps., center comp. strong
4871.512	?	1	0.336	0.420		0.80	M:L	Comps. blurred, probably 7 or more
4878.407	triple	3	1.092	0.400		2.73	L:L	
4919.174	sextuple ?	2	0.591	0.375		1.58	L:L	
5171.778	triple	3	0.521	0.075		6.95	L:M	
5195.113	triple	3	0.457	0.080		5.71	L:M	
5269.723	triple	2	0.501	0.083		6.04	L:M	
5328.236	triple ?	2	0.470	0.100		4.70	L:M	

5371.734	triple ?	2	0.413	0.095		4.35	L:M	
5397.344	quadruple	2	0.630	0.080		7.88	L:M	
5405.989	quadruple ?	2	0.281	0.100		2.81	S:M	
5429.911	sextuple ?	2	0.607	0.085		7.14	L:M	
5434.740	unseparated			0.120			O:L	
5447.130	sextuple ?	2	0.536	0.095		5.64	L:M	
5455.834	quintuple	2	0.680	0.105		6.48	L:L	3 *n*, 2 *p* comps.
5497.735	octuple	2	1.040	0.110		9.45	L:L	5 *n*, 3 *p* comps.; $\Delta\lambda$ mean of 2 pairs
5501.683	sextuple ?	2	1.001	0.095		10.54	L:M	
5507.000	septuple ?	2	1.026	0.120		8.55	L:L	Probably 4 *n*, 3 *p* comps.
5615.877	triple	2	0.586	0.080		7.33	L:M	

TABLE II

ZEEMAN SEPARATION AND PRESSURE DISPLACEMENT FOR CHROMIUM

λ	Character of Separation	Wt.	Separation $H=17{,}500$	Displacement 42 Atm. (Humphreys)	Ratio Sep. to Displ.	Classes Sep. and Displ.	Remarks
3885.364	triple	3	0.361	0.060	6.02	M:S	
3894.165	triple	3	0.368	0.072	5.11	M:M	
3908.900	triple	3	0.356	0.049	7.27	M:S	
3916.383	triple	3	0.368	0.062	5.94	M:M	
3919.399	triple	3	0.367	0.052	7.06	M:S	
3921.188	triple	3	0.368	0.076	4.84	M:M	
3928.783	triple	3	0.370	0.050	7.40	M:S	
3941.637	triple	3	0.368	0.046	8.00	M:S	
3963.831	triple ?	2	0.283	0.080	3.54	S:M	All Zeeman comps. displ. toward violet
3969.899	sextuple	2	0.543	0.063	8.62	L:M	Δλ for outer n comps., inner very close
3976.839	sextuple ?	2	0.478	0.058	8.24	L:S	All Zeeman comps. displ. toward red
3984.059	triple	3	0.208	0.140	1.49	S:L	All Zeeman comps. displ. toward violet
3990.140	triple ?	2	0.297	0.056	5.30	S:S	All Zeeman comps. displ. toward violet
3991.333	triple ?	2	0.172	0.070	2.46	S:M	All Zeeman comps. displ. toward violet
3992.950	triple	3	0.130	0.066	6.52	L:M	
4012.631	triple	2	0.238	0.080	2.98	S:M	
4039.244	triple	2	0.263	0.067	3.93	S:M	
4048.910	triple ?	2	0.243	0.070	3.47	S:M	
4058.915	triple	2	0.250	0.076	3.29	S:M	
4126.673	triple	2	0.380	0.040	9.50	M:S	
4254.505	?	1	0.458	0.056	8.18	L:S	Numerous n comps.
4274.958	triple ?	2	0.576	0.076	7.58	L:M	
4280.556	triple ?	2	0.248	0.061	4.07	S:M	
4289.885	?	1	0.544	0.087	6.25	L:M	Many comps.
4295.914	triple	2	0.513	0.056	9.16	L:S	
4297.908	triple ?	1	0.217	0.061	3.56	S:M	
4301.332	triple	1	0.406	0.052	7.81	L:S	Faint in spark
4323.772	triple	2	0.376	0.050	7.52	M:S	
4344.670	triple	2	0.364	0.057	6.39	M:S	

4351.930	triple	3	0.389	0.065	5.98	M:M	
4359.784	octuple ?	2	0.364	0.066	5.52	M:M	Probably 6 *n*, 2 *p* comps.
4363.267	triple ?	3	0.253	0.064	3.95	S:M	
4371.442	sextuple ?	2	0.422	0.060	7.03	L:S	
4497.023	?	2	0.455	0.040	11.38	L:S	*n* comps. have outer fringes
4526.632	triple	3	0.443	0.080	5.54	L:M	
4535.879	quintuple	2	0.737	0.075	9.83	L:M	3 *n*, 2 *p* comps., very unsymmetrical
4546.129	triple	3	0.646	0.060	10.77	L:S	
4580.228	?	2	0.564	0.040	14.10	L:S	*n* comps. have outer fringes.
4600.932	quadruple ?	2	0.549	0.085	6.46	L:M	
4613.544	triple	3	0.880	0.050	17.60	L:S	
4616.305	sextuple ?	2	0.572	0.053	10.79	L:S	
4626.358	sextuple	3	0.698	0.056	12.46	L:S	$\Delta\lambda$ mean of 2 pairs
4646.347	quadruple ?	2	0.450	0.065	6.92	L:M	Very strong in spark
4651.461	9 comps.	2	0.536	0.095	5.64	L:M	6 *n* comps. almost equally spaced, 3 *p* comps., $\Delta\lambda$ is mean of *n* comps.
4652.343	?	2	0.398	0.058	6.86	M:S	*n* comps. fringed. Probably 3 *p* comps.
4680.658	triple ?	1	0.232	0.056	4.14	S:S	Faint in spark
4729.864	triple	1	0.390	0.129	3.02	M:L	Focus poor for this region of plate
4730.897	triple	1	0.402	0.101	3.98	L:L	Focus poor for this region of plate
4756.300	triple	1	0.397	0.146	2.72	M:L	Focus poor for this region of plate
5204.680	9 comps.	2	0.553	0.164	3.37	L:L	$\Delta\lambda$ is mean of 3 pairs
5206.215	sextuple	2	0.701	0.156	4.49	L:L	$\Delta\lambda$ is mean of 2 pairs
5208.596	many comps.	2	0.524	0.092	5.70	L:M	
5247.737	triple	3	0.960	0.132	7.27	L:L	
5348.511	quadruple	2	0.622	0.196	3.17	L:L	

TABLE III

Zeeman Separation and Pressure Displacement for Titanium

λ	Character of Sep.	Wt.	Separation $H=17,500$	Displacement 42 Atm. (Humphreys)	Ratio Sep. to Displ.	Classes Sep. and Displ.	Remarks
3685.339	triple	2	0.232	0.012	1.03	S:S	Very strong
3904.920	triple	3	0.238	0.073	3.26	S:M	
3948.818	triple	2	0.191	0.045	4.24	S:S	
3956.476	triple ?	2	0.224	0.030	7.47	S:S	
3958.355	triple	3	0.282	0.045	6.27	S:S	
3981.917	quadruple	2	0.201	0.056	3.59	S:S	
3989.912	triple	3	0.286	0.049	5.84	S:S	
3998.790	triple	3	0.325	0.047	6.91	M:S	
4009.079	triple ?	2	0.359	0.055	6.53	M:S	
4286.168	sextuple	2	0.410	0.103	3.98	L:L	
4287.566	sextuple	2	0.436	0.087	5.01	L:M	
4291.114	quintuple	2	0.450	0.115	3.91	L:L	3 n, 2 p comps.
4295.914	unseparated			0.100		O:M	
4300.732	triple	2	0.367	0.104	3.53	M:L	
4301.158	triple	2	0.299	0.110	2.72	M:L	
4306.078	sextuple	2	0.366	0.104	3.52	M:L	
4318.830	triple	3	0.340	0.042	8.10	M:S	*Ca?* λ given by Hasselberg; Δλ for 4318.817 0.037 for 60 atm.
4427.266	triple	3	0.316	0.024 ?	1.32	M:S	
4533.419	triple	3	0.462	0.176	2.63	L:L	
4534.953	triple	2	0.457	0.124	3.77	L:L	
4544.864	octuple	2	0.513	0.080	6.41	L:M	5 n, 3 p comps., Δλ mean of n separations
4682.088	triple	3	0.395	0.077	5.13	M:M	
4691.523	triple	2	0.422	0.080	5.28	L:M	
4758.308	triple	3	0.376	0.067	5.61	M:M	
4759.463	triple	3	0.435	0.092	4.73	L:M	
4841.074	triple	3	0.390	0.029	13.45	M:S	
4981.912	triple	2	0.487	0.077	6.32	L:M	Very strong
4991.247	triple	2	0.467	0.135	3.46	L:L	Very strong
4999.689	triple	2	0.418	0.120	3.48	L:L	Very strong
5007.398	triple	2	0.346	0.150 ?	2.31	M:L	0.225 at 60 atm.
5013.479	triple	3	0.469	0.056	8.38	L:S	

PROBABLE ACCURACY OF MEASURED SEPARATIONS AND DISPLACEMENTS

While the experience of the writer has been confined to the Zeeman side of the material used in this paper, a few general considerations familiar to everyone accustomed to handling spectrum photographs give a basis of judging the degree of precision to be expected in the measurement of both separations and displacements. On a given plate only a small percentage of the lines are as a rule altogether satisfactory. For measurements of high accuracy, the exposure time, circuit conditions governing the sharpness of lines, complex separations on Zeeman plates, the presence or absence of reversals, and unsymmetrical broadening in pressure photographs are a few of the factors which require a large number of plates, each adapted to the treatment of a certain class of lines, unless there are to be wide differences in the measurement of lines on different plates and by different observers even when the experimental conditions have been made as nearly alike as possible. On this account, Zeeman measurements by different workers do not agree well except for those lines whose components are clearly resolved and sharply defined. In the same way, this difference in quality of lines explains why observers using a series of increasing pressures do not obtain closely proportional displacements for the majority of lines, and also why the measurements of Humphreys and Duffield in columns five and six, of Table I, made for nearly equal pressures, occasionally show large disagreement. As the weight of the measurement is not given for the pressure displacements, a comparison of numerical values for individual lines must take account chiefly of the order of magnitude of the quantities used, since the probable error of one or both of them is frequently large.

DISCUSSION

The question as to whether there is a close proportionality between the numerical values of Zeeman separation and pressure-shift is decided in a definite manner by the column in Tables I, II, and III giving the numerical ratio of separation to displacement. The separations for each spectrum are taken for a constant field and the displacements for a constant pressure. What has been said concerning probable errors in measurement can explain only in a very small

degree the large differences in these ratios. The lack of a constant ratio is very evident. For iron the ratio-values run from 0.78 to 15.5, with such a distribution between these limits that any range which might reasonably be assumed as due to poor measurements covers but a fraction of the lines. Thus in Table I ratios ranging from 3.00 to 6.00 take in 84 out of 166 lines, or 52 per cent. The range from 3.00 to 5.00 includes 64 lines, or 38 per cent. The ratios for titanium and chromium in Tables II and III show divergences of the same order. The lack of constancy in the ratio being apparent, the question arises as to whether there is any real connection between separation and displacement. A broad classification of the values in order of magnitude may be of service in this connection. For this purpose the separation and displacement values are classified as small, medium, and large, the range for each class being as given in the explanation of the tables. The ratios showing the comparative magnitudes of separation and displacement for each line are given in the tables in the column preceding the remarks. The following summary of the data, taking the spectra in turn, will show to what extent a general agreement exists between the Zeeman and pressure phenomena.

1. *Iron.*—The ratios of classes from Table I enable us to place the 173 iron lines in three main groups. Group 1 consists of the ratios S:S, M:M, and L:L, and shows that the separation and displacement for the corresponding lines are relatively of the same order. Group 2 contains those lines for which separation and displacement are not in the same, but in adjacent, classes; while for Group 3 the separation and displacement are of very different magnitudes, one small and the other large. The four lines which show no Zeeman effect but distinct pressure displacement are also in Group 3, the letter O being associated with S, M, or L according to the magnitude of the displacement. The number of lines in these groups is given in Table IV.

From Table IV we see that 44 per cent of the iron lines are in good agreement as to order of magnitude, 42 per cent show a probable discordance, while 14 per cent strongly contradict the hypothesis of equality of relative magnitude. This shows clearly that the two phenomena are not very closely related as regards size of one increas-

ing with size of the other. The large number of lines in Group 2 renders any positive conclusion difficult on account of the possible influence of errors of measurement. Trials with other limits for the small, medium, and large classes have shown that the group percentages are not materially altered, as this results in a transfer back and forth of lines near the limits chosen. An attempt to reduce Group 2

TABLE IV

SUMMARY OF CLASSES—IRON

Group	1			2				3					
Ratio of magnitude....	S:S	M:M	L:L	S:M	M:S	M:L	L:M	S:L	L:S	O:S	O:M	O:L
Number of lines.......	24	30	22	26	20	12	15	10	10	1	1	2
Group total...........		76			73					24		
Group percentage......		44			42					14		

was made by taking all those lines which had one or both values so near the limit of the class that the error of measurement, if in the favorable direction, might have put the two values into the same class and so brought the line into Group 1. Lines of complex Zeeman separation were also treated in this way. Thirty-five iron lines were thus selected, which when added to Group 1 as given in Table IV raised its total to 64 per cent of the whole. This number, then, may be in fair agreement as to order of magnitude, while the remaining 36 per cent are divergent beyond the errors of measurement and in some distances widely different. This last device is of course not a fair treatment of the data, since the error of measurement is as likely to move the values wider apart as closer together, and if the same treatment had been applied to the lines of Group 1, some of them would have moved into Group 2. However, giving the agreement hypothesis the benefit of the doubt, the proportions of 64 and 36 per cent appear to be the most favorable that can be gotten out of the list of iron lines.

The lines in Group 3 deserve special notice. Besides twenty lines for which either separation or displacement is small and the other large, we have the four lines λ 3746.058, λ 3767.341, λ 3850.118, λ 5434.740 which show no Zeeman separation, while one has small,

one medium, and two of them large pressure displacements. None of these lines shows any perceptible widening under a field of 20,000 gausses. They appear to be a striking example of ability to respond to one displacing agency and not to the other. There is a bare possibility that these lines may be of complex structure, with a strong central component and very weak side components which have not been observed. This is very improbable, however, as the lines have been obtained on very strong photographs, and all lines which approach this type, having a strong central *n* component and weaker ones at the sides, have their *p* component also divided into two or more parts.

A comparison of averages for large groups of lines is given in Tables V and VI. The method in forming Table V was to make a list of all pressure displacements classified as small, place opposite them the Zeeman separations for the same lines, and take the mean of each list for comparison of the magnitude of the two effects for all lines of small pressure displacement. Means were formed in the same way for lines of medium and of large displacement. The ratios of mean separation to mean displacement can be then compared. In obtaining the results for each class, means were formed for the lines in three groups according to wave-length. The whole table thus gives a comparison of the means for the several groups, and also an indication as to how the means for both separation and displacement change with the wave-length.

Table VI was made in the same way as Table V, except that here the class of Zeeman separation, small, medium, or large, was taken as the basis, and the corresponding pressure displacements used for a comparison of means.

In Table V the ratios of classes for the three magnitudes of displacement are M:S, M:M, and L:L. Table VI gives for the three magnitudes of separation the ratios S:M, M:M, L:L. There is thus good agreement as to magnitudes except for the first class in each table. Two-thirds of the lines for this class come from the region 3600–4000 and there is a sufficient scattering of high values for both separation and displacement to put the means into different classes when formed in this way. The behavior of the ratios of weighted means in the two tables is interesting. Those in Table V

decrease very nearly in the ratio 3:2:1 for the three classes, showing that the displacements increase in size much faster than the separations. This is not shown so well in Table VI, where the same material is used. While the separation means increase nearly as 4:6:9 the displacement means change slowly. It is probable that the change as shown in Table V is a real one and that it is obscured in Table VI

TABLE V

MEANS OF SEPARATION AND DISPLACEMENT CLASSIFIED ACCORDING TO AMOUNT OF DISPLACEMENT

	Range of λ	No. Lines	Means		Ratio Sep. Displ.
			Sep.	Displ.	
Displacement: small	3660–4000	35	0.203	0.046	6.37
	4000–4500	18	0.376	0.051	7.38
	4500–5600	1	0.242	0.060	4.33
Total no. lines and weighted means		54	0.320	0.048	6.68
Displacement: medium	3660–4000	30	0.270	0.084	3.32
	4000–4500	22	0.310	0.080	3.89
	4500–5600	19	0.489	0.085	5.75
Total no. lines and weighted means		71	0.345	0.083	4.16
Displacement: large	3660–4000	8	0.276	0.109	2.54
	4000–4500	24	0.437	0.207	2.11
	4500–5600	9	0.684	0.245	2.80
Total no. lines and weighted means		41	0.460	0.196	2.34

by the large difference in range of values of separations and displacements. The limits of this range are in the ratio of about 1 to 3 for the separations (omitting a few extreme values) and about 1 to 10 for the displacements. Thus, in Table V, when the displacements are grouped so as to increase in magnitude, there is a much smaller variation among corresponding values of separation than we have among the displacement values when the separations are graded as in Table VI. The widely divergent values of displacement scattered through Table VI would thus act to make the ratios of means more or less discordant.

In Tables V and VI the division into regions of wave-lengths shows the distribution of magnitudes in these regions. Following down the column headed "No. Lines" in each table, we see that the region of shortest wave-length gives the largest number of small values for both separation and displacement. For the medium and large values in each table, the proportion of lines increases in the region of

TABLE VI

MEANS OF SEPARATION AND DISPLACEMENT CLASSIFIED ACCORDING TO AMOUNT OF SEPARATION

	Range of λ	No. Lines	Means		Ratio Sep./Displ.
			Sep.	Displ.	
Separation: small	3660–4000	41	0.239	0.073	3.20
	4000–4500	17	0.257	0.099	2.59
	4500–5600	2	0.261	0.080	3.26
Total no. lines and weighted means		60	0.245	0.081	3.02
Separation: medium	3660–4000	30	0.340	0.064	5.31
	4000–4500	25	0.348	0.101	3.44
	4500–5600	6	0.317	0.135	2.35
Total no. lines and weighted means		61	0.346	0.086	4.02
Separation: large	3660–4000	2	0.464	0.041	11.32
	4000–4500	22	0.501	0.156	3.21
	4500–5600	21	0.617	0.138	4.47
Total no. lines and weighted means		45	0.554	0.143	3.88

greater wave-length, this being very decided for the "large" group. Thus there is a clear increase in magnitude of both separation and displacement as the wave-length increases. The lines here compared seem to be representative of the spectrum, as the same relation holds in the complete Zeeman tables, which contain a much larger number of lines for this range of wave-length.

A classification by Duffield[1] may be used in comparing the displacements measured by him with the corresponding Zeeman separations. He forms three main groups according to amount of displace-

[1] *Op. cit.*, p. 160.

ment. Table VII gives the mean separation and displacement for each of these groups, at first singly, then combined so as to form two groups with more lines in each.

TABLE VII

MEANS OF SEPARATION AND DISPLACEMENT FOR DUFFIELD'S DISPLACEMENT GROUPS

Group Number	I (Unreversed)	I (Reversed)	II	III	I (Total)	II and III
No. lines	26	13	6	10	39	16
Mean sep	0.349	0.307	0.529	0.376	0.335	0.433
Mean displ.	0.064	0.077	0.168	0.319	0.068	0.262
Classes sep. and displ.	M:M	M:M	L:L	M:L	M:M	L:L

We see that separation and displacement are of the same order of magnitude except for Group III where some very large displacements correspond to medium separations. The material is better presented in the last two columns, where the larger number of lines give means of higher weight. These means show as before that a much larger range is covered by the displacements than by the separations.

Duffield has found that the displacement of the lines belonging to his three groups have very different rates of increase with increase of pressure, the lines of Group III showing the most rapid change. I have plotted Humphreys' measurements for lines taken at two and three pressures and find for them a great difference in steepness of the curves. There seems to be no general relation for individual lines between the rapidity of increase of displacement with pressure and their behavior as to Zeeman separation. Indeed, the fact that this difference exists for pressure effects is to some extent against a relation of the two phenomena, since a corresponding action in the Zeeman effect would mean a different rate of increase of displacement with field strength for different lines, which is contrary to experiment.

2. *Chromium.*—In the chromium spectrum 54 lines were available for a comparison of separation and displacement. The classes into which the values fall are tabulated in Table VIII in the same manner as in Table IV for iron.

Following the same method as with iron, we find 16 lines in Group 2 for which the magnitudes of separation and displacement are nearly

enough in the same class to come within possible errors of measurement, giving 52 per cent in fair agreement for this spectrum. The agreement of magnitudes for chromium is thus not so good as for iron. The few lines beyond λ 4700 show large values for the displacement, and have also as a rule large separations. A rough classification of magnitudes is given farther on.

TABLE VIII

SUMMARY OF CLASSES—CHROMIUM

Group	1			2				3	
Ratio of magnitude	S:S	M:M	L:L	S:M	M:S	M:L	L:M	S:L	L:S
Number of lines	2	5	5	9	9	2	10	1	11
Group total	12			30				12	
Group percentage	22			56				22	

3. *Titanium.*—A large amount of material is on hand for the Zeeman effect, but as the pressure measurements are scanty we have at present but 31 titanium lines in Table III for comparison. The magnitude groups for titanium are given in Table IX, corresponding to Tables IV and VIII.

TABLE IX

SUMMARY OF CLASSES—TITANIUM

Group	1			2				3		
Ratio of magnitude	S:S	M:M	L:L	S:M	M:S	M:L	L:M	S:L	L:S	O:M
Number of lines	6	2	6	1	5	4	5	0	1	1
Group total	14			15				2		
Group percentage	45			48				7		

The agreement for magnitude classes is much the same as for iron. Six lines of Group 2 may be within the error of measurement, giving 64 per cent in possible agreement as to magnitude. λ 4295.914 is a displaced line which shows no Zeeman separation.

ROUGH CLASSIFICATION FOR CHROMIUM AND TITANIUM

The small number of lines available for chromium and titanium does not justify an extended comparison of means such as is given

for iron in Tables V and VI; but a division according to large and small displacement, each of these classes including some lines regularly classed as medium, may be of some value. Table X is formed on this plan.

TABLE X

CHROMIUM AND TITANIUM LINES CLASSED ACCORDING TO DISPLACEMENT

	Chromium		Titanium	
Displacement range	<0.080	>0.080	<0.080	>0.080
No. lines	42	12	16	14
Mean separation	0.409	0.532	0.319	0.418
Mean displacement	0.061	0.127	0.049	0.113
Ratio of magnitudes	L:M	L:L	M:S	L:L

This table shows a fair agreement as to magnitudes, the mean separation corresponding to small displacement being somewhat large for both elements.

GENERAL REMARKS AND CONCLUSION

It might be expected that the very complex Zeeman separations which occur for many lines listed in Tables I, II, and III would not show as good an agreement with the pressure displacement as those of a simpler type. There appears, however, to be no general rule of this sort. Throughout the spectra many complex lines are found to agree in magnitude with the pressure displacement, while some of the largest discrepancies are found for triplets weighted 3, which is the simplest type of separation with components most sharply defined.

Until more complete pressure measurements are at hand, we cannot say how many lines may show large Zeeman separation and no perceptible displacement. While Humphreys' tables cover most of the strong lines for the region measured, there are some lines of considerable strength missing which are of large separation. It may be that blends and other disturbing features interfere in some cases on the pressure plates.

A much better comparison could be made if measurements for a large range of wave-length for both phenomena were available. It will be shown when the complete Zeeman table is published that the mean separation for all iron lines from λ 3700 to λ 6700 increases

closely with the square of the wave-length. The pressure displacements are found to have higher values in the green than in the blue and violet, but the number of measurements is not sufficient to give an accurate value for the rate of increase. This agreement of Zeeman and pressure effects in respect to increase with wave-length, at present qualitative, may furnish us the best evidence as to the real relation of the two phenomena. The matter presented in this paper, while it shows that for a majority of the lines considered there is a fair agreement as to magnitude of separation and displacement, shows also from the number and character of the lines not in agreement that this line of evidence, even if made very extensive, will scarcely be convincing as to pressure displacement being due to magnetic perturbations. The degree of concordance which we have could perhaps result entirely from the fact that the magnitude of each effect increases with the wave-length. This does not prove a close physical relation, since any theory of the pressure effect that might be offered would probably involve a change with the wave-length. The theory of Richardson requires an increase with the third power of the wave-length, that of Larmor a decrease with increasing wave-length. If, however, the investigation of both magnetic field and pressure effects can be made to cover all but the weakest lines of several many-lined spectra through a large range of wave-length, and if the rate of increase of mean pressure and mean displacement is found to be the same, we shall have established a high degree of probability that the causes of the two phenomena are fundamentally alike.

Mount Wilson Solar Observatory
April 25, 1910

Minor Contributions and Notes

NOTE ON THE PRESSURE-SHIFT OF VIOLET-SIDED SPECTRAL LINES

In that part of their excellent paper in which they discuss the comparison of the solar spectrum with that of iron in the air, Fabry and Buisson[1] say:

> Les raies à élargissement vers le violet ne se trouvent pas parmi celles que l'on a étudiées au point de vue de l'action de la pression; ces raies subissent, pour la variation de pression de 1 atmosphère lorsqu'on passe du vide à la pression atmosphérique, un déplacement apparent vers le violet; il serait très intéressant de savoir comment elles se comportent aux pressions élevées.

The first part of this statement applies to seven of the eight violet-sided lines listed by Fabry and Buisson, but not to the line λ 4250.78. The behavior of this line under pressure has been repeatedly examined, and its increase in wave-length found to be almost exactly 0.002 Ångström unit per atmosphere.[2]

Probably the best examples of the violet-sided unsymmetrical lines that have been examined under pressure are the sodium lines λ 3302.5 and λ 3303.1, both of which spread much more toward the violet than toward the red end of the spectrum. Under pressure, however, their wave-lengths increase approximately 0.007 Ångström unit per atmosphere.[3]

Unsymmetrical broadening, so marked in the case of certain lines with increase of material in the arc, and pressure-shift, appear to be due to different causes. There is, therefore, no a priori reason for thinking that unsymmetrical lines, whether red- or violet-sided, will give pressure-shifts very different from those of symmetrical lines. At any rate, such differences in the shifts of the several types as may appear to exist decrease as the measurements are confined to narrower reversals and finer lines.

[1] *Astrophysical Journal*, **31**, 111–12, 1910.

[2] Humphreys, *Ibid.*, **6**, 200, 1897; **22**, 218, 1905; **26**, 24, 1907; Duffield, *Phil. Trans.*, A, **208**, 136 and 138, 1908.

[3] *Astrophysical Journal*, **6**, 183 and 210, 1897.

Nevertheless, it is of distinct advantage to know which of the hundreds of lines most need to be examined in the laboratory; and it is now expected that the behavior under pressure of many additional lines will soon be determined—the selection being made in part to meet the requirements of recent papers by Evershed,[1] Adams,[2] Fabry and Buisson,[3] and others.

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ADDITIONAL NOTES ON RADIAL VELOCITIES WITH OBJECTIVE-PRISM

Various salts of neodymium have been tried, but the chloride appears to be the best. The 4273 line in the nitrate has a broad, hazy extension in the side of longer wave-length, which makes it unsuitable for measurements. The sulphate shows the band as narrow as the chloride, but it is not nearly as soluble, and the cell must be 3 cm thick for a saturated solution.

I have photographed the 4273 line with the 21-foot concave grating, with an iron comparison spectrum. The center of the line can be determined easily to within 0.1 Ångström unit. Its width is a little less than 3 Å.U. The photograph is reproduced on Plate XII, Fig. 1. To determine whether any change or shift was produced by a change of temperature, two spectra were made with the 21-foot grating, one with the solution partly frozen, the other at a temperature of about 35° C. No difference in the position of the band could be detected, though it appeared to be a trifle narrower at the lower temperature, probably about 10° below the freezing point of water. Its wave-length referred to the iron lines 4271.30 and 4271.90 (Kayser and Runge) is 4272.90.

Through the courtesy of Professor E. C. Pickering, the facilities of the Harvard Observatory were placed at my disposal, and, aided by Mr. King, I made some experiments with other absorbing media, which previous examination had shown to be promising.

[1] *Memoirs Kodaikanal Observatory*, **1**, 1, 1909.

[2] *Astrophysical Journal*, **31**, 30, 1910.

[3] *Ibid.*, **31**, 97, 1910.

PLATE XII

1
2
3
4

Fe
C

1. Neodymium chloride with *Fe* comparison.
2. *a Leonis* with and without filter of peroxide of chlorine
3. Peroxide of chlorine with iron comparison
4. Peroxide of chlorine with carbon arc

The first substance tried was manganese perchloride, a yellowish gas, prepared by adding a few fragments of fused chloride of sodium to a solution of permanganate of potash in concentrated sulphuric acid, and warming gently. Care should be used in preparing this gas, as the reaction is sometimes explosive. It shows some narrow bands in the green and greenish yellow, when used with an isochromatic plate. They are not much narrower than the λ 4273 line of neodymium, as subsequently observed with the 21-foot grating, though they appeared much narrower in the star photographs, owing to the smaller dispersion of the prism in this region. I doubt if much is to be gained by the use of this substance.

Peroxide of chlorine was next tried, but it attacked the sealing-wax with which the glass plates of the cell were cemented, rapidly disappearing, and we obtained only one or two rather unsatisfactory photographs.

This substance I have since examined with the 21-foot grating, and find it to be admirably adapted to the purpose. The bands are as sharp on one edge as the iron lines, but shade off toward the ultra-violet. The position of the edge of the shaded bands does not vary with the density of the gas, and their positions can be determined to within 0.02 Å.U. on the photographs made with the grating. Unfortunately the absorption bands cover most of the hydrogen lines, so that it cannot be used very well for stars of the first type. It is possible, however, that the lines λ 4863 and λ 3837 will appear. For stars of other types the peroxide of chlorine seems to be all that can be desired. It is easily prepared by adding a few crystals of chlorate of potash to a little concentrated sulphuric acid contained in a test tube, and warming. The cell should be made of a glass ring about 1 cm thick and two circular glass plates, fastened together with some non-organic cement. Probably a mixture of water glass and powdered asbestos will answer the purpose, though some cement which never becomes quite hard would be preferable, as the cell could then be easily taken apart and cleaned.

Photographs made with the 21-foot grating are reproduced on Plate XII, Figs. 3 and 4.

The heads of the bands have been determined to within 0.02 Å.U. They are as follows: $\lambda\lambda$ 4273.20, 4324.38, 4402.93, 4458.15, 4199.42,

4062.23, 3880 (only roughly determined as yet). I see no reason why velocities cannot be determined to within 2 or 3 kilometers by means of these bands, provided of course that the rest of the apparatus is brought to such a degree of perfection as to enable them to be used to their full advantage.

To do this, with long exposures, it will of course be necessary to keep the prism at a constant temperature; atmospheric disturbances will probably put the final limit to the accuracy of the determinations.

A photograph of *a Leonis* made by Mr. King with and without the screen of peroxide of chlorine is reproduced in Fig. 2. The gas was not sufficiently diluted, however, and the extreme violet and ultra-violet were absorbed.

R. W. Wood

Johns Hopkins University
April 22, 1910

SIR WILLIAM HUGGINS

It is with the deepest regret that we record the death, on May 13, of our eminent collaborator Sir William Huggins, whose pioneer discoveries in astrophysics have made his name immortal in the annals of science.

Sir William had on February 7 completed his eighty-sixth year, in the fullest possession of his remarkable mental faculties, and with the liveliest interest in every step of progress in the branch of science which he had so largely helped to found.

His skill as an observer was equaled by his philosophical judgment in the interpretation of the results of observations, and his simple dignity was the crowning evidence of his greatness.

In offering our sincerest sympathy to his bereaved consort and coadjutor, we can remotely sense the magnitude of her loss, after many years of the closest association; for we measure it in terms of the real attachment to him resulting from his generous friendship to younger men, even when personal meetings were infrequent and communication was chiefly by letters.

A photogravure from Sir John Collier's fine portrait of Sir William Huggins was published in this *Journal* in September 1907 (Vol. **26**, facing p. 128). An appropriate account of his life and work will be published in a subsequent number of this *Journal*.

NOTICE

The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention is given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right unless the author requests that the reverse procedure be followed.

Accuracy in the proof is gained by having manuscripts type-written, provided the author carefully examines the sheets and eliminates any errors introduced by the stenographer. It is suggested that the author should retain a carbon or tissue copy of the manuscript, as it is generally necessary to keep the original manuscript at the editorial office until the article is printed.

All drawings should be carefully made with India ink on stiff paper, usually each on a separate sheet, on about double the scale of the engraving desired. Lettering of diagrams will be done in type around the margins of the cut where feasible. Otherwise printed letters should be put in lightly with pencil, to be later impressed with type at the editorial office, or should be pasted on the drawing where required.

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INDEX TO VOLUME XXXI

SUBJECTS

INDEX TO VOLUME XXXI

AUTHORS

The Astrophysical journal